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1000
Solved Problems in Fluid Mechanics

Includes Hydraulic Machines

K Subramanya

مادة محمية بموجب

1000
Solved Problems in Fluid Mechanics
(Includes Hydraulic Machines)

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Contents

Preface *xi*

Chapter 1
Properties of Fluids **1**

1.1 Introduction *1*
1.2 Density and Specific Weight *1*
1.3 Pressure *2*
1.4 Shear Stress and Viscosity *2*
1.5 Surface Tension *3*
1.6 Compressibility *4*
1.7 Vapour Pressure *5*
Worked Examples *5*
Objective Questions *12*
Problems *15*

Chapter 2
Fluid Statics **17**

2.1 Measurement of Pressure *17*
2.2 Forces on Plane Surfaces *19*
2.3 Forces on Curved Surfaces *20*
2.4 Buoyancy *20*
2.5 Rigid Body Motion *21*
Worked Examples *22*
Objective Questions *53*
Problems *59*

Chapter 3
Fluid Flow Kinematics **69**

3.1 Introduction *69*
3.2 Classification of Flow *69*
3.3 Streamline *69*
3.4 Acceleration *69*
3.5 Continuity Equation *70*
3.6 Rotational and Irrotational Motion *71*

3.7 Stream Function *71*
3.8 Potential Function *72*
3.9 Relation Between ψ and ϕ for 2-Dimensional Flow *72*
3.10 Some Common Formulae in Cylindrical Co-ordinates *72*
3.11 Elementary Inviscid Plane Flows *73*
Worked Examples 74
Objective Questions 85
Problems 89

Chapter 4
Energy Equation and Its Applications **93**

4.1 Bernoulli Equation *93*
4.2 Practical Applications of Bernoulli Equation *93*
4.3 Energy Equation *94*
4.4 Power *95*
Worked Examples 95
Objective Questions 109
Problems 112

Chapter 5
Momentum Equation and Its Applications **117**

5.1 Linear Momentum Equation *117*
5.2 The Moment of Momentum Equation *119*
Worked Examples 121
Objective Questions 132
Problems 135

Chapter 6
Dimensional Analysis and Similitude **139**

6.1 Introduction *139*
6.2 Common Variables in Fluid Flow *139*
6.3 Dimensional Homogeneity *139*
6.4 Dimensional Analysis *140*
6.5 Similitude *140*
6.6 Important Dimensionless Flow Parameters *141*
6.7 Model Scales *141*
6.8 Distorted Models *142*
Worked Examples 142
Objective Questions 152
Problems 153

Chapter 7
Laminar Flow **158**

7.1 Introduction *158*
7.2 Basic Equations *158*
7.3 Viscometers *161*
Worked Examples 161
Objective Questions 169
Problems 172

Chapter 8

Boundary Layer Concepts 175

8.1 Introduction *175*
8.2 Boundary Conditions *176*
8.3 Laminar Boundary Layer Over a Flat Plate *176*
8.4 Karman Momentum Integral Formulation *176*
8.5 Boundary Conditions for a Proper $\phi(\eta)$ *177*
8.6 Transition from Laminar Boundary Layer *177*
8.7 Turbulent Boundary Layer *177*
8.8 Laminar Sublayer *178*
8.9 Establishment of Flow in a Pipe *178*
8.10 Boundary Layer Separation *179*
Worked Examples 180
Objective Questions 188
Problems 191

Chapter 9
Drag and Lift on Immersed Bodies 194

9.1 Introduction *194*
9.2 Drag *194*
9.3 Lift *196*
Worked Examples 198
Objective Questions 205
Problems 207

Chapter 10
Turbulent Pipe Flow 210

10.1 Turbulent Flow *210*
10.2 Pipe Flow *210*
10.3 Commercial Pipes *212*
10.4 Aging of Pipes *213*
10.5 Simple Pipeline Design Problems *213*
10.6 Velocity Distribution in the Neighbourhood of Flat Surfaces *215*
Reference 215
Worked Examples 216
Objective Questions 225
Problems 228

Chapter 11
Pipe Flow Systems 232

11.1 Introduction *232*
11.2 Minor Losses *232*
11.3 Simple Pipe Problems *233*
11.4 Pipe Network *237*
11.5 Miscellaneous Problems *238*
Worked Examples 238
Objective Questions 263
Problems 265

Chapter 12
Flow in Open Channels 271

12.1 Open Channel Flow *271*
12.2 Classification *271*
12.3 Uniform Flow *271*
12.4 Rapidly Varied Flow *274*
12.5 Gradually Varied Flow *275*
12.6 Flow Measurement 276
Reference 277
Worked Examples 277
Objective Questions 300
Problems 304

Chapter 13
Flow Measurement 314

13.1 Introduction *314*
13.2 Orifices *314*
13.3 Orifice Meter *315*
13.4 Venturimeter *316*
13.5 Pitot Tube *317*
13.6 Weirs *318*
13.7 Rotameter *320*
Worked Examples 320
Objective Questions 338
Problems 342

Chapter 14
Unsteady Flow 349

14.1 Introduction *349*
14.2 Surges in Open Channels *349*
14.3 Water Hammer *350*
14.4 Establishment of Flow *352*
Worked Examples 353
Objective Questions 364
Problems 366

Chapter 15
Compressible Flow 370

15.1 Introduction *370*
15.2 Thermodynamic Principles *370*
15.3 Basic Definitions *371*
15.4 Basic Equations for Compressible Fluid Flow *372*
15.5 Application of Energy Equation *372*
15.6 Sonic Velocity *372*
15.7 Flow in a Nozzle Discharging from a Tank *373*
15.8 Normal Shock Wave *375*
References 376
Worked Examples 376

Objective Questions 386
Problems 388

Chapter 16
Fluid Flow Machines 391

16.1 Fluid Flow Machines *391*
16.2 Turbines *391*
16.3 Rotodynamic Pumps *395*
16.4 Reciprocating Pump *399*
Worked Examples 404
Objective Questions 426
Problems 429

Appendices 436

Appendix A Multiple-Choice Objective Questions *436*
Appendix B-1 Answers to Objective Questions *450*
Appendix B-2 Answers to Multiple-Choice Objective Questions of Appendix A *452*

Index 453

Preface

Fluid mechanics is an important constituent of the undergraduate syllabi of many engineering disciplines. The subject is given considerable importance in Civil, Mechanical and Chemical engineering programmes at the core as well as at professional levels. While problem-solving is an important aspect of learning fluid mechanics, a typical textbook in this subject does not provide adequate space for this aspect due to various constraints. Therefore, the students who are required to solve problems in various tests and examinations, find the subject of fluid mechanics difficult to cope with in their studies.

The book, primarily based on my earlier book, *Theory and Applications in Fluid Mechanics* (TMH, 1993) is designed as an essential supplement to any textbook in fluid mechanics with the needs of undergraduate engineering students in mind. The typical undergraduate syllabus in Fluid Mechanics is covered in sixteen chapters. In each of the chapters, an outline of the basic theory is given and is followed by a large number of carefully chosen worked examples covering all the sub-areas of the chapter theme. A set of objective questions, with answers provided at the end of the book, will help the students in quick review of the chapter and also in thorough understanding of the concepts. Further, a set of tested problems with answers is provided in each chapter to help the students in their practice. The contents of each chapter are designed to help the students in learning all aspects of the subject matter, viz. theory, application, information manipulation and processing. The set of multiple-choice objective questions provided at the end of the book in Appendix A cuts across various chapters and will be specially be useful to those preparing for competitive examinations. Every attempt has been made to make the book user-friendly.

SI units are used throughout the book and a logical sequence of topics is followed. In sixteen chapters, the book covers the syllabus of Fluid Mechanics and Hydraulic Machinery for the undergraduate programmes in Civil, Mechanical and Chemical engineering disciplines. The topics covered include Fluid Properties; Fluid Statics; Kinematics and Inviscid Flows; Energy and Momentum Equations and Their Applications; Dimensional Analysis and Similitude; Laminar and Turbulent Flows; Boundary Layer Concepts; Drag and Lift on Submerged Bodies; Pipe Flow Systems; Open Channel Flows; Flow Measurements; Unsteady Flows; Compressible Flows; and Hydraulic Machines comprising Turbines, Rotodynamic and Reciprocating Pumps. With this coverage, the book will be extremely useful to the engineering and AMIE students. Further, students preparing for competitive examinations such as UPSC examinations and GATE will find the book of invaluable help. Teachers will find the book beneficial as source material for designing question papers for various examinations.

Suggestions for improvement of the book would be gratefully appreciated.

K Subramanya

1

Properties of Fluids

CONCEPT REVIEW

1.1 INTRODUCTION

Fluid mechanics is the science of the mechanics of fluids. The terms fluid is used to denote matter in liquid or gaseous phase. As opposed to a solid, a fluid will undergo continuous deformation when subjected to shearing forces and the deformation will continue so long as the shearing force continues to act on the fluid body.

1.1.1 Continuum

In engineering problems dealing with fluids, one generally deals with dimensions that are very large compared to molecular sizes. The space between the molecules is not considered and the fluid properties are considered to vary continuously in space. The density of fluid is thus a point function. This method of considering fluid as a continuous mass is called as *continuum principle*. Except in dealing with rarified gases, all normal fluid mechanics analysis deals with fluid as a continuum.

1.1.2 Units

In fluid mechanics four fundamental dimensions namely mass, length, time and temperature are involved. These days the SI units are adopted in describing the various parameters of fluid flow. In this system the fundamental units are:

Mass [M]	kilogram	kg
Length [L]	metre	m
Time [T]	second	s
Temperature [θ]	Kelvin	K
	(for thermodynamic calculations) or °Celsius	°C

Based upon these fundamental units a number of derived units are developed. The most commonly used derived units is the unit of force which is *newton* (N). A *newton* of force corresponds to an acceleration of 1 m/s^2 of a mass of 1 kg. The commonly used derived terms and their units are listed in Table 1.1.

Table 1.1 *Commonly Used Derived Terms in Fluid Mechanics*

Derived term	*Dimension*	*SI unit*	*Abbreviation*
Area	(L^2)	m^2	
Volume	(L^3)	m^3	
Velocity	(LT^{-1})	m/s	
Acceleration	(LT^{-2})	m/s^2	
Force	(MLT^{-2})	N	
Pressure (or stress)	($ML^{-1}T^{-2}$)	N/m^2	Pascal Pa = N/m^2
Energy (or work)	(ML^2T^{-2})	N.m	Joule J = N.M.
Power	(ML^2T^{-3})	J/s	Watt W = J/s
Density	(ML^{-3})	kg/m^3	
Viscosity	($ML^{-1}T^{-1}$)	kg/m.s (= N.s/m^2)	Pa.s
Surface tension	(MT^{-2})	N/m	

1.2 DENSITY AND SPECIFIC WEIGHT

The *density* ρ of a fluid is its mass per unit volume. The units are kg/m^3. In general the density of a fluid depends upon temperature and pressure. For incompressible fluids (liquids) the variation of density with pressure is however small.

The *specific weight* γ of a fluid is its weight per unit volume. Thus,

$$\boxed{\gamma = \rho g \text{ (in units of N/m}^3\text{)}}$$

The standard value of acceleration due to gravity is 9.806 m/s^2 and is usually taken as 9.81 m/s^2. At standard temperature (20°C) and pressure (1 atmospheric pressure or 760 mm of mercury), the density of water is 998 kg/m^3. Thus, the specific weight of water (at 20°C and 1 atmospheric pressure) is $\gamma = 998 \times 9.81 = 9790$ N/m^3 = 9.79 kN/m^3.

Relative Density (RD) of a fluid is the ratio of its density to that of pure water at standard conditions. Hence it is dimensionless. Thus,

$$\text{RD} = \frac{\text{Density of substance}}{\text{Density of pure water at standard temperature and pressure}}$$

$$= \frac{\text{Density of the substance (kg/m}^3\text{)}}{998\,(\text{kg/m}^3)} \text{ at } 20°\text{C}$$

The term *specific gravity* used in CGS and FPS units is the same as relative density.

If the relative density of a liquid is 0.850, it means that its density is 0.850 × 998 = 848.3 kg/m^3 at 20°C.

Commonly used values of approximate relative densities in fluid flow calculations are 1.0 and 13.6 for water and mercury respectively. When no other information is available, the following standard values (corresponding to 20°C and standard atmospheric pressure) are used:

Item	*water*	*Air*
Density ρ	998 kg/m^3	1.205 kg/m^3
Relative density (RD)	1.00	1.204×10^{-3}
Specific weight γ	9790 N/m^3 (= 9.79 kN/m^3)	11.82 N/m^3

(Unless otherwise stated the above standard values for ρ and γ will be used in this book).

[***Note:*** For approximate/quick calculations for water, $\rho = 1000$ kg/m^3 and $\gamma = 9.8$ kN/m^3 can be used].

1.3 PRESSURE

Pressure is the compressive stress on the fluid and is given by

$$\text{Pressure } p = \frac{\text{Force } F}{\text{Area } A} \text{ for uniform pressure}$$

$$= \frac{dF}{dA} \text{ for variable pressure}$$

The units are N/m^2 = Pa.

Other commonly used units are

kPa = kilo pascals = 1000 N/m^2

bar = 100 kPa = 10^5 N/m^2

Sometimes the pressure is expressed in terms of the height h of an equivalent column of fluid of density ρ. Thus,

$$\boxed{p = \rho g h = \gamma h} \tag{1.1}$$

$$\text{and } h \text{ (meteres of fluid)} = \frac{p}{\rho g} = \frac{p}{\gamma}\frac{(\text{N/m}^2)}{(\text{N/m}^3)}$$

In such cases h is called the pressure head.

For example: (i) A pressure head of 5 m of water is equivalent to a pressure of

$$5.0 \times 9790 = 48950 \text{ Pa} = 48.95 \text{ kPa}$$

(ii) A pressure head of 3 cm of mercury = $h\gamma$

$$= \frac{3}{100} \times (13.6 \times 9790) \text{ Pa}$$
$$= 3994 \text{ Pa} = 3.994 \text{ kPa}$$

1.4 SHEAR STRESS AND VISCOSITY

While the pressure, a normal stress, is encountered in both fluid static and dynamic situations, the shear stress (τ) is encountered in the motion of all real fluids. The unit of shear stress is N/m^2 and is designated in Pa or kPa depending on the magnitude.

1.4.1 Viscosity

Viscosity is the resisting property of a fluid to shearing force. The shear stress τ is related to the deformation rate by the *Newton's law of viscosity*, as

$$\boxed{\tau = \mu \frac{du}{dy}} \tag{1.2}$$

where $\frac{du}{dy}$ = velocity gradient in the y direction and

μ = coefficient of viscosity.

The fluids which obey Newton's law of viscosity are known as *Newtonian fluids.*

The coefficient of viscosity μ (also known variously as coefficient of dynamic viscosity, absolute viscosity or simply viscosity) has the units

$$\mu = \frac{\tau}{(du/dy)} = \frac{\text{N/m}^2}{\left(\frac{\text{m}}{\text{s}}\cdot\frac{1}{\text{m}}\right)} = \text{N.s/m}^2 = \text{Pa.s}$$

The ratio of the coefficient of dynamic viscosity to the density of the fluid is designated by Greek letter (nu) ν. Hence,

$$\nu = \frac{\mu}{\rho} = \frac{\text{N.s/m}^2}{\text{kg/m}^3} = \frac{\text{kg.m}^{-1}\,\text{s}^{-1}}{\text{kg. m}^{-3}} = \text{m}^2/\text{s}$$

This term ν has the dimensions of $[L^2/T]$ and is known as *kinematic viscosity* of the fluid.

Sometimes the coefficient of dynamic viscosity μ is designated by *poise* (abbreviated as P) or *centipoise* (abbreviated as CP) where

$$1 \text{ poise} = 1\ \frac{\text{gm}}{\text{cm.s}} = \frac{1\,\text{dyne. second}}{\text{cm}^2}$$

$$= \frac{10^{-5}}{(10^{-2})^2}\ \frac{\text{N.s}}{\text{m}^2} = \frac{1}{10}\ \text{Pa.s}$$

$$1 \text{ centipoise} = \frac{1}{100}\ \text{poise} = \frac{1}{1000}\ \text{Pa.s}$$

The kinematic viscosity ν is sometimes expressed in *stoke* or *centi stoke* where

$$1 \text{ stoke} = 1\ \frac{\text{cm}^2}{\text{s}} = 1\ (10^{-2})^2\ \frac{\text{m}^2}{\text{s}} = 10^{-4}\ \text{m}^2/\text{s}$$

$$1 \text{ centistoke} = \frac{1}{100}\ \text{stoke} = 10^{-6}\ \text{m}^2/\text{s}$$

The coefficient of viscosity μ depends upon the temperature. Generally, for liquids the value of μ decreases with an increase in the temperature, and for gases the value of μ increases with an increase in the temperature.

Table 1.2 gives the viscosities of some commonly used fluids at 20°C and standard presssure.

1.5 SURFACE TENSION

A liquid forms an interface with a second liquid or gas. The surface energy per unit area of interface is known as *surface tension* or *coefficient of surface tension* σ. The most common interfaces and values of σ, for clean surface at 20°C, are:

$\sigma = 0.073$ N/m for air-water interface.

and $\sigma = 0.480$ N/m for air-mercury interface.

Note that the surface tension σ has the dimension of force/unit length (N/m).

When a liquid interface interacts with a solid surface, a contact angle θ is formed. For water-clean glass surface $\theta \approx 0°$ and for mercury-clean glass $\theta \approx 130°$.

Due to surface tension, pressure changes occur across a curved interface. The pressure difference between inside and outside of a curved surface Δp is related to the radius of curvature R and surface tension σ as:

Table 1.2 *Properties of Some Common Fluids at 20°C and 1 atm Pressure*

Fluids	*Density* $\rho(kg/m^3)$	*Dynamic viscosity* $\mu(Ns/m^2)$	*Kinematic viscosity* $\nu(m^2/s)$	*Surface tension* $\sigma(N/m)$	*Bulk modulus* $K\ (N/m^2)$
(a) Liquids					
Water	998	1.00×10^{-3}	1.00×10^{-6}	7.28×10^{-2}	2.19×10^{9}
Sea water	1025	1.07×10^{-3}	1.04×10^{-6}	7.28×10^{-2}	2.28×10^{9}
Petrol	680	2.92×10^{-4}	4.29×10^{-7}	2.16×10^{-2}	9.58×10^{8}
Kerosene	804	1.92×10^{-3}	2.39×10^{-4}	2.80×10^{-2}	1.43×10^{9}
Glycerine	1260	1.49	1.18×10^{-3}	6.33×10^{-2}	4.34×10^{9}
Mercury	13550	1.56×10^{-3}	1.15×10^{-7}	4.84×10^{-1}	2.55×10^{10}
SAE 10 oil	917	1.04×10^{-1}	1.13×10^{-4}	3.60×10^{-2}	1.31×10^{9}
SAE 30 oil	917	2.90×10^{-1}	3.16×10^{-4}	3.50×10^{-2}	1.38×10^{9}
Castor oil	960	9.80×10^{-1}	1.02×10^{-3}	3.92×10^{-2}	1.44×10^{9}
	$\rho\,(\text{kg/m}^3)$	$\mu\,(\text{Ns/m}^2)$	$\nu\,(\text{m}^2/\text{s})$		Specific heat ratio, $k = C_p/C_v$
(b) Gases					
Air	1.205	1.80×10^{-5}	1.494×10^{-5}		1.40
Carbon dioxide	1.840	1.48×10^{-5}	0.804×10^{-5}		1.28
Hydrogen	0.084	0.90×10^{-5}	10.714×10^{-5}		1.40
Nitrogen	1.160	1.76×10^{-5}	1.517×10^{-5}		1.40
Methane	0.668	1.34×10^{-5}	2.000×10^{-5}		1.30
Oxygen	1.330	2.00×10^{-5}	1.504×10^{-5}		1.40
Water vapour	0.747	1.01×10^{-5}	1.352×10^{-5}		1.33

(1) For the interior of a liquid cylinder

$$\boxed{\Delta p = \frac{\sigma}{R}} \tag{1.3}$$

(2) For a spherical droplet

$$\boxed{\Delta p = \frac{2\sigma}{R}} \tag{1.4}$$

(3) A soap bubble has two surfaces and the pressure difference is given by

$$\boxed{\Delta p = \frac{4\sigma}{R}} \tag{1.5}$$

Thus the pressure inside a droplet or a soap bubble will be higher than the surrounding atmosphere. The pressure inside will be higher, the smaller the size of the droplet or bubble.

1.5.1 Capillarity

Liquids have both cohesion and adhesion, which are forms of molecular attraction. Capillarity, the rise (or fall) of liquid in small-diameter tubes is due to this attraction. Liquids, such as water, which wet a surface cause capillary rise. In nonwetting liquids (e.g. mercury) capillary depression is caused.

For a cylindrical glass tube the capillary rise (or depression) h (Fig. 1.1) is given by

$$\boxed{h = \frac{2\sigma \cos\theta}{\gamma R}} \tag{1.6}$$

where θ = contact angle,
γ = unit weight of the liquid (= ρg),
R = radius of curvature of the glass tube
σ = coefficient of surface tension.

(***Note:*** Capillary rise is usually measured to the bottom of the meniscus)

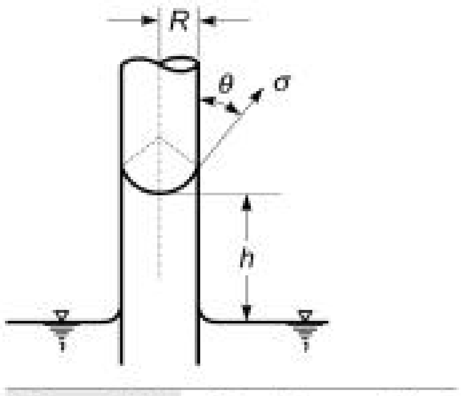


Fig. 1.1 *Capillary Rise*

For clean glass and water θ can be assumed to be zero. For clean mercury–air–glass interface, θ = 130°.

1.6 COMPRESSIBILITY

Gasses are highly compressible and their relaionship between pressure, temperature and volume is expressed by perfect gas equation

$$\boxed{\frac{p}{\rho} = pv = RT} \tag{1.7}$$

where p = absolute pressure
ρ = mass density
v = specific volume = $1/\rho$
T = absolute temperature in Kelvin
= (273 + °C)
R = characteristic gas constant = $\left(\frac{\text{m}^2}{\text{s}^2\text{K}}\right)$

For any gas $$R = \frac{8312}{M} \tag{1.8}$$

where M = molecular weight of gas.
For air M = 28.97, giving R_{air} = 287 m^2/(s^{2}K).
Another fundamental equation of a perfect gas between two state points is

$$\boxed{\frac{p_1}{\rho_1^n} = \frac{p_2}{\rho_2^n}} \tag{1.9}$$

where p is absolute pressure.

(i) If the process is *isothermal* (i.e. at constant temperature),

$$n = 1.0$$

(ii) If the process is *adiabatic* (i.e. without heat transfer) and without friction (*isentropic*)

$$n = k$$

where $k = C_p/C_v$ = ratio of specific heat at constant pressure (C_p) and that at constant volume (C_v). For air and diatomic gases $k \approx 1.4$. Values of k for various gases are given in Table 1.2 (b).

Combining Eq. 1.7 and Eq. 1.9

$$\boxed{\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}} \tag{1.10}$$

1.6.1 Bulk Modulus of Elasticity

The compressibility of a fluid is expressed by the *bulk modulus of elasticity K* as

$$\boxed{K = \frac{\mathrm{d}p}{(-\mathrm{d}\forall/\forall)}} \tag{1.11}$$

where dp = change in pressure
and d$\forall$ = change in the volume $\forall$.

Since $\left(-\frac{d\forall}{\forall}\right)$ is dimensionless, the dimension of K is the same as that of p, viz. N/m^2 = (Pa).

For a perfect gas

(i) $K = p$ for isothermal process, and
(ii) $K = kp$ for isentropic process.

By definition, the compressibility of a fluid is *inversely* proportional to its value of K.

1.6.2 Velocity of Propagation of Sound (C)

Sound is propagated in a fluid due to compressibility of the medium, and the speed of sound C is given by

$$\boxed{C = \sqrt{K/\rho}} \qquad (1.12)$$

where K = bulk modulus of elasticity of the medium and ρ = mass density of the fluid.

1.7 VAPOUR PRESSURE

Vapour Pressure is the pressure at which a liquid boils and is in equilibrium with its own vapor. In many liquid flow situations, low pressures approaching vapor pressure of the liquid may occur. When this happens, the liquid flashes into vapour, forming rapidly expanding cavity. This phenomenon, known as *Cavitation,* has serious implications on the operating performance of hydraulic machines and passages of high-speed flows, (see Chapter 16, Sec. 16.3.6 for further details).

Vapour Pressure of a liquid depends upon temperature and increases with it. At 20°C, water has a vapour pressure (P_v) of 2,34 kPa (i.e. vapour pressure head (P_v/γ) = 0.24 m).

WORKED EXAMPLES

1.1 A mass of liquid weighs 500 N when exposed to standard earth's gravity $g = 9.806$ m/s^2.

(a) What is its mass?
(b) What will be its weight in a planet with acceleration due to gravity of (i) 3.5 m/s^2 and (ii) 18.0 m/s^2?

Solution: Let W = weight of the liquid and m = its mass.

(a) $W = mg$

$500 = m \times 9.806$

$$m = \frac{500}{9.806} = \mathbf{50.99\ kg}$$

(b) The mass of the fluid remains constant regardless of its location. Hence m = 50.99 kg at all locations.

(i) If $g_1 = 3.5$ m/s^2

$W_1 = mg_1 = 50.99 \times 3.5 = \mathbf{178.46\ N}$

(ii) If $g_2 = 18.0$ m/s^2

$W_2 = mg_2 = 50.99 \times 18.0 = \boxed{917.8\ \text{N}}$

1.2 A body has a weight of 400 N at standard earth's gravity of 9.806 m/s^2. When a force of 800 N is applied to the body, what is the value of the acceleration (i) on earth and (ii) on moon where the gravity is 1.6 m/s^2?

Solution: The mass of the body,

$$m = \frac{\text{weight}}{\text{gravity}} = \frac{400}{9.806} = 40.791 \text{ kg}$$

This mass is constant and does not change with location. Hence, when a force F is applied, by Newton's second law,

$$F = ma$$

or acceleration $\quad a = F/m$

which is independent of g. Hence, both on the earth, as well as on moon,

$$a = \frac{800}{40.791} = \mathbf{19.612\ m/s^2}$$

1.3 A liquid at 20°C has a relative density of 0.80 and a kinematic viscosity of 2.3 centistoke. Determine its (i) unit weight and (ii) dynamic viscosity in Pa.s.

Solution:

(i) Unit weight $\gamma = \rho g = (\rho_{\text{water}} \times \text{RD}) \times g$

Taking ρ_{water} at 20° as 998 kg/m^3,

$\gamma = 998 \times 0.8 \times 9.81$

$= 7832.3$ N/m^3 = $\mathbf{7.832\ kN/m^3}$

(ii) Dynamic viscosity $\mu = \nu\rho$

$\nu = 2.3$ centistoke $= 2.3 \times 10^{-6}$ m^2/s

$\rho = 998 \times 0.8 = 798.4$ kg/m^3

$\mu = 2.3 \times 10^{-6} \times 798.4 = \mathbf{1.836 \times 10^{-3}\ Pa.s.}$

1.4 The space between two parallel plates kept 3 mm apart is filled with an oil of dynamic viscosity 0.2 Pa.s. What is the shear stress on the lower fixed plate, if the upper one is moved with a velocity of 1.50 m/s? (Refer Fig 1.2).

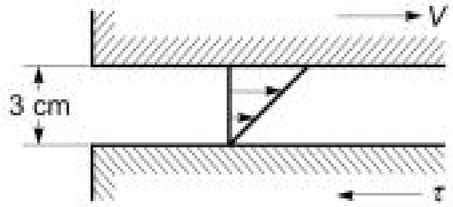


Fig. 1.2

Solution: Since the gap between the plates is very small, a linear variation of velocity can be assumed.

$$\frac{\mathrm{d}u}{\mathrm{d}y} = \frac{V}{h} = \frac{1.50}{3\times10^{-3}} = 500(\mathrm{s}^{-1})$$

τ = Shear stress on the bottom plate

$$= \mu\,\frac{\mathrm{d}u}{\mathrm{d}y} = 0.2 \times 500 = 100\ \mathrm{N/m^2}$$

1.5 The velocity distribution in a viscous flow over a plate is given by

$$u = 4y - y^2 \text{ for } y \le 2 \text{ m}$$

where u = velocity in m/s at a point distant y from the plate. If the coefficient of dynamic viscosity is 1.5 Pa.s determine the shear stress at $y = 0$ and at $y = 2.0$ m.

Solution: Given

$$u = 4y - y^2$$

Therefore $\dfrac{\mathrm{d}u}{\mathrm{d}y} = 4 - 2y$

Shear stress $\tau = \dfrac{\mathrm{d}u}{\mathrm{d}y} = \mu\,(4 - 2y)$

At $y = 0$, $\tau_0 = 4\mu = 4 \times 1.5 =$ **6.0 Pa.s**
At $y = 2.0$ m, $\tau_2 = \mu(4 - 4) =$ **0**

1.6 The velocity distribution near the solid wall at a section in a laminar flow is given by

$$u = 5.0 \sin(5\pi y)$$

for $y \le 0.10$ m. Compute the shear stress at a section at (i) $y = 0$, (ii) $y = 0.05$ m and (iii) $y = 0.10$ m. The dynamic viscosity of the fluid is 5 poise.

Solution: Given,

$$\mu = 5 \text{ poise} = \frac{5}{10}\ \text{Pa.s}$$

Since $u = 5.0 \sin(5\pi y)$

$$\frac{\mathrm{d}u}{\mathrm{d}y} = 5.0 \times 5\pi \cos(5\pi y)$$

Shear stress $\tau = \mu\,\dfrac{\mathrm{d}u}{\mathrm{d}y} = \dfrac{5}{10} \times 25\pi \cos(5\pi y) = 12.5\pi \cos(5\pi y)$

(i) At $y = 0$, $\tau = 12.5\pi \cos(0) = 12.5\pi =$ **39.27 N/m²**

(ii) At $y = 0.05$ m, $\tau = 12.5\pi \cos(5\pi \times 0.05)$
$= 12.5\pi \times 0.707 =$ **27.76 N/m²**

(iii) At $y = 0.10$ m, $\tau = 12.5\pi \cos(5\pi \times 0.1) =$ **0**

1.7 A 90 N rectangular solid block slides down a 30° inclined plane. The plane is lubricated by a 3 mm thick film of oil of relative density 0.90 and viscosity 8.0 poise. If the contact area is 0.3 m², estimate the terminal velocity of the block (Refer. Fig. 1.3).

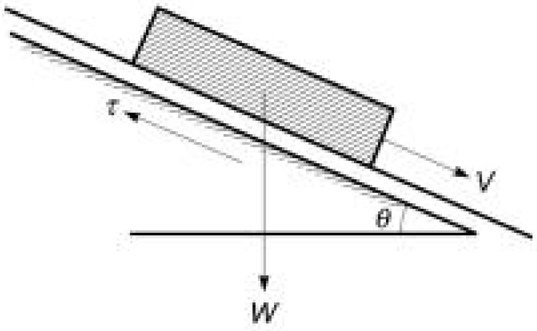


Fig. 1.3

Solution: Given

$W = 90$ N
V = terminal velocity
$\theta = 30°$

At the terminal velocity, the sum of the forces acting on the block in the direction of its motion is zero. Hence

$$W \sin\theta - \tau A = 0$$

where τ = shear stress on the block and
A = area of the block.

$\tau = \mu\,\dfrac{\mathrm{d}u}{\mathrm{d}y} = \mu\,\dfrac{V}{h}$ where h = thickness of oil film

$\mu = 8 \text{ poise} = \dfrac{8}{10}\ \text{Pa.s} = 0.8$ Pa.s
$h = 3 \text{ mm} = 3 \times 10^{-3}$ m
$A = 0.3\ \mathrm{m^2}$

Substituting the various values in the above equation,

$$90 \sin 30° - \frac{(0.8\,V)}{3\times10^{-3}} \times (0.3) = 0$$

$$\therefore \qquad V = \frac{45}{80} = \mathbf{0.5625\ m/s}$$

1.8 A thin plate is placed between two flat surfaces h cm apart such that the viscosity of liquids on the top and bottom of the plate are μ_1 and μ_2 respectively. Determine the position of the thin plate such that the viscous resistance to uniform motion of the thin plate is minimum. [Assume h to be very small].

Solution: Let y be the distance of the thin flat plate from the top flat surface (Fig. 1.4) and V = velocity of the thin plate.

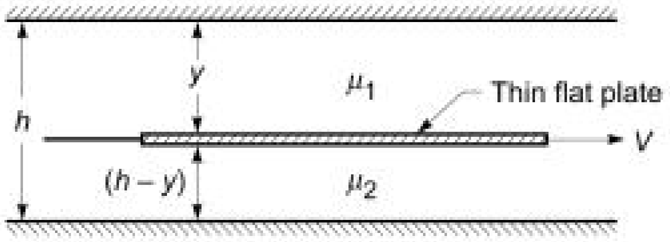


Fig. 1.4

Shear stress on the top portion $\tau_1 = \mu_1 \dfrac{du}{dy} = \mu_1 \dfrac{V}{y}$

Since the gap in the bottom portion = $h - y$

Shear stress, $\tau_2 = \mu_2 \dfrac{V}{(h-y)}$

Force on both sides of the plate

$$F = A(\tau_1 + \tau_2) = VA\left[\frac{\mu_1}{y} + \frac{\mu_2}{h-y}\right]$$

where A = area of the thin plate.

For F to be minimum $\dfrac{dF}{dy} = 0$

$$-\frac{\mu_1}{y^2} + \frac{\mu_2}{(h-y)^2} = 0$$

$$\frac{y^2}{(h-y)^2} = \left(\frac{\mu_1}{\mu_2}\right)$$

or
$$\boldsymbol{\frac{y}{(h-y)} = \sqrt{\mu_1/\mu_2}}$$

1.9 A cylindrical shaft of 90 mm diameter rotates about a vertical axis inside a fixed cylindrical tube of length 50 cm and 95 mm internal diameter. If the space between the tube and the shaft is filled by a lubricant of dynamic viscosity 2.0 poise, determine the power required to overcome viscous resistance when the shaft is rotated at a speed of 240 rpm (Refer. Fig. 1.5).

Solution:

V = Circumferential velocity of the shaft

$$= \omega r = \frac{2\pi N}{60} \times r$$

$$= \frac{2\pi \times 240}{60} \times \left(\frac{0.090}{2}\right) = 1.131 \text{ m/s}$$

Clearance $h = \dfrac{95-90}{2} = 2.5$ mm

Assuming linear variation of velocity across the gap,

Velocity gradient $\dfrac{du}{dr} = \dfrac{V}{h}$

$$= \frac{1.131}{2.5 \times 10^{-3}} = 452.4 \text{ s}^{-1}$$

μ = 2.0 poise = 0.2 Pa.s

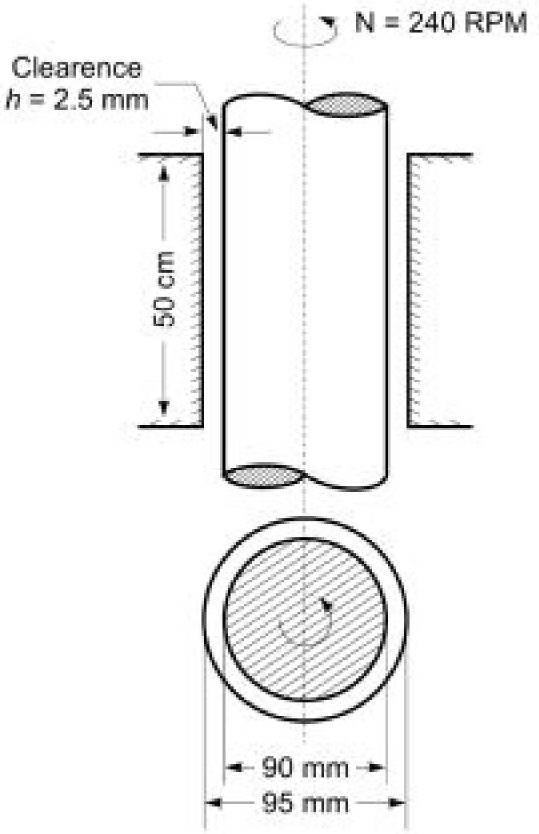


Fig. 1.5

Shear stress on the shaft $\tau = \mu \dfrac{du}{dr}$

$$= 0.2 \times 452.4 = 90.48 \text{ Pa}$$

Shear force $F_s = \tau \times 2\pi r \times L$

$$= 90.48 \times \left(2\pi \times \frac{0.09}{2}\right) \times 0.50$$

$$= 12.791 \text{ N}$$

Torque $T = F_s r$

$$= 12.791 \times \frac{0.09}{2} = 0.6756 \text{ N.m}$$

Power required $= P = \dfrac{2\pi N}{60} \cdot T$

$$= \frac{2\pi \times 240}{60} \times 0.6756 = \boxed{\textbf{14.5 W}}$$

1.10 A sleeve 10 cm long encases a vertical metal rod 3.0 cm in diameter with a radial clearance of 0.02 mm. If when immersed in an oil of viscosity 6.0 poise, the effective weight of the sleeve is 7.5 N, will the sleeve slide down the rod and if so at what velocity? (Refer Fig 1.6)

Solution: Given

μ = 6 poise = 0.6 Pa.s
h = 0.02 mm = 0.02×10^{-3} m

Let V = Velocity of the sleeve when sliding down.

Shear stress $\tau = \mu \dfrac{du}{dr} = \mu \dfrac{V}{h}$

$$= (0.6) \times \frac{V}{2 \times 10^{-5}} = 3 \times 10^4 \text{ V}$$

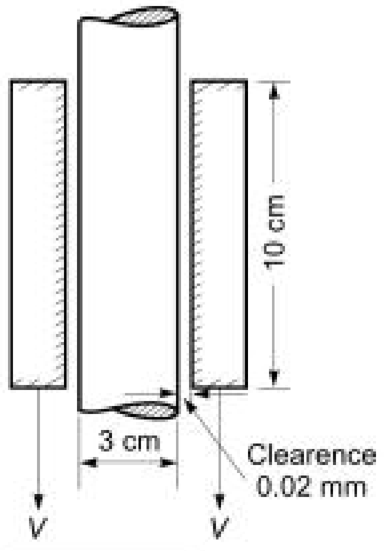


Fig. 1.6

At terminal velocity

Shear force = Submerged weight of the sleeve.

$\therefore \quad (2\pi rL) \times \tau = W_s$

$$\left(2\pi \times \frac{0.03}{2} \times \frac{10}{100} \times 3 \times 10^4\right) V = 7.5$$

$$282.74\ V = 7.5$$

$$V = 0.02653 \text{ m/s} = \mathbf{2.66\ cm/s}$$

1.11 A hydraulic lift used for lifting automobiles has a 25 cm diameter ram which slides in a 25.018 cm diameter cylinder, the annular space being filled with oil having a kinematic viscosity of 3.7 cm^2/s and relative density of 0.85. If the rate of travel of the ram is 15 cm/s find the frictional resistance when 3.3 m of ram is engaged in the cylinder.

Solution:

$$\mu = \nu\rho = 3.7 \times 10^{-4} \times 0.85 \times 998$$
$$= 0.3139 \text{ Pa.s}$$

$$\text{Shear stress } \tau = \mu \frac{du}{dy} = \mu \frac{V}{h}$$
$$= 0.3139 \times \frac{0.15}{(25.018 - 25.00)/(2 \times 10^2)}$$
$$= 523.1 \text{ N/m}^2$$

Frictional resistance $F_s = A\tau$

$$F_s = (\pi \text{DL}) \times \tau$$
$$= \pi \times \frac{25}{100} \times 3.3 \times 523.1$$
$$= 1356 \text{ N} = \mathbf{1.356\ kN}$$

1.12 A circular disc of radius R is kept at a small height above a fixed bed by means of a layer of oil of viscosity μ. If the disc is rotated at an angular velocity ω, obtain an expression for viscous torque on the disc (Refer Fig 1.7).

Solution: Consider an element of disc of width dr at a radial distance r.

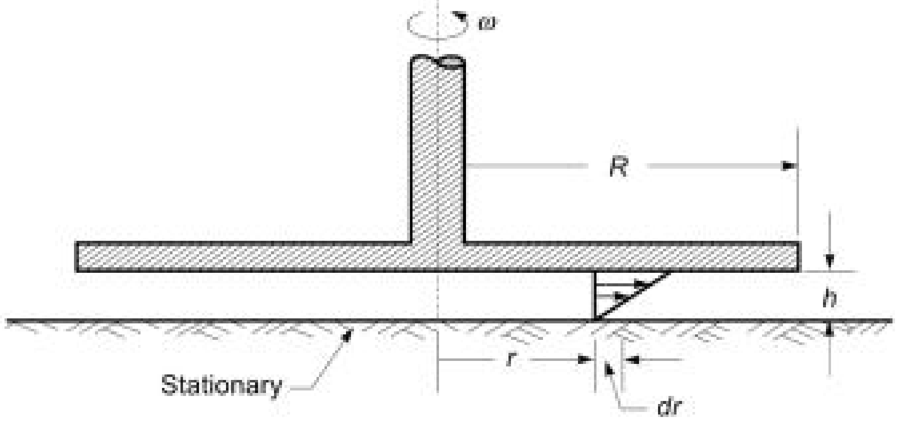


Fig. 1.7

Velocity at this radius = $V = r\omega$.

Assuming linear variation of velocity with depth in the gap h,

$$\text{Shear stress } \tau = \mu \frac{V}{h} = \frac{\mu}{h}\omega r$$

Viscous torque on the element

$$dT = \frac{\mu}{h}\omega r\,(2\pi r dr)r$$
$$= \frac{\mu\omega}{h} \cdot 2\pi r^3 dr$$

$$\text{Total torque} \quad T = \int_0^R dT = \int_0^R \frac{\mu\omega}{h}\,2\pi r^3 dr$$
$$= \left[\frac{\mu\omega}{h} \cdot 2\pi \frac{r^4}{4}\right]_0^R$$

or
$$\mathbf{T} = \frac{\mathbf{1}}{\mathbf{2}} \cdot \frac{\pi\mu\omega}{h} \cdot \boldsymbol{R^4}$$

1.13 Two coaxial cylinders 10 cm and 9.75 cm in diameter and 2.5 cm high have both their ends open and have a viscous liquid filled in between. A torque of 1.2 N.m is produced on the inner cylinder when the outer one rotates at 90 rpm. Determine the coefficient of viscosity of the liquid (Refer Fig 1.8).

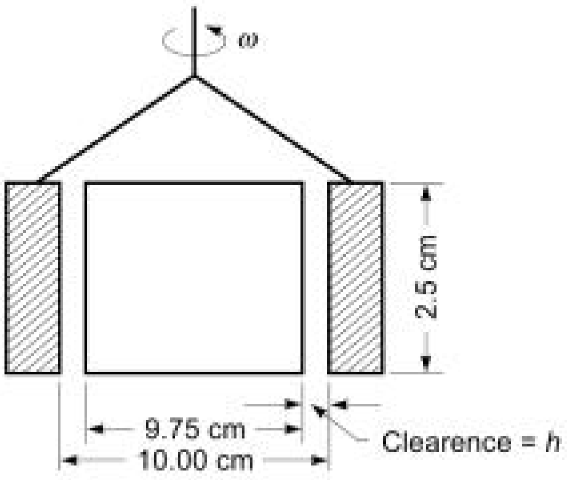


Fig. 1.8

Solution: Tangential velocity

$$V = \frac{\pi DN}{60} = \frac{\pi \times 0.10 \times 90}{60}$$
$$= 0.4712 \text{ m/s}$$

$$\text{Radial clearance } h = \frac{(10.00 - 9.75)}{2} \text{ cm}$$
$$= 0.125 \text{ cm} \quad \text{or} \quad 0.00125 \text{ m}$$

$$\text{Velocity gradient } \frac{du}{dr} = \frac{V}{h} = \frac{0.4712}{0.00125} = 377$$

$$\text{Shear stress } \tau = \mu \frac{du}{dr} = 377\mu$$

$$\text{Shear force } F_s = \tau \times \text{area}$$
$$= 377\mu \left(2\pi \times \frac{0.1}{2} \times \frac{2.5}{100}\right)$$
$$= 2.961\mu$$

$$\text{Torque} = F_s \times \text{lever arm} = F_s\, r$$
$$T = 2.961\, \mu \times \frac{(0.1)}{2} = 0.14805\, \mu$$

$$\text{But Torque } T = 1.2 \text{ N.m}$$

Therefore $0.14805\, \mu = 1.2$

$\therefore$ $$\mu = \frac{120}{0.14805} = \mathbf{8.106 \text{ Pa.s}}$$

1.14 A solid cone of radius r_0, and vertex angle 2θ is to rotate at an angular velocity ω. An oil of viscosity μ and thickness h fills the gap between the cone and the housing. Determine the torque T required to rotate the cone (Refer Fig. 1.9).

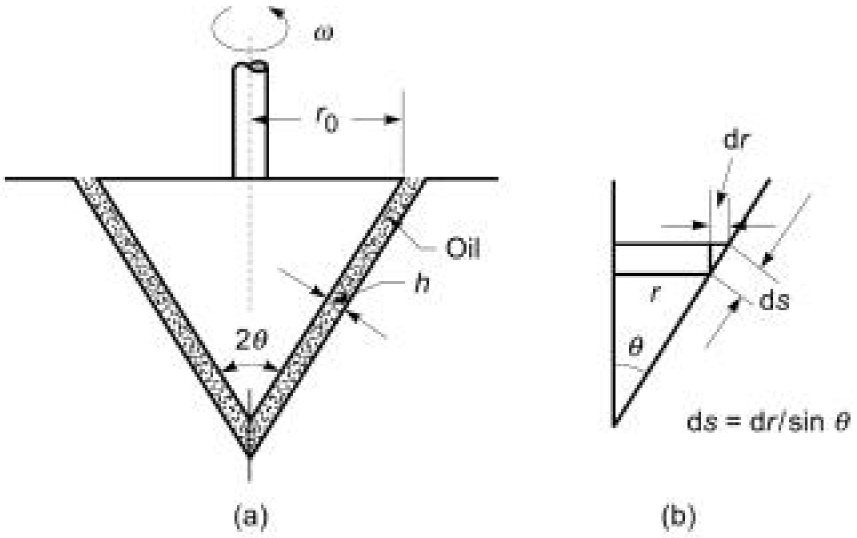


Fig. 1.9

Solution: At any radius $r \le r_0$,

$$u = \omega r$$

Shear stress on the inclined wall $\tau = \mu \dfrac{du}{dy}$

$$= \mu \frac{V}{h} = \mu \frac{\omega r}{h}$$

Considering an elemental area $(2\pi r\, ds)$

$$= 2\pi r \cdot \frac{dr}{\sin\theta}$$

$$d\,(\text{torque}) = dT = r\, d(\text{force})$$
$$= r\tau 2\pi r \cdot \frac{dr}{\sin\theta}$$
$$= r\mu \frac{\omega r}{h} 2\pi r \frac{dr}{\sin\theta}$$
$$= \mu \frac{2\pi\omega}{h} \frac{1}{\sin\theta} \cdot r^3\, dr$$

$$\text{Torque } T = \int_0^{r_0} dT = \frac{2\pi\omega\mu}{h\sin\theta} \int_0^{r_0} r^3\, dr$$

$$\boldsymbol{T = \frac{\pi\omega\mu}{2h\sin\theta} r_0^4}$$

1.15 If the surface tension at air-water interface is 0.073 N/m, what is the pressure difference between inside and outside of an air bubble of diameter 0.01 mm?

Solution: An air bubble has only one surface. Hence

$$\Delta p = \frac{2\sigma}{R}$$
$$= \frac{2 \times 0.073}{\left(\frac{0.01}{2}\right) \times 10^{-3}} = 29200 \text{ N/m}^2$$
$$= \mathbf{29.2 \text{ kPa}}$$

1.16 If the surface tension at the soap-air interface is 0.088 N/m, calculate the internal pressure in a soap bubble of 3 cm diameter.

Solution: In a soap bubble, there are two interfaces. Hence

$$\Delta p = \frac{4\sigma}{R} = \frac{4 \times 0.088}{\left(\frac{3}{2} \times 10^{-2}\right)}$$

= 23.47 N/m² above atmospheric pressure

1.17 A U-tube has two limbs of internal diameter 6 mm and 16 mm respectively and contains some water. Calculate the difference in water levels in the two limbs. Surface tension of water = 0.073 N/m.

Solution: Let h = difference in water levels in the two limbs. By assuming the angle of contact $\theta = 0°$,

$$\Delta p = \rho g h = \left(\frac{2\sigma}{R_1} - \frac{2\sigma}{R_2}\right)$$

$$h \times 998 \times 9.81 = 2 \times 0.073 \left(\frac{1}{3 \times 10^{-3}} - \frac{1}{8 \times 10^{-3}}\right)$$

$$h = 3.1 \times 10^{-3} \text{ m} = \mathbf{3.1 \text{ mm}}$$

1.18 A clean tube of internal diameter 3 mm is immersed in a liquid with a coefficient of surface tension = 0.48 N/m. The angle of contact of the liquid with the glass can be assumed to be 130°. The density of the liquid is 13,600 kg/m^3. What would be the level of the liquid in the tube relative to the free surface of the liquid outside the tube.

Solution: The liquid in the tube rises (or falls) due to capillarity. The capillary rise (or fall)

$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

Here $R = \frac{3}{2}$ mm $= 1.5 \times 10^{-3}$ m

$\theta = 130°$, $\sigma = 0.48$ N/m

$\gamma = \rho g = 13.6 \times 10^3 \times 9.81$

$$\therefore \quad h = \frac{2 \times 0.48 \times \cos 130°}{(13.6 \times 10^3 \times 9.81) \times (1.5 \times 10^{-3})}$$

$$= -3.08 \times 10^{-3} \text{ m} = \mathbf{-3.08\ mm}$$

Therefore, there is a capillary depression of 3.08 mm.

1.19 Assuming that the interstices in a clay are of size equal to one tenth the mean diameter of the grain, estimate the height to which water will rise in a clay soil of average grain diameter 0.05 mm. Assume surface tension at air-water interface as 0.073 N/m.

Solution:

Diameter of pores = $2R = 0.005$ mm

$R = 0.00025$ mm $= 2.5 \times 10^{-6}$ m

$$\Delta h = \frac{2\sigma}{\gamma R} \text{ by assuming } \theta = 0°$$

$$= \frac{2 \times 0.073}{(9.81 \times 998) \times (2.5 \times 10^{-6})}$$

$$= \mathbf{5.95\ m}$$

1.20 Calculate the work done in blowing a soap bubble of diameter 12 cm. Assume the surface tension of soap solution = 0.040 N/m.

Solution: The soap bubble has two interfaces.

Work done = Surface tension × total surface area

$$= 0.040 \times 4\pi\left(\frac{12}{2} \times 10^{-2}\right)^2 \times 2$$

$$= \mathbf{36.2 \times 10^{-4}\ N.m}$$

1.21 What force is necessary to lift a thin platinum wire ring of 3.0 cm diameter from a water surface. Assume the surface tension of water = 0.0728 N/m and neglect the weight of the wire.

Solution: Two water surfaces at the ring resist the lifting force. Referring to Fig. 1.10, by assuming $d << D$

$$F = 2[\pi D \sigma]$$

$$= 2 \times \pi \times \frac{3.0}{100} \times 0.0728 = \mathbf{0.0137\ N}$$

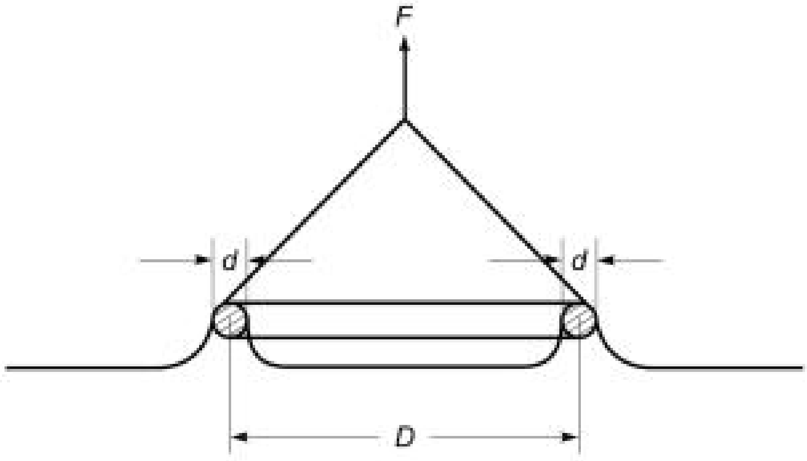


Fig. 1.10

1.22 Using perfect gas law, estimate the density of (i) air and (ii) CO$_2$, at 60°C and 120 kPa (abs) pressure.

Solution:

(i) For air M = Molecular weight = 28.97

Gas constant $R = \frac{8312}{28.97} = 287$

$p = 120 \times 10^3$ Pa. (abs)

$T = 273 + 60 = 333$ K

Density $\rho = \frac{p}{RT} = \frac{120 \times 10^3}{287 \times 333}$

(ii) For CO$_2$, $M = 44$

Gas constant $R = \frac{8312}{44} = 189$

Density $\rho = \frac{120 \times 10^3}{(189)(333)} = \mathbf{1.907\ kg/m^3}$

1.23 Air at 20°C and 200 kPa (abs) contained in a cylinder is compressed to half its volume. Find the pressure and temperature inside the cylinder if the process is (a) Isothermal and (b) Isentropic with $k = 1.4$.

Solution:

(a) In *isothermal process* $p_1 v_1 = p_2 v_2$

$$\therefore \quad p_2 = p_1 \frac{v_1}{v_2} = 200 \times \left(\frac{1}{0.5}\right)$$

$$= \mathbf{400\ kPa\ (abs)}$$

The temperature will remain constant and hence

$$T_2 = T_1 = 20°\text{C}$$

(b) In *isentropic process*:

pv^k = constant

$$p_2 = p_1\left(\frac{v_1}{v_2}\right)^k = 200\left(\frac{1}{0.5}\right)^{1.4}$$

$$= \mathbf{527.8\ kPa\ (abs)}$$

For temperature,

$$pv = RT \text{ and } pv^k = \text{constant}$$

$$RTv^{k-1} = \text{constant}$$

$$\text{i.e.} \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$T_1 = 20°\text{C} = 273 + 20 = 293 \text{ K}$$

$$\frac{v_1}{v_2} = 2.0 \text{ and } (k - 1) = 0.4$$

$$T_2 = 293\ (2)^{0.4} = 386.6 \text{ K}$$

$$= (386.6 - 273) = \mathbf{113.6°C}$$

1.24 Gas A at 100 kPa (abs) is compressed isothermally and Gas B at 80 kPa (abs) is compressed isentropically ($k = 1.4$). What is the bulk modulus of elasticity of each gas and which is more compressible?

Solution:

Gas A: In isothermal change $K = p$

Hence $K_A = p =$ **100 kPa**

Gas B: In adiabatic change $K = kp$

Hence $K_B = kp = 1.4 \times 80 =$ **112 kPa**

Since $K_A < K_B$, gas A is more compressible than gas B, in the notified situation.

1.25 What change in pressure is required to compress a gas under isothermal conditions to quarter of its original volume?

Solution: If V is the original volume

$$\Delta V = (V_2 - V_1) = \left(\frac{1}{4}V - V\right) = -\frac{3}{4}V$$

$$dp = \left(-\frac{dV}{V}\right)K$$

where K is the bulk modulus of elasticity.

At isothermal conditions $K = p$. Hence change in pressure

$$\boldsymbol{\Delta p = \left(\frac{3}{4}\right)p}$$

where p = initial pressure of the gas.

1.26 At 20°C, the bulk modulus of elasticity of kerosene is 1.43×10^9 Pa and its relative density 0.804. Calculate the velocity of propagation of sound in kerosene at this temperature.

Solution:

Density of kerosene $\rho = 0.804 \times 998$

$$= 802.4 \text{ kg/m}^3$$

Velocity of sound $C = \sqrt{K/\rho}$

$$= \sqrt{\frac{1.43 \times 10^9}{802.4}} = \mathbf{1335\ m/s}$$

1.27 Determine the pressure increase required to reduce the volume of water by 1.5%, if its bulk modulus of elasticity is 2.2×10^9 Pa.

Solution: Let V = Volume of water

$$\text{Change in volume} = dV = -\frac{1.5}{100}V = -0.015\,V$$

$$-\frac{dV}{V} = 0.015$$

$$\text{Increase in pressure} = \Delta p = \left(-\frac{dV}{V}\right)K$$

$$= 2.2 \times 10^9 \times 0.015$$

$$= 3.3 \times 10^7 \text{ Pa}$$

$$= \mathbf{3.3 \times 10^4\ kPa}$$

1.28 The velocity of propagation of sound in air is calculated by assuming the process to be isentropic. What is the velocity of sound at 80°C?

Solution:

For air, gas constant $R = 287$

$$T = 80°\text{C} = 273 + 80 = 353 \text{ K}$$

Since the process is isentropic, $K = kp$.

Assuming the air as perfect gas

$$p = \rho R T$$

Hence the speed of sound

$$C = \sqrt{K/\rho} = \sqrt{\frac{k\rho RT}{\rho}} = \sqrt{kRT}$$

For air $k = 1.4$ and hence

$$C = \sqrt{1.4 \times 287 \times 353} = \mathbf{376.6\ m/s}$$

OBJECTIVE QUESTIONS

1.1 A perfect fluid (also known as an ideal fluid) is
(a) a real fluid
(b) the one which obeys perfect gas laws
(c) compressive and gaseous
(d) incompressible and frictionless

1.2 The concept of continuum in fluid flow assumes that the characteristics length of the flow is
(a) smaller than the mean free path of the molecules
(b) larger than the mean free path of the molecules
(c) larger than the dimensions of the suspended particles
(d) larger than the wavelength of sound in the medium

1.3 When a shear stress is applied to a substance it is found to resist it by static deformation. The substance is a
(a) liquid (b) solid
(c) gas (d) fluid

1.4 When subjected to shear force, a fluid
(a) deforms continuously no matter how small the shear stress may be
(b) deforms constinuously only for large shear forces
(c) undergoes static deformation
(d) deforms continuously only for small shear stresses

1.5 An object with a mass of 5 kg as determined on earth is placed in a planet which has an acceleration due to gravity of 3 m/s^2. Its mass and weight in this planet are respectively
(a) 5 kg and 15 N
(b) 1.53 kg and 5 N
(c) 15 kg and 15 N
(d) 15 kg and 5 N

1.6 The condition of "No Slip" at rigid boundaries is applicable to
(a) flow of Newtonian fluids only
(b) flow of ideal fluids only
(c) flow of all real fluids
(d) flow of non-Newtonian fluids only

1.7 Newton's law of viscosity for a fluid states that the shear stress is
(a) proportional to angular deformation
(b) proportional to rate of angular deformation
(c) inversely proportional to angular deformation
(d) inversely proportional to rate of angular deformation

1.8 The viscosity of
(a) liquids increases with temperature
(b) gases increases with temperature
(c) fluids decreases with temperature
(d) fluids increases with temperature

1.9 For a fluid, the shear stress was found to be directly proportional to the rate of angular deformation. The fluid is classified as
(a) Newtonian (b) Non-Newtonian
(c) Dilatant fluid (d) Thixotropic

1.10 If the relationship between the shear stress τ and the rate of shear strain du/dy is expressed as

$$\tau = k\left[\frac{du}{dy}\right]^n$$

the fluid with the exponent $n < 1$ is known as
(a) Pseudoplastic fluid
(b) Bingham fluid
(c) Dilatant fluid
(d) Newtonian plastic

1.11 A real fluid is any fluid which
(a) has surface tension and is incompressible
(b) has zero shear stress
(c) has constant viscosity and density
(d) has viscosity

1.12 A fluid indicated the following shear stress and deformation rates:

du/dy (units)	0	1	2	4
τ (units)	10	15	20	30

This fluid is classified as
(a) Newtonian (b) Bingham plastic
(c) Dilatant (d) Pseudoplastic

1.13 If the shear stress τ and shear rate (du/dy) relationship of a material is plotted with τ on the Y-axis and du/dy on the X-axis, the behaviour of an ideal fluid is exhibited by
(a) a straight line passing through the origin and inclined to the X-axis
(b) the positive X-axis

(c) the positive Y-axis
(d) a curved line passing through the origin

1.14 In the following Fig. 1.11 the line A describes the rheological behaviour of a fluid. The fluid can be classified as

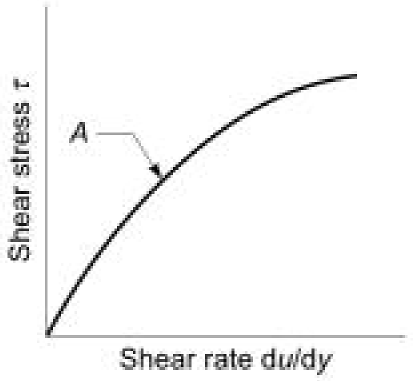


Fig. 1.11

(a) Newtonian (b) Bingham plastic
(c) Ideal (d) Non-Newtonian

1.15 Shear stress for a general fluid motion is respresented by

$$\tau = \mu \left(\frac{du}{dy} \right)^n + B$$

where, n and B are constants. A Newtonian fluid is given by
(a) $n > 1$ and $B = 0$
(b) $n = 1$ and $B = 0$
(c) $n > 1$ and $B \neq 0$
(d) $n < 1$ and $B = 0$

1.16 The following shear stress-shear rate relationship was obtained for a fluid:

du/dy (units)	0	1	3	5
τ (units)	0	6	18	30

The fluid is classified as
(a) Bingham plastic (b) Dilatant
(c) Newtonian (d) Ideal

1.17 The dimensions of the coefficient of dynamic viscosity in [M, L, T] notation system are
(a) $M L^{-1} T$ (b) $M L^{-1} T^{-1}$
(c) $M^{-1} L T$ (d) $M L T^{-1}$

1.18 The dimensions of the coefficient of dynamic viscosity in [F, L, T] notation system are
(a) $F T L^{-2}$ (b) $F L^{-1} T^{-1}$
(c) $F L^2 T^{-1}$ (d) $F T^{-2} L$

1.19 A Newtonian fluid fills the clearance between a shaft and a sleeve. When a force of 800 N is applied to the shaft, parallel to the sleeve, the shaft attains a speed of 1.5 cm/s. If a force of 2.4 kN is applied instead, the shaft would move with a speed of
(a) 1.5 cm/s (b) 13.5 cm/s
(c) 0.5 cm/s (d) 4.5 cm/s

1.20 Typical example of a non-Newtonian fluid of pseudoplastic veriety is
(a) Water (b) Air
(c) Blood (d) Printing ink

1.21 The kinematic viscosity ν is related to the dynamic viscosity μ and density ρ as $\nu =$
(a) μ/ρ (b) $\mu\rho$
(c) ρ/μ (d) $\mu/\rho g$

1.22 A flow of a viscous fluid with $\mu = 1.0$ Ns/m^2 has a velocity distribution given by $u = 0.90\ y - y^2$. The shear stress at $y = 0.45$ m is
(a) 0.90 N/m^2 (b) ∞
(c) zero (d) $-$ 0.90 N/m^2

1.23 A perfect gas
(a) is same as an ideal fluid
(b) satisfies pv^k = constant
(c) has zero velocity
(d) satisfies $p/\rho = RT$ relation

1.24 A perfect gas
(a) is a perfect fluid
(b) does not have viscosity
(c) is incompressible
(d) does not really exist

1.25 In an isentropic process
(a) pv = constant (b) p/T = constant
(c) p/v^k = constant (d) pv^k = constant

1.26 The bulk modulus of elasticity for a liquid, K
(a) is a function of both temperature and pressure
(b) at any given temperature decreases continuously with pressure
(c) at any pressure increases continuously with temperature
(d) is a constant

1.27 In a sample of water an increase of pressure by 18 MN/m^2 caused 1% reduction in the volume. The bulk modulus of elasticity of this sample, in MN/m^2, is
(a) 1.80 (b) 180
(c) 1800 (d) 0.18

1.28 Broadly speaking water is
(a) 10 times more compressible than steel
(b) 80 times more compressible than steel
(c) 80 times less compressible than steel
(d) 800 times less compressible than steel

1.29 The bulk modulus of elasticity K for a gas at constant temperature is

(a) p/ρ (b) ρT
(c) ρRT (d) p

1.30 The bulk modulus of elasticity for a gas undergoing adiabatic process pv^k = constant is,

(a) p/ρ (b) p
(c) kp (d) constant

1.31 Kerosene is known to have a bulk modulus of elasticity $K = 1.43 \times 10^9$ N/m^2 and a relative density of 0.806. The speed of sound in kerosene, in m/s, is

(a) 1333 (b) 1075
(c) 1197 (d) 184

1.32 The bulk modulus of elasticity K of

(a) a gas is larger than that of a solid
(b) a liquid is smaller than that of a gas
(c) a liquid is larger than that of a solid
(d) a solid is larger than that of a liquid

1.33 If the capillary rise of water in a 2 mm diameter tube is 1.5 cm, the height of capillary rise in a 0.5 mm diameter tube, in cm, will be

(a) 10.0 (b) 1.5
(c) 6.0 (d) 24.0

1.34 If the surface tension of water-air interface is 0.073 N/m, the gauge pressure inside a rain drop of 1 mm diameter is,

(a) 146.0 N/m^2 (b) 0.146 N/m^2
(c) 73.0 N/m^2 (d) 292.0 N/m^2

1.35 The excess pressure (above atmospheric) inside a soap bubble of diameter 1 cm, by assuming the surface tension of soap solution to be 0.04 N/m is,

(a) 32.0 N/m^2 (b) 16.0 N/m^2
(c) 160.0 N/m^2 (d) 0.32 N/m^2

1.36 The capillary rise in a 3 mm tube immersed in a liquid is 15 mm. If another tube of diameter 4 mm is immersed in the same liquid the capillary rise would be

(a) 11.25 mm (b) 20.00 mm
(b) 8.44 mm (d) 26.67 mm

1.37 An apparatus produces water droplets of size 70 μm. If the coefficient of surface tension of water in air is 0.07 N/m, the excess pressure in these droplets, in kPa, is,

(a) 5.6 (b) 4.0
(c) 8.0 (d) 13.2

1.38 If the coefficient of surface tension of water in air is 0.07 N/m, the diameter of a tube that can be used to keep the capillary height between 1.80 cm to 2.00 cm is,

(a) 1.65 mm (b) 3.33 cm
(c) 1.65 cm (d) 1.40 cm

1.39 At a liquid-air-solid interface the contact angle θ measured in the liquid is less than 90°. The liquid is,

(a) Wetting
(b) Non-wetting
(c) Ideal
(d) Does not form a stable bubble

1.40 If σ_{Lg} is surface tension at liquid-gas interface, σ_{gs} is surface tension at gas-solid interface, and σ_{Ls} is surface tension at liquid-solid interface, a small drop of liquid when dropped on to a solid surface will remain in equilibrium without spreading if

(a) $|\sigma_{Lg} - \sigma_{Ls}| > \sigma_{gs}$ (b) $|\sigma_{gs} - \sigma_{Ls}| > \sigma_{Lg}$
(c) $|\sigma_{Lg} - \sigma_{Ls}| < \sigma_{gs}$ (d) $|\sigma_{gs} - \sigma_{Ls}| < \sigma_{Lg}$

1.41 The predominant fluid property associated with cavitation phenomenon is

(a) Surface tension
(b) Vapour pressure
(c) Mass density
(d) Bulk modulus of elasticity

1.42 At 20°C, pure water will have a vapour pressure, in kPa, of about

(a) 0.5 (b) 2.34
(c) 101.3 (d) 8.67

1.43 At 100°C, at sea level, pure water will have a vapour pressure, in kPa, of about

(a) 0.50 (b) 2.3
(c) 10.1 (d) 101.3

PROBLEMS

1.1 The relative density of a fluid is 1.26 and its dynamic viscosity is 1.50 Pa.s. Calculate its (i) specific weight and (ii) kinematic viscosity.

(*Ans.* $\gamma = 12.336\ \text{kN/m}^3$, $\nu = 1.193 \times 10^{-3}\ \text{m}^2/\text{s}$)

1.2 From a table of properties of liquids it was found that, at 20°C, carbon tetrachloride has a dynamic viscosity of 9.67×10^{-4} Pa.s and a kinematic viscosity of $6.08 \times 10^{-7}\ \text{m}^2/\text{s}$. Calculate its (i) relative density and (ii) specific weight.

(*Ans.* $RD = 1.5936$, $\gamma = 15.602\ \text{kN/m}^3$)

1.3 A flat plate 60 cm × 120 cm slides on SAE 10 oil ($\mu = 1.04 \times 10^{-1}$ Pa.s) over a large plane surface. What force is required to drag the plate at 3.5 m/s if the oil film is 3 mm thick?

(*Ans.* $F = 87.36$ N)

1.4 Glycerin has a density of $1260\ \text{kg/m}^3$ and a kinematic viscosity of $0.00183\ \text{m}^2/\text{s}$. What shear stress is required to deform this fluid at a strain rate of $10^4\ \text{s}^{-1}$?

(*Ans.* $\tau = 14.9$ kPa)

1.5 The velocity distribution in the flow of a thin film of oil down an inclined channel is given by

$$u = \frac{\gamma}{2\mu}(d^2 - y^2)\sin\alpha$$

where d = depth of flow, α = angle of inclination of the channel to the horizontal, u = velocity at a depth y below the free surface, γ = unit weight of oil and μ = dynamic viscosity of oil. Calculate the shear stress: (i) on the bottom of the channel, (ii) at mid-depth and (iii) at the free surface.

(*Ans.* At $y = d$, $\tau_0 = -\gamma d \sin\alpha$;
at $y = d/2$, $\tau_{1/2} = -\gamma (d/2) \sin\alpha$;
at $y = 0$, $\tau_t = 0$)

1.6 Two large plane surfaces are 20 mm apart and the gap contains oil of dynamic viscosity 0.60 Pa.s. A thin plate of cross sectional area $0.50\ \text{m}^2$ is to be pulled through the gap at a constant velocity of 0.60 m/s. The location of the plate will have to be such that it is 8 mm from one of the surfaces. Neglecting edge effects, estimate the force required for pulling the plate as above.

(*Ans.* $F = 37.5$ N)

1.7 A flat plate 30 cm × 50 cm slides on oil ($\mu = 0.75$ Pa.s) over a large plane surface. What force is required to drag the plate at a uniform velocity of 1.6 m/s, if the separating oil film is 0.2 mm thick?

(*Ans.* $F = 900$ N)

1.8 A square plate 50 cm × 50 cm weighing 200 N slides down an inclined plane of slope 1 vertical: 2.5 horizontal with a uniform velocity of 0.40 m/s. If a thin layer of oil of thickness 0.5 cm fills the space between the plate and the inclined plane, determine the coefficient of viscosity of the oil.

(*Ans.* $\mu = 1.4$ Pa.s)

1.9 A flat plate of area $0.15\ \text{m}^2$ is to be towed up an inclined plane of slope 1 vertical : 3 horizontal on a thin layer of oil ($\mu = 0.75$ Pa.s.) of thickness 0.2 mm. If the weight of the plate is 250 N, estimate the force required to pull the plate at 1.6 m/s velocity.

(*Ans.* $F = 979$ N)

1.10 A block with a base of 15 cm × 20 cm and weighing 20 N is allowed to slide down a long inclined plane of slope 1 vertical : 5 horizontal. A thin film of oil ($\mu = 1.5$ poise) of thickness 0.3 mm exists on the surface of the inclined plane. Estimate the terminal speed of the block.

(*Ans.* $V = 26.15$ cm/s)

1.11 A piston of 7.95 cm diameter and 30 cm long works in a cylinder of 8.0 cm diameter. The annular space of the piston is filled with an oil of viscosity 2 poise. If an axial load of 10 N is applied to the piston, calculate the speed of movement of the piston.

(*Ans.* $V = 16.68$ cm/s)

1.12 Two discs of 20 cm diameter are placed 1 mm apart and the gap is filled with an oil of viscosity 0.8 kg/m.s. Determine the power required to rotate the upper disc at 600 rpm while holding the lower one stationary.

(*Ans.* $P = 496$ W)

1.13 A metal plate of size 0.60 m × 0.60 m and 1 mm thick and weighing 25 N is to be lifted up edgewise with a uniform velocity of 0.2 m/s in the gap between two flat surfaces. The plate is in the middle of the gap of width 2 mm and the gap contains oil of relative density 0.85 and viscosity 1.6 poise. Calculate the vertical force required for this job.

(*Ans.* $F_v = 88.08$ N)

1.14 A 90 mm diameter shaft rotates at 1200 rpm in a 100 mm long journal bearing of 90.5 mm internal diameter. The annular space in the bearing is filled with oil having a dynamic viscosity of 0.12 Pa.s. Estimate the power dissipated as heat.

(*Ans.* $P = 434$ W)

1.15 A vertical shaft has a hemispherical bottom of radius R which rotates inside a bearing of identical shape at its end. An oil film of thickness h and viscosity μ is maintained in the bearing. Estimate the viscous torque in the shaft when it rotates with an angluar velocity ω.

$$\left(Ans.\ T = \frac{4\pi\omega\mu R^4}{3h} \right)$$

1.16 A liquid drop of radius R breaks up into 64 smaller drops, all of equal size. Calculate the work done.

(*Ans.* $W = 12\ \pi\sigma R^2$)

1.17 A tube of internal diameter 2 mm is dipped vertically into a vessel containing mercury. The lower end of the tube is 2 cm below the mercury surface. Estimate the pressure of air inside the tube required to blow a hemispherical bubble at the lower end. Assume surface tension of mercury = 0.40 N/m.

(*Ans.* $\Delta p = 3.468$ kPa)

1.18 Obtain an expression for the height of capillary rise for a fluid of surface tension σ and contact angle θ between two parallel vertical plates at a distance B apart.

$$\left(Ans.\ h = \frac{2\sigma \cos\theta}{\gamma B} \right)$$

1.19 If the surface tension of water in contact with air is 0.075 N/m, what correction need be applied towards capillary rise in the manometric readings in tubes of 3 mm diameter?

(*Ans.* Correction: –10.2 mm to the readings).

1.20 Air is introduced through a nozzle into a tank of water to form a stream of bubbles of size 2 mm. Calculate by how much the pressure in the nozzle must exceed that of the surrounding water? Assume $\sigma = 0.073$ N/m.

(*Ans.* $\Delta p = 143.4$ pa)

1.21 Estimate the pressure inside a water droplet of size 0.3 mm. Assume $\sigma = 0.0728$ N/m.

(*Ans.* $\Delta p = 0.97$ kPa)

1.22 In measuring the unit surface energy of an oil of relative density (RD) = 0.85 by the bubble method, a tube with an internal diameter of 1.5 mm is immersed to a depth of 1.25 cm in the oil. Air is forced through the tube forming a bubble at the lower end. What magnitude of unit surface energy is indicated by a maximum bubble pressure of 1470 kPa?

(*Ans.* $\sigma = 0.01613$ N/m)

1.23 A perfect gas has its pressure doubled and its specific volume decreased by one third. If the initial temperature was 30°C what is the final temperature?

(*Ans.* $T_2 = 131$°C)

1.24 Chlorine at 100 kPa (abs) and 15°C is isentropically compressed to one fifth of its volume. Assuming $k = 1.4$, estimate the final temperature and pressure.

(*Ans.* $T_2 = 275.2$°C,
$p_2 = 951.8$ kPa)

1.25 Oxygen at 50°C and 90 kPa (abs) is compressed to 40% of its volume. What is the temperature and pressure of the gas if the process is (a) adiabatic ($k = 1.4$) and (b) isothermal.

(*Ans.* (a) $p_2 = 324.6$ kPa (abs), $T_2 = 193$°C
(b) $p_2 = 225$ kPa (abs), $T_2 = 50$°C)

1.26 (i) If the pressure on a sample of water is increased by 10^3 kPa above atmospheric pressure what percentage reduction in the volume of water is observed? Assume $K_{\text{water}} = 2.2 \times 10^9$ Pa.

(ii) To achieve the same reduction in volume as above in a sample of air by isothermal process what increase in the pressure is needed?

(*Ans.* (i) 0.0455% (ii) 0.0461 kPa)

1.27 If $K_{\text{mercury}} = 2.62 \times 10^{10}$ Pa, determine the velocity of propagation of sound in mercury.

(*Ans.* $C = 1388$ m/s)

1.28 The velocity of propagation of sound in air (C) is calculated by assuming the process to be isentropic. What error in C is involved if the sound propagation is assumed to occur isothermally ?

(*Ans.* 15.5% less)

1.29 Air at 60°C and standard atmospheric pressure has a density of 1.060 kg/m^3. What is the velocity of propagation of sound in this medium? Assume the process to be isentropic and $k = 1.4$.

(*Ans.* $C = 365.8$ m/s)

1.30 A liquid with a volume of 0.2 m^3 at 300 kPa is subjected to a pressure of 3000 kPa and its volume is found to decrease by 0.2%. Calculate the bulk modulus of elasticity of the liquid.

(*Ans.* $K = 1.35 \times 10^9$ Pa)

2

Fluid Statics

CONCEPT REVIEW

2.1 MEASUREMENT OF PRESSURE

Different aspects of pressure distribution in incompressible fluids and compressible fluid when at rest are described in this section. Use of manometers to measure pressure differences are highlighted.

2.1.1 Pressure in Incompressible Fluids

Fluid pressure at a point is the same in all directions. In a fluid at rest the pressure intensities in a horizontal plane are equal. For incompressible fluids (i.e. for liquids and gases where compressibility effects can be ignored) the variation of pressure in vertical direction (Fig. 2.1) is given by

$$\boxed{\frac{\mathrm{d}p}{\mathrm{d}z} = -\gamma} \tag{2.1}$$

$$(p_2 - p_1) = \gamma (Z_1 - Z_2) \tag{2.1-a}$$

$$\boxed{(p + \gamma Z) = (p_1 + \gamma Z_1) = (p_2 + \gamma Z_2) = \text{Constant}} \tag{2.2}$$

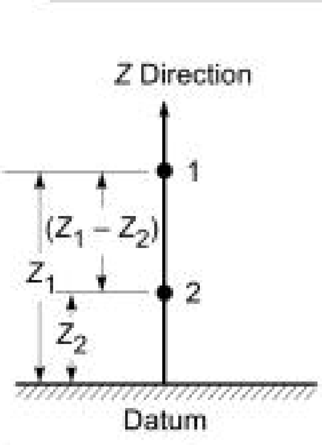


Fig. 2.1

where γ = Specific weight of the fluid

and Z = Vertical distance measured from a datum (positive upward).

At a free surface the pressure is atmospheric. If h is the depth below the free surface of a point M, the absolute pressure at M (Fig. 2.2) is

$$\boxed{p_m(\text{abs}) = \gamma h + p_{\text{atm}}}$$

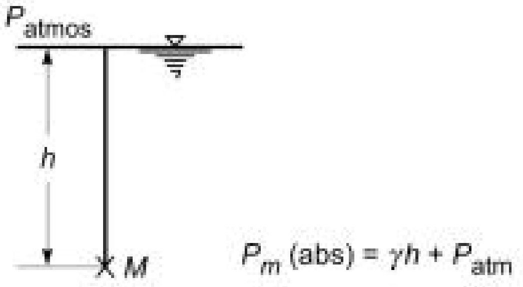


Fig. 2.2

If the pressure *in excess of atmosphere* is recorded then

$$\boxed{p_m(\text{abs}) - p_{\text{atm}} = p_{\text{m}} = \gamma h} \tag{2.3}$$

(Note that h is measured positive downwards from the liquid surface).

The pressure p_{m} is then called *gauge pressure*.

The linear variation of pressure with depth below the free surface is known as *hydrostatic pressure distribution*.

The variation of gauge pressure in a liquid below the free surface is shown in Fig. 2.3. From this $p_1 = \gamma h_1$ and $p_2 = \gamma h_2$, or

$$\boxed{(p_2 - p_1) = \gamma (h_2 - h_1)}$$

Note that in the above the atmospheric pressure was assumed as the datum, i.e. reference with a zero value.

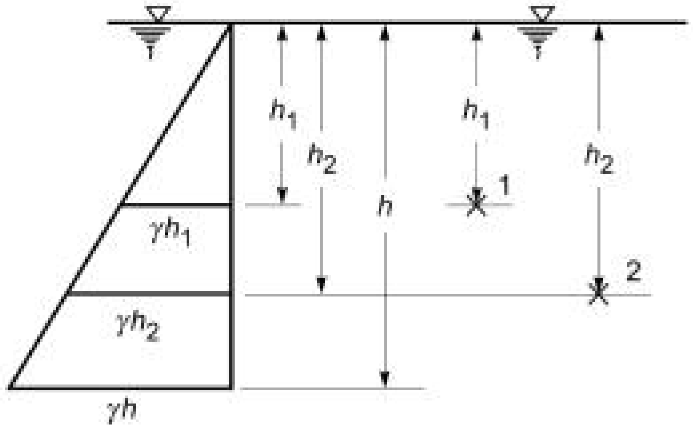


Fig. 2.3

Different references can be taken and depending upon the reference pressures we have:

Absolute pressure is the pressure measured above the absolute zero, a thermodynamic concept. Absolute pressures are designated with (abs) following the symbol or numeral to distinguish from other forms. Thus, for example, 12.0 kPa(abs); p_m (abs) are absolute pressures. Absolute pressures cannot be negative.

Gauge pressure is the pressure measured with respect to local *atmospheric pressure*. Gauge pressures are extensively used in engineering practice and as such are indicated with a symbol or a numeral without any other explanatory notation, e.g. 14.0 kPa, – 3.2 kPa, p_m are gauge pressures. Note that gauge pressures can be positive or negative. Negative gauge pressures are also called *vacuum pressures*.

It is seen that

$$\text{Absolute pressure} = (\text{Local atmospheric pressure}) + (\text{gauge pressure})$$

Pressure has the dimension of [Force/Area]= $[F\,L^{-2}]$ and is usually expressed in *pascals* Pa (= N/m^2); kilo pascals kPa (= 10^3 N/m^2); height h of a column of a fluid of specific weight γ, in *bars* (= 10^5 Pa) or *atmospheres* (= number of standard atmospheric pressure value). The pressures are commonly indicated as gauge pressures and unless a pressure is specifically marked absolute the pressure is treated as gauge pressure. The *atmosphere*, however, is an exception and is an absolute pressure unit. Gauge pressures are commonly measured by a *Bourdon gauge*. Differences in pressures are measured by manometers.

Local atmospheric pressure (i.e. the absolute pressure of the atmosphere at a place) is measured by a mercury *barometer*. The local atmospheric pressure varies with the elevation above mean sea level and local meteorological conditions. For engineering application, a standard atmospheric pressure at mean sea level at 15°C is often used. The value of this standard atmospheric pressure (called 1 atmosphere) is

$$\begin{aligned} 1\text{ atm} &= 10.3\text{ m of water} \\ &= 760\text{ mm of mercury} \\ &= 101.3\text{ kPa} \\ &= 10130\text{ mbar} \end{aligned}$$

Aneroid barometer is another instrument commonly used to measure local atmospheric pressure.

2.1.2 Pressure in Compressible Fluids

For a compressible fluid the density changes with change in pressure and temperature. For a perfect gas we have

$$\boxed{p = \rho RT} \tag{2.4}$$

where p = absolute pressure
T = temperature in Kelvin
R = gas constant
ρ = mass density

Since $\dfrac{dp}{dz} = -\rho g$

$$\boxed{\int dp = -g\int \rho\, dz} \tag{2.5}$$

Depending upon the process involved, i.e. isothermal or adiabatic, the corresponding variation of pressure can be evaluated.

Isothermal Process In an isothermal process $T = T_0$ = constant

Hence

$$\frac{dp}{dz} = -\frac{p}{RT_0}g$$

$$\int_1^2 \frac{dp}{p} = -\frac{g}{R}\int_{Z_1}^{Z_2}\frac{dz}{T_0}$$

$$\boxed{\frac{p_2}{p_1} = \exp\left[\frac{-g(Z_2 - Z_1)}{RT_0}\right]} \tag{2.6}$$

Adiabatic Process For the case of adiabatic process (zero heat transfer), if there is no friction (isentropic)

$$\frac{p}{\rho^k} = \text{Constant} = C_s \tag{2.7}$$

where k = adiabatic constant for the gas. Combining with perfect gas law (Eq. 2.4) we get

$$\frac{T}{\rho^{k-1}} = \text{Constant};\quad \frac{T}{p^{(k-1)/k}} = \text{Constant}$$

Substituting in Eq. 2.5 and simplifying yields

$$\frac{p_2}{p_1} = \left[1 - \frac{(k-1)}{k}\,g\,(Z_2 - Z_1)\,\frac{\rho_1}{p_1}\right]^{k/(k-1)} \tag{2.8}$$

or

$$\boxed{\frac{p_2}{p_1} = \left[1 - \frac{(k-1)}{k}\,g\,\frac{(Z_2 - Z_1)}{RT_1}\right]^{k/(k-1)}} \tag{2.9}$$

The variation of the temperature with Z is given by

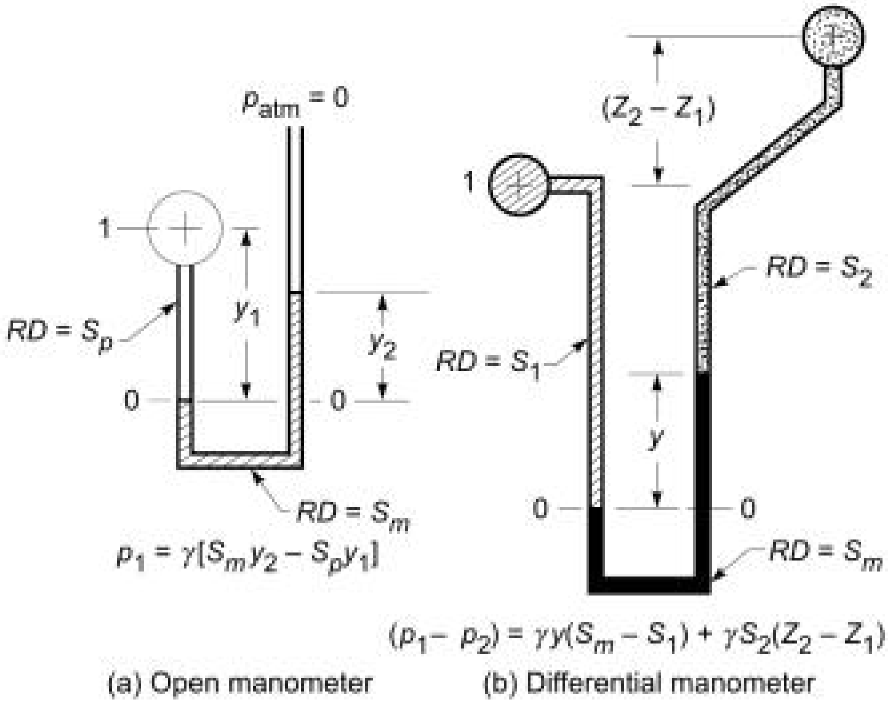
$$\boxed{\frac{T_2}{T_1} = \left[1 - \frac{(k-1)}{k} \cdot \frac{g\,(Z_2 - Z_1)}{RT_1}\right]} \tag{2.10}$$

The rate of variation of the temperature with elevation (dT/dZ) is known as *lapse rate* and is given by

$$\boxed{L = \frac{dT}{dZ} = -\frac{g}{R}\left(\frac{k-1}{k}\right)} \tag{2.11}$$

2.1.3 Manometers

For a fluid at rest it is seen from Eq. 2.1(a) that $(p_2 - p_1) = \gamma\,(Z_2 - Z_1)$. The pressure difference between two points can thus be measured by a static column of a liquid. A manometer is a device to determine the pressure in a fluid by balancing it against a column of liquid. Figure 2.4 shows some commonly used forms of manometers. In a manometer the basic point to remember is that, in a continuous mass of the same static fluid, the pressure at the points in any horizontal plane will be the same.



(a) Open manometer (b) Differential manometer

Fig. 2.4 *Manometers*

2.2 FORCES ON PLANE SURFACES

An important problem in the design of hydraulic structures and other structures which interact with fluids is the computation of hydrostatic forces on plane surfaces. Computations of magnitude and point of application of hydrostatic forces on plane surfaces are described.

2.2.1 Magnitude of Force on a Plane

When a plane area is immersed in a static liquid with its plane making an angle θ with the free liquid surface (Fig. 2.5) the total hydrostatic force on *one side* of the area is

$$\boxed{F = \gamma \bar{h} A} \tag{2.12}$$

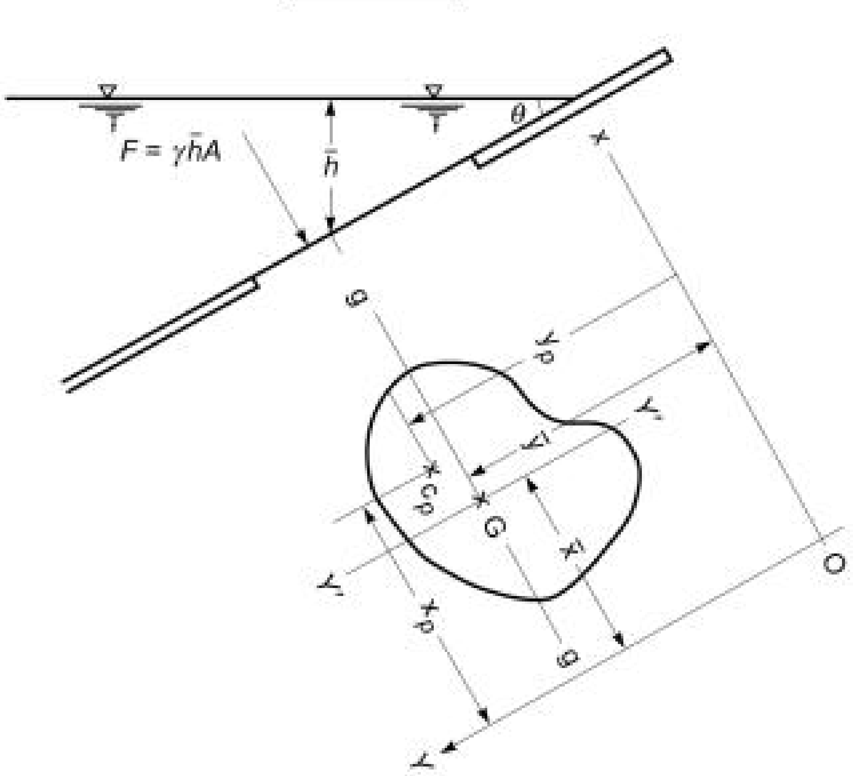


Fig. 2.5 *Centre of Pressure*

where γ = specific weight of the liquid
$\bar{h}$ = depth of the centre of gravity of the area below the free surface
A = area of the immersed plane.

It may be noted that the force F is independent of the angle of inclination θ so long as the depth of the centroid $\bar{h}$ is unchanged.

2.2.2 Centre of Pressure

The point of application of the force F on the submerged area is called the *centre of pressure*. Considering the line of intersection of the plane area with the liquid surface (Line OX) as the reference axis, the centre of pressure is located along the plane at

$$\boxed{y_p = \bar{y} + \frac{I_{gg}}{A\bar{y}}} \tag{2.13}$$

where I_{gg} = moment of inertia about an axis parallel to OX and passing through the centre of gravity of the area
$\bar{y}$ = location of the centre of gravity with respect to the axis OX
A = area of the plane area

Note that the distances y are measured *along the plane* from the axis OX.

The lateral position of the centre of pressure with respect to any axis OY perpendicular to OX and lying in the plane of the lamina is

$$\boxed{x_p = \bar{x} + \frac{I_{xy}}{A\bar{y}}} \tag{2.14}$$

where I_{xy} = product of inertia $\left(= \int_A xy \, dA\right)$ of the area about axis GY′, passing through the centre of gravity of the area and parallel to OY and OX.

When either of the centroidal axes $x = \bar{x}$ or $y = \bar{y}$ is an axis of symmetry, $I_{xy} = 0$ and $x_p = \bar{x}$.

2.3 FORCES ON CURVED SURFACES

When the fluid static force on a curved submerged surface is desired, it is convenient to consider the horizontal and vertical components of the force separately.

2.3.1 Horizontal Component

The horizontal component of hydrostatic force in any chosen direction on any area (plane or curved) is equal to the projection of the area on a vertical plane normal to the chosen direction. The horizontal force acts through the centre of pressure of the vertical projection.

2.3.2 Vertical Component

The vertical component of the hydrostatic force on any surface (plane or curved) is equal to the weight of volume of liquid extending above the surface of the object to the level of the free surface. This vertical component passes through the centre of gravity of the volume considered. The volume and the free surface can be real or imaginary.

2.3.3 Tensile Stress in a Pipe or Shell

In a circular pipe subjected to high pressure, the pressure centre can be taken to be at the centre of the pipe. The tensile circumferential stress (*hoop stress*) in a pipe wall subjected to an internal pressure of p (Fig. 2.6) is

$$\boxed{\text{hoop stress}\, \sigma_h = \frac{pD}{2t}} \tag{2.15}$$

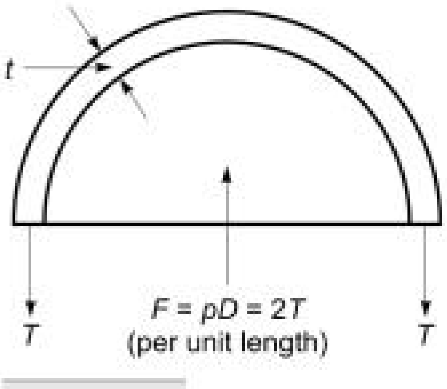


Fig. 2.6

where D = diameter of the pipe
t = thickness of pipe.

This formula assumes $t/D < 0.1$ and hence is based on thin cylinder theory.

If the ends of a cylinder are closed and the cylinder has a fluid under pressure, a longitudinal stress σ_L is produced in the cylinder. This stress is given by

$$\boxed{\sigma_L = \frac{1}{2}\sigma_h = \frac{pD}{4t}} \tag{2.16}$$

For thin spherical shells the tensile stress is

$$\boxed{\sigma_s = \frac{pD}{4t}} \tag{2.17}$$

2.4 BUOYANCY

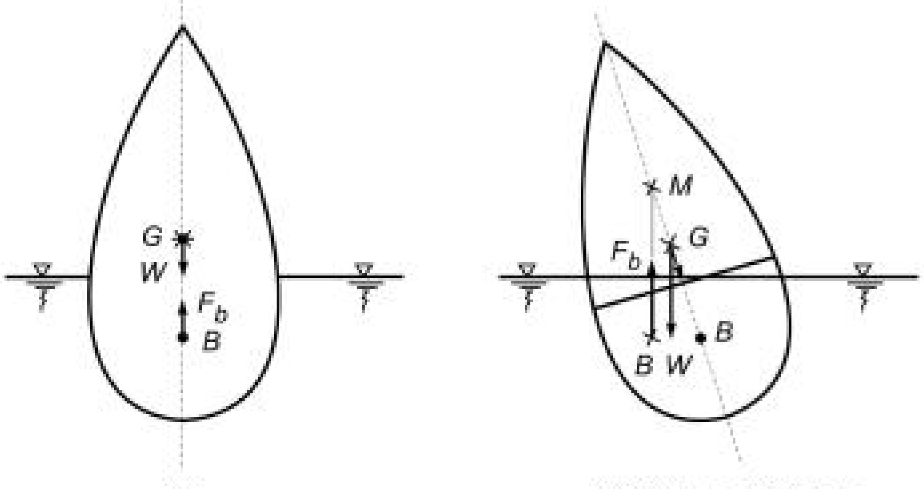
When a body is submerged or floating in a static fluid the resultant force exerted on it by the fluid is called *buoyancy force*. This buoyancy force is always vertically upwards, and has the following characteristics.

1. The buoyancy force is equal to the weight of the fluid displaced by the solid body.
2. The buoyancy force acts through the centre of gravity of the displaced volume, called the *centre of buoyancy*.
3. A floating body displaces a volume of fluid whose weight is equal to the weight of the body.

2.4.1 Stability

A submerged body is stable if the centre of gravity of the body lies below the centre of buoyancy.

For a floating body the stability depends upon the type of couple that is formed for small angular displacements. For a body shown in Fig. 2.7(a) the centre of gravity is G and the centre of buoyancy is B. Initially it is stable with G above B. Figure 2.7 (b) shows the same body with a small displacement. If B′ is the new centre of buoyancy a vertical from B′ intersects the line of symmetry through



(a)

(b) Stable equilibrium
(*M* is above *G*)

Fig. 2.7

G at M. M is known as the *meta centre*. If M is above G, then MG the metacentric height is positive and the equilibrium is *stable*. If M is below G, MG is negative and equilibrium is *unstable*. The *metacentric height* MG is independent of magnitude of angular rotation (so long as it is small) and is given by

$$\text{MG} = \frac{I}{V} - \text{BG} \tag{2.18}$$

In this equation

I = Moment of inertia of the *water line area* about an axis through the centre of the area and perpendicular to the axis of tilt (longitudinal axis).

BG = Vertical distance between the centre of gravity and centre of buoyancy.

V = Volume of the fluid displaced by the body.

If M coincides with G, MG is zero, the body is said to be in *neutral equilibrium*.

2.5 RIGID BODY MOTION

When a fluid mass in a container is subjected to a motion such that there is no relative motion between the particles, such a motion is known as *rigid body motion*. The motion can be either translation or rotation at constant acceleration or a combination of both. As there is no relative motion there is no shear stress in such a motion and the pressure distribution is similar to that in fluids at rest, of course modified by the combined action of gravity and fluid acceleration.

2.5.1 Translation

If a container with a fluid is given a translation (a linear motion) with a uniform acceleration the piezometric head will have a gradient in the direction of motion.

If the motion is in the x-direction with a constant acceleration a_x then

$$-\frac{\mathrm{d}h}{\mathrm{d}x} = \tan\theta = \frac{a_x}{g} \tag{2.19}$$

where $h = (p/\gamma + z)$ = piezometric head above datum

θ = Inclination of hydraulic grade line.

= Inclination of water surface, measured clockwise with respect to the x-direction.

Thus, if a vessel containing a liquid is given an acceleration a_x in x-direction (Fig. 2.8.) the surface will back up against the farthest side, i.e. it will have increasing depth in (–x) direction.

If a closed tank without a free surface is involved, an imaginary free surface equivalent to the piezometric head line can be considered. This piezometric head line will be inclined to the x-direction such that

$$\boxed{\tan\theta = a_x/\text{g}}$$

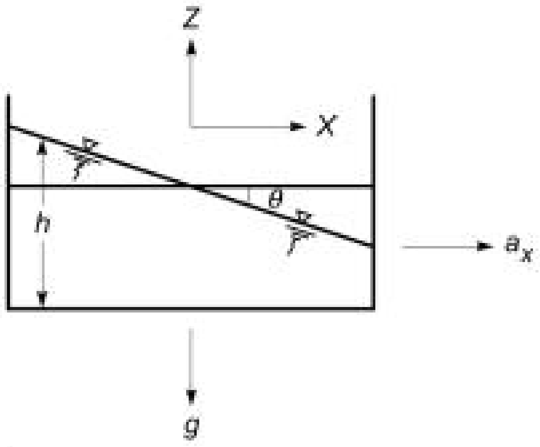


Fig. 2.8

It follows from the above that if acceleration is solely in the vertical direction (+ z direction) then $a_x = 0$ and tan $\theta = 0$. This means that the liquid surface will remain horizontal. However, the pressure p_h at any depth h below the free surface will now be

$$\boxed{p_h = \gamma h\left(1 + \frac{a_z}{\text{g}}\right)} \tag{2.20}$$

In this a_z = vertical acceleration in + z direction (if the acceleration is vertically downwards, a_z is taken as negative).

In vertical acceleration the liquid suffers an apparent gravity equal to (g + a_z).

If the acceleration is a_s in any direction s, then the components a_x and a_z in x- and z-directions are considered. The fluid surface will now have an inclination tan θ given by

$$\boxed{-\frac{\mathrm{d}h}{\mathrm{d}x} = \tan\theta = \frac{a_x}{(\text{g} + a_z)}} \tag{2.21}$$

2.5.2 Rigid Body Rotation

When a vessel containing a liquid with a free surface is rotated about an axis, the free surface will be a paraboloid of revolution given by

$$\boxed{y = \frac{\omega^2 r^2}{2\text{g}}} \tag{2.22}$$

where ω = angular velocity

y = height of the free surface above the vertex at a radial distance r from the axis.

At any two points r_1 and r_2 from the axis

$$(y_2 - y_1) = \frac{\omega^2}{2\text{g}}(r_2^2 - r_1^2)$$

Since $\omega r = V$ = tangential velocity.

$(y_2 - y_1) = \Delta y$ = difference in the liquid surface elevation between the points 2 and 1 (Fig. 2.9)

$$= \frac{V_2^2}{2\text{g}} - \frac{V_1^2}{2\text{g}} = \Delta\,(\text{V}^2/2\text{g})$$

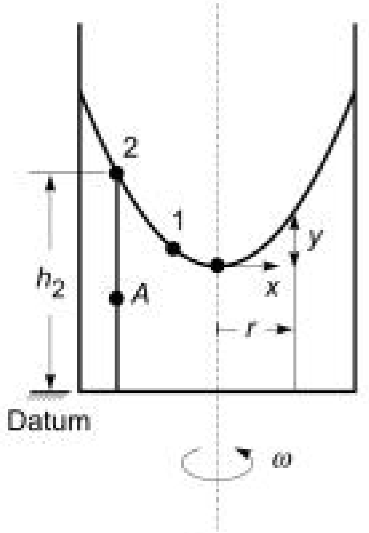


Fig. 2.9

= difference in the velocity head at these two points

The pressure distribution in any vertical line at a radial distance r will, however, remain hydrostatic. At point 2,

$h_2 = \frac{p_A}{\gamma} + z_A$ for all values of A on this vertical line.

If the free surface does not exist, the piezometric head will follow the relation for y (Eq. 2.22) as:

$$\boxed{(h - h_0) = \frac{\omega^2 r^2}{2g}}$$

where h = piezometric head above a datum at any radial distance r from the axis
h_0 = value of h at $r = 0$, i.e. on the axis
ω = angular velocity.

The piezometric head $h = \left(\frac{p}{\gamma} + z\right)$ will vary with r as a paraboloid of revolution and this surface can be considered as an imaginary liquid surface.

The volume of a paraboloid of revolution is one half the volume of the circumscribing cylinder.

WORKED EXAMPLES

A. Measurement of Pressure

2.1 Find the pressure represented by a column of
(i) 10 cm of water.
(ii) 5 cm of oil of relative density 0.75.
(iii) 2 cm of mercury.

Solution:

Pressure $p = \gamma h$

(i) For water column: $\gamma = 9790$ N/m^3

$$p = 9790 \times \frac{10}{100} = \mathbf{979\ N/m^2}$$

(ii) For the oil column: $\gamma = 0.75 \times 9790$
$= 7342.5$ N/m^3

$$p = 7342.5 \times \frac{5}{100} = \mathbf{367.13\ N/m^2}$$

(iii) For mercury column: $\gamma = 13.6 \times 9790$
$= 133144$ N/m^3

$$p = 133144 \times \frac{2}{100} = \mathbf{2662.9\ N/m^2}$$

2.2 Find the absolute pressure at a depth of 5 m below the surface of a liquid of relative density 0.85. The barometer reading on the surface is 750 mm of mercury.

Solution:

For the liquid: $\gamma = 0.85 \times 9790 = 8321.5$ N/m^3

Atmospheric pressure $= 0.750 \times 13.6 \times 9790$
$= 99858$ N/m^2

Absolute pressure in the liquid at 5.0 m depth
= pressure due to 5 m of liquid + Atmospheric pressure
$= (5 \times 8321.5) + 99858$
$= 141465.5$ N/m^2 = **141.466 kPa**

2.3 A 6 m deep tank contains 4 m of water and 2 m of oil of relative density 0.88. Determine the pressure at bottom of the tank (Refer Fig 2.10).

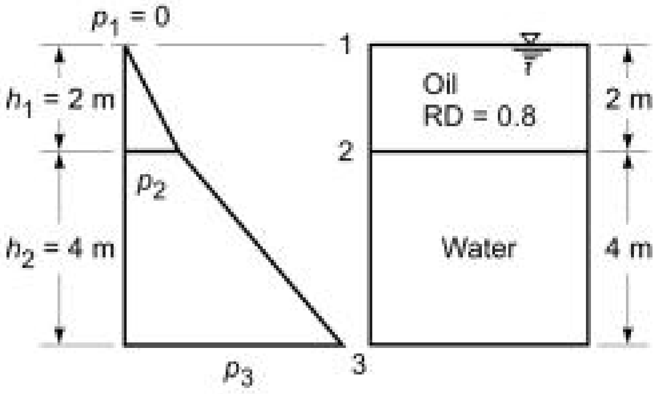


Fig. 2.10

Solution:

First determine the pressure at the oil-water interface.

$p_2 = p_1$ + pressure due to 2 m of oil
$= p_1 + (\gamma_0 \times 2)$
$p_1 = 0,\ \gamma_0 = 0.88 \times 9790 = 8615.2$ N/m^3

$$\therefore \qquad p_2 = 8615.2\ (2) = 17230.4\ \text{N/m}^2$$

For water $\gamma_w = 9790\ \text{N/m}^3$

$$\begin{aligned} p_3 &= p_2 + \text{pressure due to 4 m of water} \\ &= 17230.4 + (9790 \times 4) \\ &= 56390.4\ \text{N/m}^2 = \mathbf{56.390\ kPa} \end{aligned}$$

2.4 Assuming an isothermal process, find the pressure at an elevation of 2500 m above sea level. The mass density and atmospheric pressure at an elevation of 500 m above sea level are known to be 1.1677 kg/m^3 and 95480 Pa. What is the density of air at that level?

Solution: For an isothermal atmosphere, the pressure distribution is given by

$$\frac{p_2}{p_1} = \exp\left[-\frac{g(Z_2 - Z_1)}{RT_0}\right]$$

Since $\dfrac{p_1}{\rho_1} = RT_0$

$$\frac{p_2}{p_1} = \exp\left[-\frac{\rho_1 g(Z_2 - Z_1)}{p_1}\right]$$

$$p_2 = 95480 \exp\left[-\frac{(1.1677 \times 9.81) \times 2000}{95480}\right]$$

$$= 75111\ \text{Pa}$$

$$\frac{p_2}{\rho_2} = \frac{p_1}{\rho_1}$$

$$\therefore \qquad \rho_2 = \frac{p_2}{p_1}\rho_1$$

$$= \frac{75111}{95480} \times 1.1677 = \mathbf{0.9186\ kg/m^3}$$

2.5. Calculate the atmospheric pressure and density at an elevation of 3500 m above sea level, given the density and atmospheric pressure at an elevation of 1000 m above sea level to be 1.1120 kg/m^3, and 89,890 Pa respectively. Assume the process to be adiabatic.

Solution: For an adiabatic atmosphere

$$\frac{p_2}{p_1} = \left[1 - \left(\frac{k-1}{k}\right)\frac{g(Z_2 - Z_1)}{RT_1}\right]^{\frac{k}{(k-1)}}$$

since $\dfrac{p_1}{\rho_1} = RT_1$

$$\frac{p_2}{p_1} = \left[1 - \left(\frac{k-1}{k}\right)\frac{(g\rho_1)(Z_2 - Z_1)}{p_1}\right]^{(k/k-1)}$$

k is assumed as equal to 1.4. Hence

$$\frac{k}{(k-1)} = \frac{1.4}{0.4} = 3.5$$

$$p_2 = 89{,}890\left[1 - \left(\frac{1.4-1}{1.4}\right)\frac{(9.81 \times 1.1120)\ (3500 - 1000)}{89{,}890}\right]^{3.5}$$

$$= 89{,}890\ [\ 1 - 0.086683]^{3.5}$$

$$= 65446\ \text{pa} = 65.446\ \text{kPa}$$

$$\frac{p_1}{\rho_1^k} = \frac{p_2}{\rho_2^k}$$

$$\therefore \qquad \rho_2 = \left(\frac{p_2}{p_1}\right)^{1/k}\rho_1$$

$$= \left(\frac{65446}{89890}\right)^{1/1.4}(1.1120) = \mathbf{0.88665\ kg/m^3}$$

2.6 Determine the pressure difference between points A and B, for the set up shown in Fig. 2.11

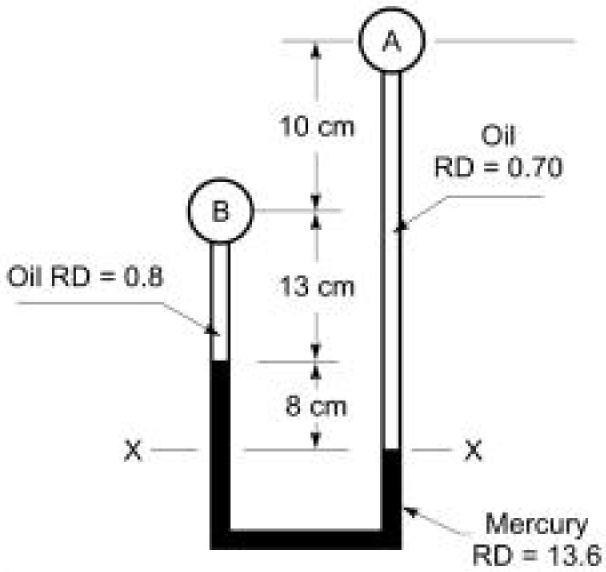


Fig. 2.11

Solution: Equating the pressures on both the limbs at the horizontal plane XX (Unit weight of water is taken as 9790 N/m^3):

$$\begin{aligned} p_A + [0.10 + 0.13 + 0.08] \times \gamma_A \\ = p_B + 0.13\ \gamma_B + 0.08\ \gamma_m \end{aligned}$$

$$\gamma_A = 0.70 \times 9790 = 6853\ \text{N/m}^3$$

$$\gamma_B = 0.80 \times 9790 = 7832\ \text{N/m}^3$$

$$\gamma_m = 13.6 \times 9790 = 133144\ \text{N/m}^3$$

$$(p_A - p_B) = -0.31 \times 6853 + 0.13 \times 7832 + 0.08 \times 133144$$

$$= 9545\ \text{Pa} = \mathbf{9.545\ kPa}$$

2.7 For the manometer shown in Fig. 2.12 calculate the pressure difference between points M and N.

Solution: Equating the pressures at both the limbs along the horizontal plane XX

$$p_m - \gamma_w \cdot (0.06 + 0.035) = p_N - \gamma_w\ (0.12 + 0.06) - \gamma_0(0.035)$$

γ_w = unit wt. of water
= 9790 N/m^3 (assumed)

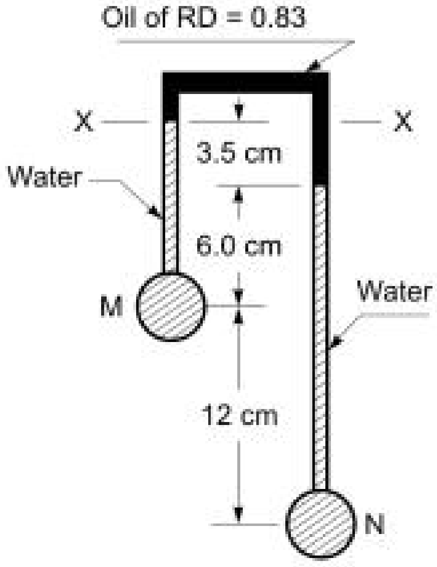


Fig. 2.12

$$\gamma_0 = \text{unit wt. of oil} = 0.83 \times 9790 = 8125.7 \text{ N/m}^3$$

$$\therefore \quad (p_M - p_N) = 9790 \times 0.095 - 9790 \times 0.18 - 8125.7 \times 0.035$$

$$= -1116.5 \text{ N/m}^2 = -1.1165 \text{ kPa}$$

Pressure at N is larger than at M by **1.117 kPa**

2.8 Two pressure tanks are built one inside the other (Fig. 2.13). A bourdon gauge M connected to the inner tank reads 20 kPa. Another bourdon gauge N connected to the outer tank reads 35 kPa. An aneroid barometer reads 750 mm of mercury. Calculate the absolute pressure recorded at M and N in of mercury.

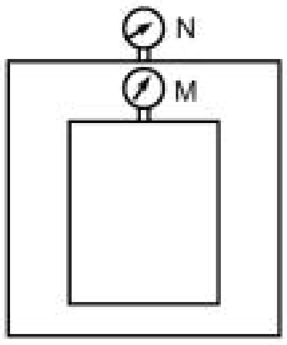


Fig. 2.13

Solution: A bourdon gauge records the gauge pressure relative to the pressure of the medium surrounding the tube.

Local atmospheric pressure is measured by the aneroid barometer.

In the present case local atmospheric pressure outside the gauge N = 750 mm.

Hence absolute pressure at N =

$$p_N(\text{abs}) = 750 + \left(\frac{3.5}{13.6 \times 9.79} \times 1000\right)$$

$$= \mathbf{1012.9 \text{ mm of mercury (abs)}}$$

The gauge M reads relative to its surrounding pressure of 1012.9 mm of mercury (abs).

Hence, $p_M(\text{abs}) = 1012.9 + \left(\dfrac{20 \times 1000}{13.6 \times 9.79}\right)$

$$= \mathbf{1163.1 \text{ mm of mercury (abs)}}$$

2.9 The tank in Fig. 2.14 is closed at top and contains air at a pressure p_A. Calculate the value of p_A for the manometer readings shown.

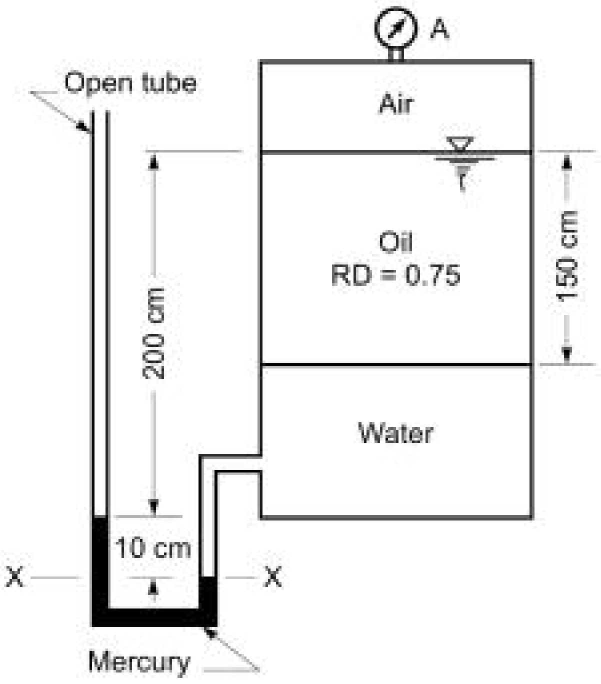


Fig. 2.14

Solution: Considering the pressure at the horizontal plane X–X:

$$p_A + 1.5 \times \gamma_0 + (2.0 - 1.5)\,\gamma_w + 0.10\,\gamma_w = 0.10\,\gamma_m$$

$$\gamma_o = \text{specific wt. of oil} = 0.75 \times 9790 = 7342.5 \text{ N/m}^3$$

$$\gamma_w = \text{specific wt. of water} = 9790 \text{ N/m}^3$$

$$\gamma_m = \text{specific wt. of mercury} = 13.6 \times 9790 = 133144 \text{ N/m}^3$$

$$\therefore \quad p_A = (0.10 \times 133144) - 0.50 \times 9790 - 1.5 \times 7342.5 - 0.1 \times 9790$$

$$= -3573.4 \text{ Pa} = \mathbf{-3.573 \text{ kPa}}$$

2.10 Determine the pressure difference $(p_M - p_N)$ when the manometer indicates as shown in Fig. 2.15.

Solution: Equating the pressure across the horizontal plane X-X:

$$p_M + 0.20\,\gamma_o + 0.16\,\gamma_o = p_N + 0.15\,\gamma_o + 0.15\,\gamma_m - 0.12\,\gamma_o + 0.12\,\gamma_m + 0.04\,\gamma_m$$

$$\gamma_o = \text{specific weight of oil} = 0.8 \times 9790 = 7832 \text{ N/m}^3$$

$$\gamma_m = \text{specific weight of mercury} = 13.6 \times 9790 = 133144 \text{ N/m}^3$$

$$(p_M - p_N) = -0.36 \times 7832 + 0.15 \times 7832 - 0.12 \times 0.7832 + (0.15 + 0.12 + 0.04) \times 133144$$

$$= -2584.6 + 41274.6 = 38690 \text{ Pa}$$

$$= \mathbf{38.690 \text{ kPa}}$$

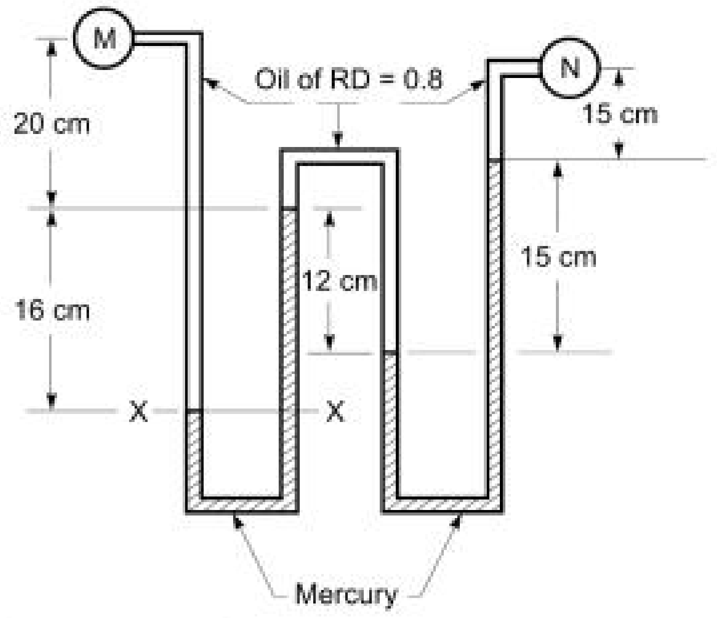


Fig. 2.15

2.11 A single leg mercury manometer (Fig. 2.16) is often used to measure differential pressure through reading of a single column only. A constant zero of the scale is adopted for all pressures.

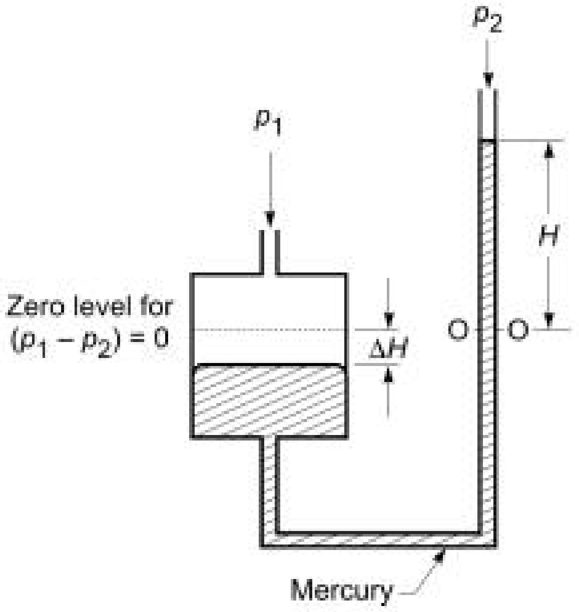


Fig. 2.16

(a) What should be the diameter of the cistern if the indicated mercury column H is to be within 0.5% the true height corresponding to a pressure differential of $(p_2 - p_1)$.

(b) If the ratio of the areas of cistern to that of the tube is 1/500, what percentage error is involved in the difference in pressure by reading only limb of height H?

Solution: The column height is measured with reference to zero level corresponding to $(p_1 - p_2) = 0$. When a pressure differential is applied the mercury in the cistern will go down by ΔH and the true column height is $(\Delta H + H)$.

(a) It is required that $H + \Delta H \le 1.005\ H$

$$\Delta H \le 0.005\ H$$

$$\frac{\Delta H}{H} \le 0.005$$

Since (Area of cistern) × ΔH = (area of tube) × H

For minimum area of cistern A_c; $\Delta H\ A_c = aH$

and $$\frac{\Delta H}{H} = 0.005 = \frac{a}{A_c} = \left(\frac{d}{D_c}\right)^2$$

Hence D_c = Minimum diameter of cistern

$$= d/\sqrt{0.005} = \mathbf{14.142\ d}$$

(b) If $\dfrac{a}{A_c} = 1/500$

then $$\frac{\Delta H}{H} = \frac{1}{500}$$

Percentage error in

$$H = \frac{\Delta H}{H} \times 100 = \frac{1}{500} \times 100 = \mathbf{0.2\%}$$

2.12 A glass tube of uniform bore is bent into the form of a square of sides a and filled with equal amounts of three immiscible liquids of densities ρ_1, ρ_2 and ρ_3. It is known that $\rho_1 < \rho_2 < \rho_3$. If the tube arrangement is placed in a vertical plane (i.e. two sides vertical) and if one of the vertical sides is completely filled with the liquid of density ρ_2.

(a) Show that

$$\frac{1}{3}(2\rho_3 + \rho_1) > \rho_2 > \frac{1}{3}(\rho_3 + 2\rho_1)$$

(b) If the relative densities of the first and third liquids are 1.0 and 1.2 respectively, find the range of the relative densities of the second liquid which makes the above arrangement possible.

Solution: Referring to Fig. 2.17, let E, F and G be the interfaces.

Let EA = x.

Then DE = DA − EA = $(a - x)$

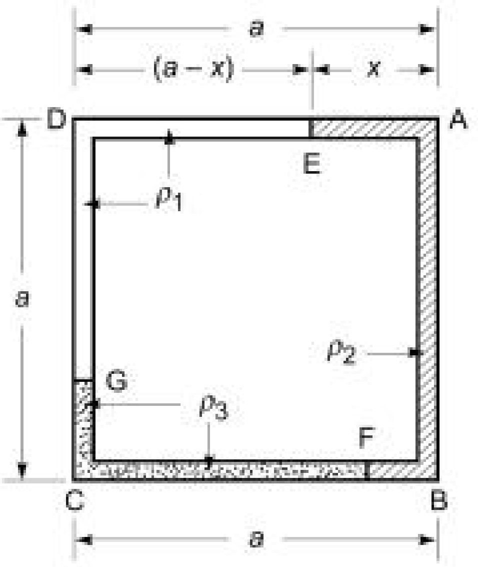


Fig. 2.17

Total tubing length = $4a$

Length of each liquid = $\frac{4}{3}a$

Hence for liquid 1:

$$EG = \frac{4}{3}a$$

$$DG = \frac{4}{3}a - (a - x) = \frac{1}{3}a + x$$

For liquid 3:

$$GC = a - \left(\frac{1}{3}a + x\right) = \frac{2}{3}a - x$$

$$FB = a - \left(\frac{2}{3}a + x\right) = \frac{1}{3}a - x$$

$$\left(\text{Check: FB + BA + AE} = \frac{1}{3}a - x + a + x = \frac{4}{3}a\right)$$

At the Interface F: The pressure balance is

Pressures of (Column DG + Column GC) = Pressure of column AB

$$\rho_1 g\left(\frac{1}{3}a + x\right) + \rho_3 g\left(\frac{2}{3}a - x\right) = \rho_2 g\, a$$

$$x(\rho_3 - \rho_1) = \frac{1}{3}a\,(2\rho_3 + \rho_1 - 3\rho_2)$$

$$x = \frac{1}{3}a\,(2\rho_3 + \rho_1 - 3\rho_2)/(\rho_3 - \rho_1)$$

It is known that $x > 0$ and also $x < \frac{1}{3}a$

Hence $\quad 0 < x < \frac{a}{3}$

$$\therefore \quad 0 < \frac{2\rho_3 + \rho_1 - 3\rho_2}{\rho_3 - \rho_1} < 1$$

Also since $\rho_1 < \rho_2 < \rho_3$ the denominator $(\rho_3 - \rho_1)$ is positive. Hence the numerator is

$$0 < (2\rho_3 + \rho_1 - 3\rho_2) < 1$$

or $(2\rho_3 + \rho_1) > 3\rho_2$

$$\text{or} \quad \rho_2 < \frac{1}{3}\,(2\rho_3 + \rho_1) \qquad \text{(i)}$$

$$\text{Also since} \quad \frac{2\rho_3 + \rho_1 - 3\rho_2}{\rho_3 - \rho_1} < 1$$

$$2\rho_3 + \rho_1 - 3\rho_2 < (\rho_3 - \rho_1)$$

$$\text{or} \quad 3\rho_2 < \rho_3 + 2\rho_1$$

$$\rho_2 > \frac{1}{3}(\rho_3 + 2\rho_1) \qquad \text{(ii)}$$

Hence from inequalities (i) and (ii)

$$\mathbf{\frac{1}{3}(2\rho_3 + \rho_1) > \rho_2 > \frac{1}{3}\,(\rho_3 + 2\rho_1)}$$

(b) $\rho_1 = 1.0$, $\rho_3 = 1.2$

$$\text{Hence} \quad \rho_2 > \frac{1}{3}\,(\rho_3 + 2\rho_1)$$

$$\text{i.e.} \quad > \frac{1}{3}\,(1.2 + 2 \times 10)$$

$$> 1.0667$$

$$\text{Also} \quad \rho_2 < \frac{1}{3}\,(2\rho_3 + \rho_1)$$

$$< \frac{1}{3}\,(2 \times 1.2 + 1.0)$$

$$< 1.1333$$

Hence $\quad \mathbf{1.0667 < \rho_2 < 1.1333}$

B. Forces On Plane Surfaces

2.13 A rectangular plate 0.6 m wide and 1.2 m deep lies within a water body such that its plane is inclined at 45° to the horizontal and the top edge is 0.70 m below the water surface. Determine the total pressure force on one side of the plate and the location of the centre of pressure (Refer Fig. 2.18).

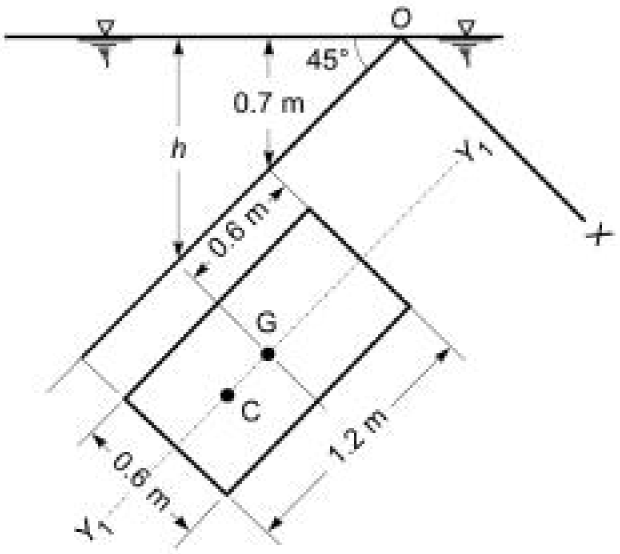


Fig. 2.18

Solution: Referring to Fig. 2.18

$\bar{h}$ = depth of CG of the plate = 0.7 + 0.6 sin 45°
= 1.1243 m

$$\text{Total pressure force } F = \gamma_A \bar{h}$$

$$= 9790 \times (0.6 \times 1.2) \times 1.1243$$

$$= 7924.9 \text{ N} = \mathbf{7.925\ kN}$$

Centre of pressure:

Because of symmetry $x_p = \bar{x}$, i.e. C lies on the axis Y_1Y_1 passing through the CG of the area.

$$y_p = \bar{y} + \frac{I_{gg}}{A\,\bar{y}}$$

$$\bar{y} = \frac{\bar{h}}{\sin 45^\circ} = \frac{1.1243}{1/\sqrt{2}} = 1.59 \text{ m}$$

$$I_{gg} = \frac{bd^3}{12} = \frac{0.6 \times (1.2)^3}{12} = 0.0864$$

$$y_p = 1.59 + \frac{0.0864}{(0.6 \times 1.2) \times (1.59)}$$

$$= \mathbf{1.665\ m}$$

[y_p is measured along the plane of the area from the axis OX]

2.14 A plane area in the form of a right angle triangle of height h is immersed vertically in water with its vertex at the water surface. Calculate the total force on one side of the triangular plane and the location of the centre of pressure.

Solution:

$$\text{Total force } F = \gamma A \bar{h}$$

$$= \gamma\left(\frac{1}{2}bh\right) \times \frac{2h}{3} = \frac{\gamma}{3}bh^2$$

To determine the centre of pressure, consider the axis OX and OY as shown in Fig. 2.19. For an element of width x and height dy at a depth y,

$$\frac{b}{h} = \frac{x}{y} \quad \text{or} \quad y = \frac{h}{b}x$$

$$\mathrm{d}y = \frac{h}{b}\mathrm{d}x$$

Taking moments of force on the element about OY and intergrating

$$F \cdot x_p = \int_0^h \gamma\, y(x\,\mathrm{d}x)\frac{x}{2}$$

$$\frac{\gamma}{3}bh^2 x_p = \int_0^b \gamma \frac{h}{b}x\, x \frac{h}{b}\mathrm{d}x \frac{x}{2}$$

$$= \int_0^b \frac{\gamma h^2}{2b^2}x^3\,\mathrm{d}x$$

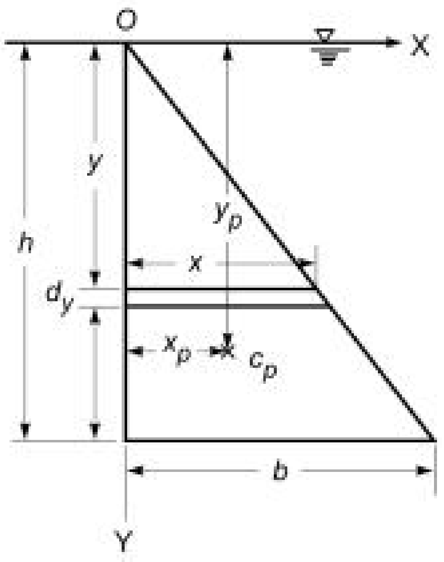


Fig. 2.19

$$= \frac{\gamma h^2 b^2}{8}$$

$$\mathbf{x_p = \frac{3}{8}b}$$

Similarly for y_p:

$$\frac{\gamma}{3}bh^2 y_p = \int_0^h \gamma\, y(x\,\mathrm{d}y)\,y = \int \gamma y^2\left(\frac{by}{h}\right)\mathrm{d}y$$

$$= \frac{\gamma b}{h}\int y^3\,\mathrm{d}y = \frac{\gamma b}{h}\frac{h^4}{4}$$

$$\mathbf{y_p = \frac{3}{4}h}$$

2.15 A circular plate of diameter 0.75 m is immersed in a liquid of relative density 0.80 with its plane making an angle of 30° with the horizontal. The centre of the plate is at a depth of 1.50 m below the free surface. Calculate the total force on one side of the plate and the location of the centre of pressure.

Solution:

Referring to Fig. 2.20,

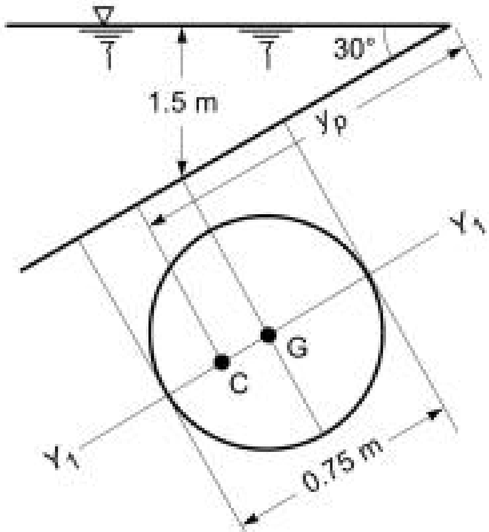


Fig. 2.20

$$\gamma = 0.80 \times 9790 = 7832\ \text{N/m}^3$$

$$\bar{h} = 1.5\ \text{m}$$

F = Force on one side of the plate

$$= \gamma A \bar{h}$$

$$= 7832 \times \left[\frac{\pi}{4} \cdot (0.75)^2\right] \times 1.5$$

$$= \mathbf{5190\ N}$$

By symmetry $x_p = \bar{x}$, i.e. the centre of pressure lies on the Y_1Y_1 axis through the CG of the area. To find y_p:

$$y_p = \bar{y} + \frac{I_{gg}}{A\bar{y}}$$

$$\bar{y} = 1.5/\sin 30° = 3.0\ \text{m}$$

$$I_{gg} = \frac{\pi}{64} D^4 = \frac{\pi}{64} \times (0.75)^4$$

$$y_p = 3.00 + \frac{\frac{\pi}{64}(0.75)^4}{\frac{\pi}{4} \times (0.75)^2 \times (3.0)}$$

$$= \mathbf{3.0117\ m}$$

2.16 A vertical isosceles triangular gate with its vertex up has a base width of 2 m and a height of 1.5 m. If the vertex of the gate is 1 m below the free water surface, find the total pressure force and the position of the centre of pressure on one side of the plate.

Solution:

Total pressure force $F = \gamma A \bar{h}$

$$= 9790 \times \left(\frac{1}{2} \times 2 \times 1.5\right)\left(1 + \frac{2}{3} \times 1.5\right)$$

$$= \mathbf{29370\ N}$$

Referring to Fig. 2.21,

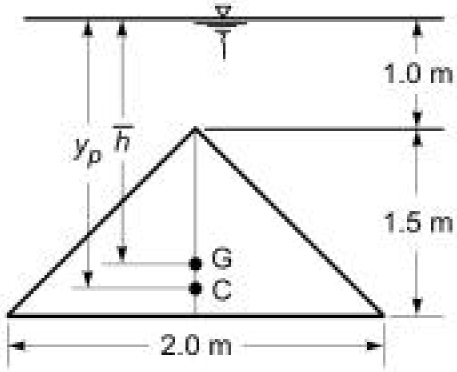


Fig. 2.21

By symmetry $x_p = \bar{x}$, i.e. the centre of pressure lies on the vertical axis passing through the CG of the plate.

$$y_p = \bar{y} + \frac{I_{GG}}{A\bar{y}}$$

Since the plane is vertical, $\bar{y} = \bar{h} = 2.0$ m

$$A = \frac{1}{2} \times 2 \times 1.5 = 1.5\ \text{m}^2$$

$$I_{GG} = \frac{1}{36} \times bh^3$$

$$= \frac{1}{36} \times 2.0\,(1.5)^3$$

$$= 0.1875\ \text{m}^4$$

$$y_p = 2.0 + \frac{0.1875}{1.5 \times 2.0}$$

$$= \mathbf{2.0625\ m}$$

2.17 A cubical tank has sides of 1.5 m. It contains water in the lower 0.6 m depth. The upper remaining part is filled with oil of relative density 0.9. Calculate for one vertical side of the tank (i) the pressure force and (ii) position of the centre of pressure.

Solution:

$\gamma_{\text{oil}} = 0.9 \times 9790 = 8811$ N

$\gamma_{\text{water}} = 9790$ N

p_0 = pressure at the top = 0

p_i = pressure at the interface = $8811 \times 0.9 = 7930$ N/m^2

p_b = pressure on the bottom = $7930 + 9790 \times 0.6 = 13804$ N/m^2

F_1 = Force on the top 0.9 m of a side.

$$= \frac{1}{2} \times 7930 \times (0.9 \times 1.5) = 5353\ \text{N}$$

acting at $\left(\frac{2}{3} \times 0.9\right) = 0.6$ m below the surface.

F_2 = Part of force on the bottom 0.6 m of a side $= 7930 \times 0.6 \times 1.5 = 7137$ N

acting at $\left(0.9 + \frac{0.6}{2}\right) = 1.2$ m below the surface.

F_3 = remaining part of force on the bottom 0.6 m of a side

$$= \frac{1}{2} \times (13804 - 7930) \times 0.6 \times 1.5$$

$$= \frac{1}{2} \times 5874 \times 0.6 \times 1.5 = 2643\ \text{N}$$

acting at $\left(0.9 + \frac{2}{3} \times 0.6\right) = 1.3$ m below the surface.

Total force $= F_1 + F_2 + F_3 = 15133$ N

$$= \mathbf{15.133\ kN}$$

Centre of pressure $= y_p$

$$= \frac{(5353 \times 0.6) + (7137 \times 1.2) + (2643 \times 1.3)}{15133}$$

$$= \mathbf{1.005\ m}$$

By symmetry $\bar{x} = x_p$, i.e. the centre of pressure acts on the vertical passing through the CG of the side.

2.18 A trapezoidal plate of parallel sides *a* and 2*a* and height *h* is immersed vertically in water with its side of length *a* horizontal and topmost. The top edge is at a depth *h* below the water surface. Determine the total force on one side of the plate and the location of the centre of pressure.

Solution: The trapezium can be considered to be made of a rectangle (width $a \times$ height h) and a triangle (base $a \times$ height h) as in Fig. 2.22.

1. Pressure force F_1 on rectangular part

$$F_1 = \gamma(ah)\left(h + \frac{h}{2}\right) = \frac{3}{2}\gamma a h^2$$

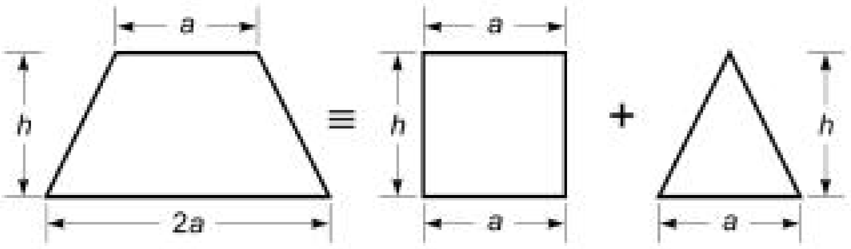


Fig. 2.22

Centre of pressure of force F_1

$$h_{c1} = \bar{y} + \frac{I_{gg}}{A\bar{y}} = \left(h + \frac{h}{2}\right) + \frac{ah^3/12}{(ah)\left(h + \frac{h}{2}\right)}$$

$$= \frac{3h}{2} + \frac{h}{12} \times \frac{2}{3} = \frac{14}{9}h$$

2. Pressure force on the triangular part:

$$F_2 = \gamma\left(\frac{ah}{2}\right)\left(h + \frac{2}{3}h\right) = \gamma ah^2\left(\frac{5}{6}\right)$$

Centre of pressure of F_2

$$h_{c2} = \left(h + \frac{2}{3}h\right) + \frac{ah^3/36}{\left(\frac{ah}{2}\right)\left(h + \frac{2}{3}h\right)}$$

$$= \frac{50+1}{30}h = \frac{51}{30}h$$

Total thrust $F = F_1 + F_2$

$$F = \gamma ah\left[\left(h + \frac{h}{2}\right) + \frac{1}{2}\left(h + \frac{2}{3}h\right)\right]$$

$$= \gamma ah^2\left(\frac{7}{3}\right) = \mathbf{\frac{7}{3}\gamma ah^2}$$

Centre of pressure of total force F

$$h_c = \frac{F_1 h_{c1} + F_2 h_{c2}}{F}$$

$$= \frac{\left(\frac{3}{2}\gamma ah^2 \times \frac{14}{9}h\right) + \left(\gamma ah^2 \times \frac{5}{6}\right)\left(\frac{51}{30}h\right)}{\frac{7}{3}\gamma ah^2}$$

$$= \frac{\frac{7}{3}h + \frac{17}{12}h}{7/3}$$

$$\mathbf{h_c = \frac{45}{28}h}$$

2.19 Figure 2.23 shows a rectangular flash board AB which is 4.5 m high and is pivoted at C. What must be the maximum height of C above B so that the flash board will be on the verge of tipping when the water surface is at A?

Solution: For critical stability, the centre of pressure must be at C.

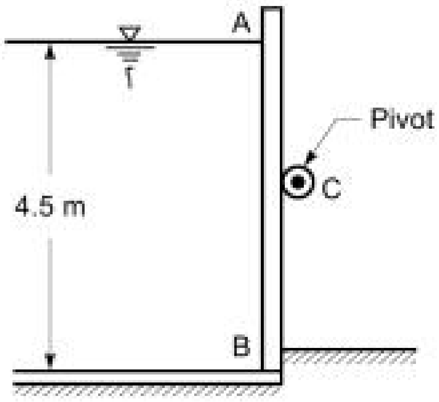


Fig. 2.23

Thus h = height of C above B.

$$= \frac{4.5}{3} = \mathbf{1.5\ m}$$

2.20 If the pivot of the flash board in Example 2.19 is at a height h = 1.5 m, determine the reactions at B and C when the water surface is 4.0 m above B.

Solution: Consider unit width of the flash board

$$\text{Total force } F = 9.790 \times \frac{4}{2} \times (4 \times 1)$$

$$= 78.32 \text{ kN/m}$$

acting at 4/3 m above B.

Let R_B and R_C be the reactions at B and C respectively. Taking moments of forces about C,

$$78.32 \times \left(1.5 - \frac{4}{3}\right) - R_B \times 1.5 = 0$$

$$\mathbf{R_B = 8.7\ kN/m}$$

$$R_C = F_1 - R_B = 78.32 - 8.70$$

$$= \mathbf{69.62\ kN/m}$$

2.21 A closed tank as shown in Fig. 2.24 contains water which is pressurized with a pressure of 10 kN on the top. A rectangular gate MN of width 0.6 m and height 1.5 m is provided on the inclined face. Calculate the total force on the gate and the location of the centre of pressure.

Solution:

From the Fig. 2.24

h_1 = depth of M below free surface = 2.0 m

h_2 = depth of N below free surface

= 2.0 + 1.5 sin 45° = 3.06 m.

$\bar{h}_{CG}$ = 2.0 + 0.75 sin 45° = 2.53 m

$\bar{y}$ = 2.53/sin 45° = 3.578 m

The force on the gate MN can be considered to be made up of two parts [see Fig. 2.24(b)].

F_1 = force due to uniform pressure p_a acting uniformly over the surface.

= $p_a A$ = (10 × 1.5 × 0.6) = 9 kN

acting at CG of the area h_{cg} = 2.53 m

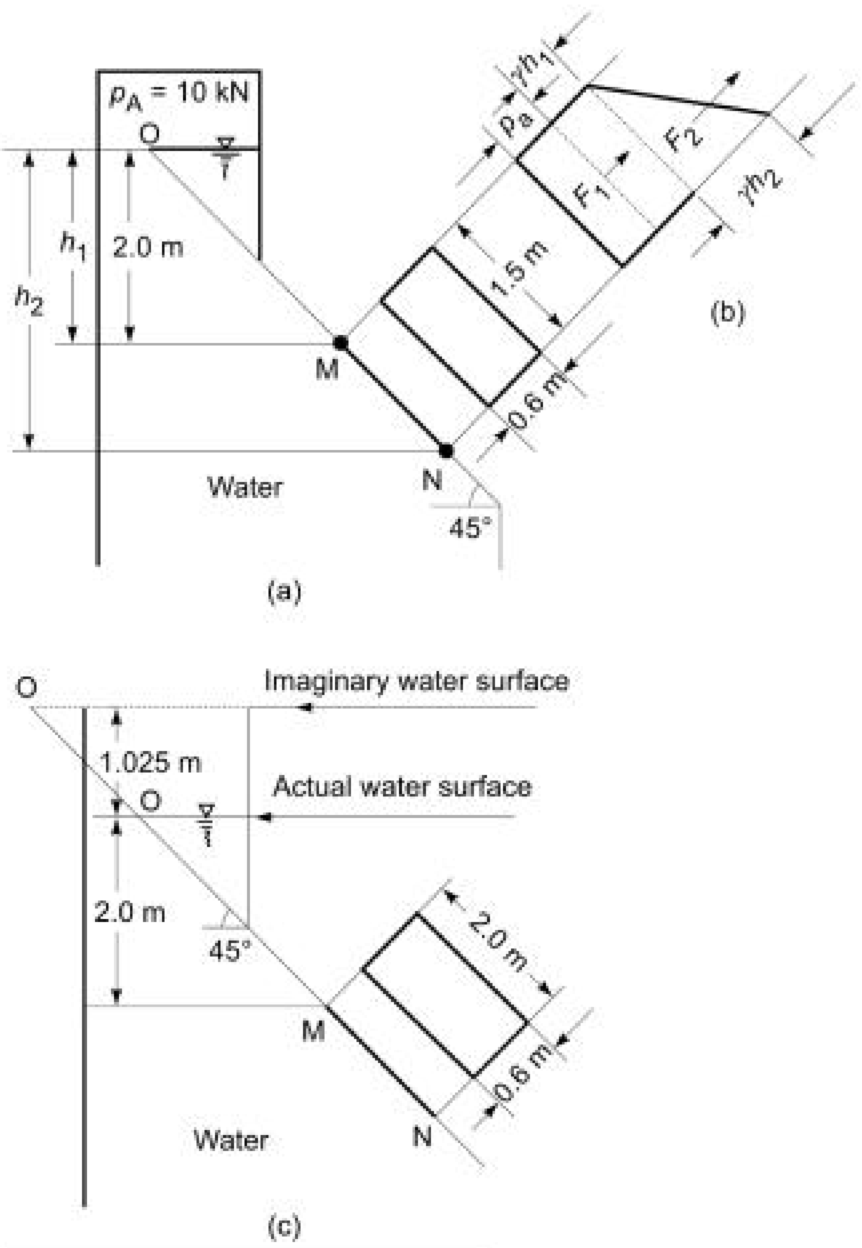

p_A = 10 kN
O
h_1
2.0 m
h_2
M
Water
N
45°
(a)
γh_1
p_a
F_2
F_1
γh_2
1.5 m
0.6 m
(b)
O
Imaginary water surface
1.025 m
O
Actual water surface
2.0 m
45°
2.0 m
M
0.6 m
Water
N
(c)

Fig. 2.24 *Example 2.21*

i.e $\bar{y}$ = 2.53/sin 45° = 3.578 m along OM axis.

F_2 = Force due to hydrostatic pressure of liquid

$= \gamma A \bar{h}$

$= 9.79 \times (1.5 \times 0.6) \times 2.53 = 22.29$ kN

This force F_2 acts at y_{c1}, where

$$y_{c1} = \bar{y} + \frac{I_{gg}}{A\bar{y}}$$

$$= 3.578 + \frac{0.6 \times (1.5)^3/12}{(0.6 \times 1.5)\,(3.578)}$$

$$= 3.6308 \text{ m}$$

h_c = vertical depth of this y_{c1}

$= 3.6308 \times \sin 45° = 2.5674$ m

Total force $F = F_1 + F_2$

$= 9 + 22.29 =$ **31.29 kN**

This force acts at y_c.

Taking moments about CG

$$F \times (y_c - \bar{y}) = F_2 \times (y_{c1} - \bar{y})$$

$$y_c - \bar{y} = \frac{22.29}{31.29} \times (3.6308 - 3.578) = 0.0376 \text{ m}$$

$$y_c = 3.578 + 0.0376 = 3.616 \text{ m}$$

Depth of centre of pressure from the water surface = h_p = 3.616 sin 45° = **2.557 m**

Alternative method:

Consider the equivalent column of water corresponding to a pressure of 10 kN

$$h_e = \frac{10.0}{9.79} = 1.0215 \text{ m}$$

Now an imaginary water surface at a height h_e above the actual water surface be considered as reference surface. Hence force on the gate

$$F = \gamma A \bar{h}$$

$$= 9.79 \times (0.6 \times 1.5) \times \left(3.0215 + \frac{15}{2}\sin 45°\right)$$

$$= 31.29 \text{ kN}$$

y'_p = location of centre of pressure on the O′M axis

$$= \bar{y}' + \frac{I_{GG}}{A\bar{y}'}$$

$$\bar{y}' = \left(\frac{3.0215}{\sin 45°} + 0.75\right) = 5.023$$

$$\frac{I_{GG}}{A\bar{y}'} = \frac{(0.6 \times (1.5)^3)/12}{(0.6 \times 1.5)\,(5.023)} = 0.0373$$

$$y'_p = 5.023 + 0.0373 = 5.060 \text{ m}$$

y_p = distance of centre of pressure from actual water surface

$= 5.060 - (1.0215/\sin 45°) = 3.616$ m

Depth of centre of pressure from actual water surface = 3.616 sin 45°

= **2.557 m**

2.22 For the system shown in Fig. 2.25 calculate the height H of oil at which the rectangular hinged gate will just begin to rotate counterclockwise.

Solution:

Force due to oil $F_1 = \gamma_0 A \bar{h}$

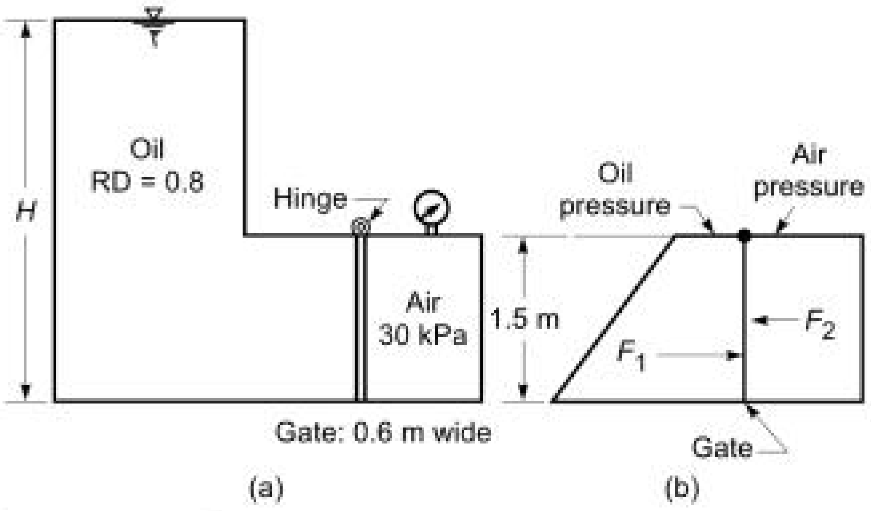

H
Oil
RD = 0.8
Hinge
Air
30 kPa
1.5 m
Gate: 0.6 m wide
(a)
Oil pressure
Air pressure
F_1
F_2
Gate
(b)

Fig. 2.25

$$\gamma_{\text{oil}} = 0.8 \times 9.79 = 7.832 \text{ kN/m}^3$$

$$A = (0.6 \times 1.5) = 0.9 \text{ m}^2$$

$$\bar{h} = (H - 1.5) + \left(\frac{1.5}{2}\right)$$

$$F_1 = 7.832 \times 0.9 \times \{(H - 1.5) + 0.75\}$$

$$= 7.0488\,H - 5.2866$$

Centre of pressure of force F_1

$$h_{c1} = \bar{h} + \frac{I_{gg}}{Ah}$$

$$= (H - 0.75) + \frac{0.6 \times (1.5)^3/12}{(0.6 \times 1.5)\,(H - 0.75)}$$

$$= (H - 0.75) + \frac{0.1875}{(H - 0.75)}$$

(2) Force due to air pressure

$$F_2 = pA$$

$$= 30 \times (0.6 \times 1.5) = 27 \text{ kN}$$

Centre of pressure of this force (below the oil surface)

$$h_{c2} = \bar{h} = (H - 0.75)$$

Taking the moments about the hinge:

$$F_1 \times [h_{c1} - (H - 1.5)] = F_2 \times [h_{c2} - (H - 1.5)]$$

$$\times \{7.0488\,H - 5.2866\}$$

$$\times \left\{(H - 0.75) - (H - 1.5) + \frac{0.1875}{(H - 0.75)}\right\}$$

$$= 27 \times \{(H - 0.75) - (H - 1.5)]\}$$

$$\{7.0488\,H - 5.2866\}\left\{0.75 + \frac{0.1875}{(H - 0.75)}\right\}$$

$$= 27 \times 0.75$$

On solving by trial and error

$$\boldsymbol{H = 4.33 \text{ m}}$$

2.23 A semicircular gate having a radius of 1.2 m and weighing 12 kN can swing about its diameter which is kept horizontal. The water surface on one side is 5.0 m above its diameter. Determine the minimum force required to keep the gate closed. The gate lies in a plane making an angle of 30° with the water surface.

Solution: In the Fig. 2.26, O_1O_2 B is the semicircular gate.

$$\bar{y} - y_0 = \text{OG} = \frac{4R}{3\pi} = \frac{4 \times 1.2}{3\pi} = 0.5093 \text{ m}$$

Since $y_0 = \dfrac{5.0}{\sin 30°} = 10.0$ m, $\bar{y} = 10.5093$ m

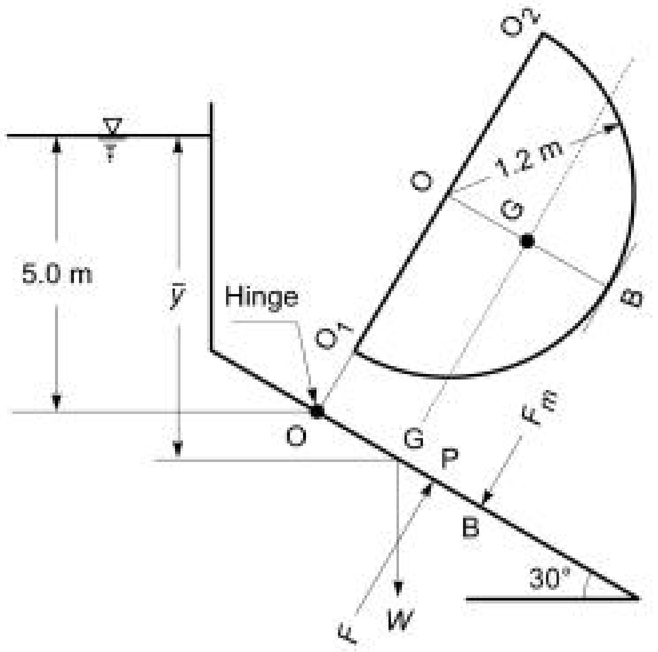


Fig. 2.26

$\bar{h}$ = depth of CG below water surface
$= 5.0 + \text{OG} \sin 30° = 5.0 + 0.5093 \sin 30°$
$= 5.2546$ m

Force on the gate (normal to the plane of the gate)

$$= F = \gamma A\bar{h} = 9.79\,\frac{(\pi \times 1.2^2)}{2} \times 5.2546$$

$$= 116.36 \text{ kN}$$

Location of centre of pressure on OB

$$y_p = \bar{y} + \frac{I_{gg}}{A\,\bar{y}} = \bar{y} + \frac{I_{0_10_2}}{A\,\bar{y}} - \frac{(\text{OG})^2}{\bar{y}}$$

$$y_p = 10.5093 + \left(\frac{\pi D^4}{128}\right)\frac{1}{\left(\dfrac{\pi D^2}{8}\right)} \times \frac{1}{10.5093} - \frac{(0.5093)^2}{10.5093}$$

Substituting $D = 2.5$ m, $y_p = 10.5218$ m

$$\text{OP} = y_p - y_o = 0.5218 \text{ m}$$

Let F_m = minimum force required to lift the gate.

By taking moments about O,

$$= W \times \text{OG} \cos 30° = F \times \text{OP} - F_m \times \text{OB}$$

$$12 \times 0.5093 \times 0.866 = 116.36 \times 0.5218 - 1.2\,F_m$$

$$5.293 = 60.717 - 1.2\,F_m$$

$$\boldsymbol{F_m = 46.19 \text{ kN}}$$

2.24 A sheet piling holds fresh water and salt water (relative density = 1.035) on either sides of it as shown in Fig. 2.27. Find the moment about the base M of the resultant force per unit length of piling.

Solution:

Consider 1 m length of piling

F_1 = force due to salt water

$$= \gamma_1 A_1 \bar{h}_1$$

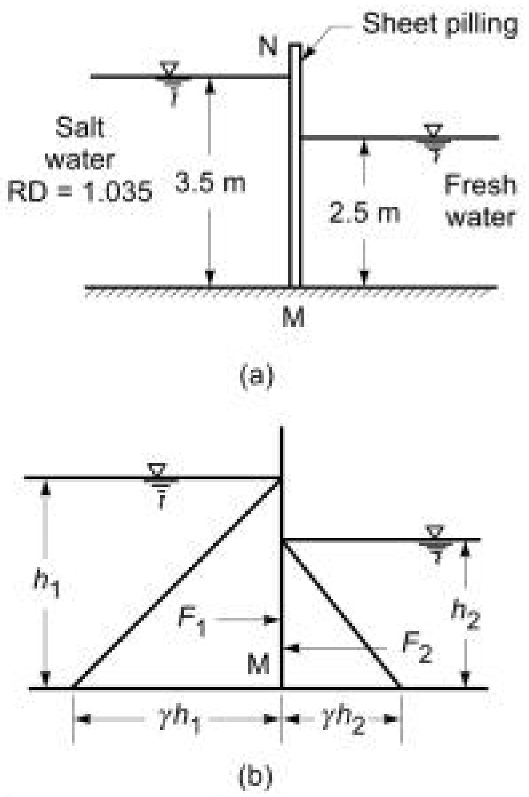


Fig. 2.27

$$= (1.035 \times 9.79)\,(3.5 \times 1.0)\left(\frac{3.5}{2}\right)$$

$$= 62.062 \text{ kN}$$

h_{c1} = Centre of pressure of this force F_1

$$= \bar{h}_1 + \frac{I_{gg}}{A_1 \bar{h}_1} = \frac{3.5}{2} + \frac{1.0 \times (3.5)^3/12}{(3.5 \times 1.0)\left(\frac{3.5}{2}\right)}$$

$$= 1.75 + 0.5833 = 2.333 \text{ m.}$$

Lever arm $a_1 = 3.5 - 2.333 = 1.167$ m

F_2 = Force due to fresh water = $\gamma_2 A_2 \bar{h}_2$

$$= (9.79)\,(2.5 \times 1)\left(\frac{2.5}{2}\right) = 30.594 \text{ kN}$$

$$h_{c2} = \bar{h}_2 + \frac{I_{gg}}{A_2 \bar{h}_2} = \frac{2.5}{2} + \frac{[1.0 \times (2.5)^3]/12}{(2.5 \times 1.0)(2.5/2)}$$

= 1.667 m. Lever arm $a_2 = 2.5 - 1.667 = 0.833$ m

Net moment about $M = M_1 = F_1 a_1 - F_2 a_2$

(Taking clockwise as positive)

$$M = (62.062 \times 1.167) - (30.594 \times 0.833)$$

$= 72.43 - 25.48 =$ **46.95 kN.m (clockwise)**

2.25 A tank of 2 m length and of cross section shown in Fig. 2.28 contains water. The tank is made of 8 mm steel plates. What is the force on the bottom due to water? What are the longitudinal tensile stresses in the side walls (AB) if (a) the tank is suspended from the top and (b) it is supported at the bottom ?

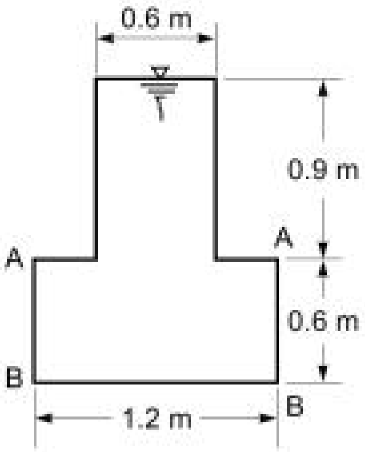


Fig. 2.28

Solution: Force on the bottom due to water

$$F_1 = \gamma A \bar{h}$$

$$= 9.79 \times (1.2 \times 2.0) \times 1.5 = 35.24 \text{ kN}$$

Force on the surface AA = $9.79 \times (0.6 \times 2.0) \times 0.9$

= 10.57 kN

When suspended from top the stress on the side walls

$$\sigma = \frac{35.24}{(1.2 + 1.2 + 2.0 + 2.0) \times 8/1000}$$

= **688.3 kPa**

When supported from bottom the stress on the side walls

$$\sigma = \frac{10.57}{(1.2 + 1.2 + 2.0 + 2.0) \times 8/1000}$$

= **206.4 kPa**

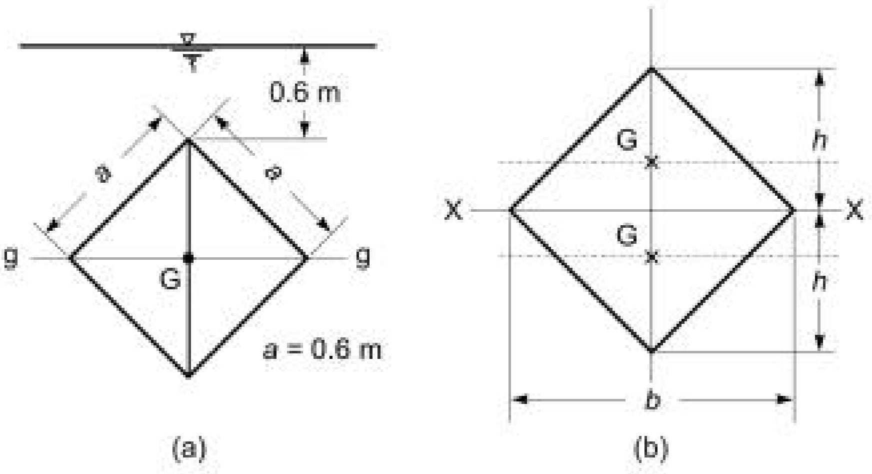
2.26 A square lamina is immersed in water with one of the diagonals vertical as shown in Fig. 2.29(a). Calculate the force on one side and the location of the centre of pressure.

Solution:

Area $A = a^2 = 0.36 \text{ m}^2$

$$\bar{h} = 0.6 + \frac{a}{\sqrt{2}} = 0.6 + \frac{0.6}{\sqrt{2}}$$

= 1.0243 m



(a) (b)

Fig. 2.29

Consider two triangular laminas of base b and height h, as shown in Fig. 2.29(b). Considering the axis XX

$$I_{xx} = \left\{\frac{1}{36}bh^3 + \left(\frac{bh}{2}\right)\frac{h^2}{9}\right\} \times 2$$

$$= \frac{1}{12}bh^3 \times 2 = \frac{bh^3}{6}$$

For the present case $I_{xx} = I_{gg}$

$$b = 2a/\sqrt{2} \text{ and } h = a/\sqrt{2}$$

$$I_{gg} = \frac{1}{6}\cdot\frac{2a}{\sqrt{2}}\,\frac{a^3}{2\sqrt{2}} = \frac{1}{12}a^4$$

$$= \frac{1}{12}(0.6)^4 = 0.0108$$

Force on one side of the lamina

$$F = \gamma A\bar{h} = 9.79 \times (0.36)\,(1.0243)$$

$$= \mathbf{3.61\ kN}$$

Centre of pressure:
By symmetry $x_p = 0$

$$h_p = y_p = \bar{h} + \frac{I_{gg}}{A\bar{h}}$$

$$= 1.0243 + \frac{0.0108}{(0.36)(1.0243)}$$

$$= \mathbf{1.0536\ m}$$

C. Forces on Curved Surfaces

2.27 A horizontal cylindrical barrier of 2.5 m diameter retains water up to its top on one side. Calculate the horizontal and vertical forces due to water per metre width of this barrier.

Solution:

Referring to Fig. 2.30 and considering 1 m width,

F_H = horizontal force

$= \gamma$ (Projected area of the cylinder MSN on a vertical M′N′) $\times \bar{h}$

$$= 9.79 \times (2.5 \times 1)\cdot\left(\frac{2.5}{2}\right) = 30.59 \text{ kN}$$

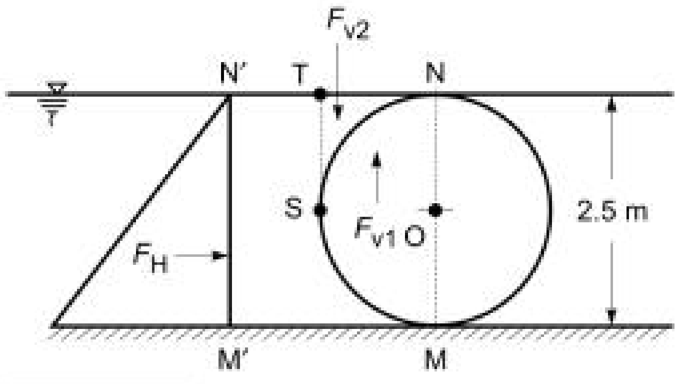


Fig 2.30

Vertical force

$$F_V = F_{V1} \text{ on MS} - F_{V2} \text{ on SN}$$

$$= \gamma\,(\text{volume NMST} - \text{volume NST})$$

$$= \gamma\left\{\left(\frac{1}{4}\times\frac{\pi\times(2.5)^2}{4}\right)+\left(\frac{2.5}{2}\times\frac{2.5}{2}\right)\right\}\times 1$$

$$-\gamma\left\{\left(\frac{2.5}{2}\times\frac{2.5}{2}\right)-\left(\frac{\pi\times(2.5)^2}{4\times 4}\right)\right\}\times 1$$

$$= \gamma\times\frac{1}{2}\times\frac{\pi}{4}(2.5)^2\times 1 = 24.03 \text{ kN}$$

Hence $\quad \mathbf{F_H = 30.59\ kN}$

$\mathbf{F_V = 24.03\ kN}$

2.28 A door in a tank is in the form of a quadrant of a cylinder of 1.5 m radius and 1.8 m wide (Fig. 2.31). Calculate the resultant force on the door and its location on the gate.

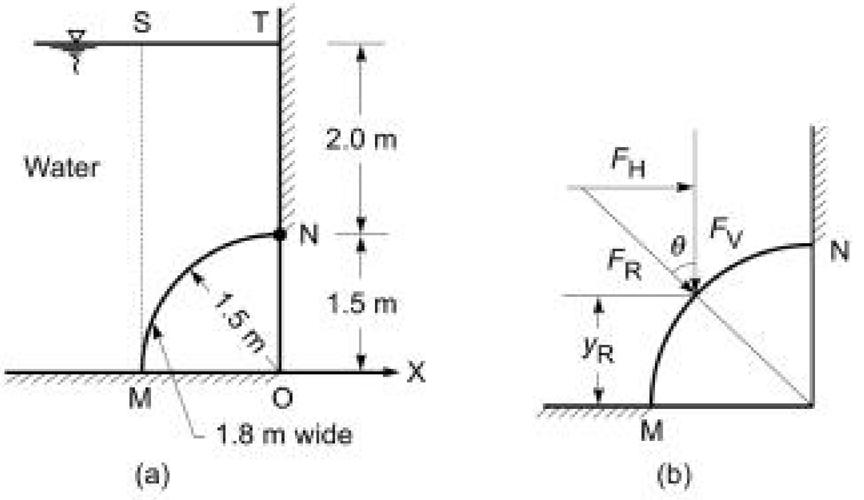


Fig. 2.31

Solution: Horizontal component of the force

$$F_H = \gamma\,(\text{Projected area}) \times \bar{h}$$

$$= 9.79 \times (1.5 \times 1.8) \times \left(2.0 + \frac{1.5}{2}\right)$$

$$= 72.69 \text{ kN}$$

Vertical component of the force

$$F_V = \gamma\,(\text{volume TNMS})$$

$$= 9.79\left[(1.5\times 1.8)\times(2.0+1.5) - \frac{1}{4}\times\pi\times(1.5)^2\times 1.8\right]$$

$$= 61.37 \text{ kN}$$

Resultant force

$$F_R = \sqrt{F_H^2 + F_V^2}$$

$$= \sqrt{(72.69)^2 + (61.37)^2} = \mathbf{95.13\ kN}$$

If θ is the inclination of F_R to vertical

$$\tan \theta = \frac{F_H}{F_V} = \frac{95.13}{61.37} = 1.55$$

$$\theta = 57.173°$$

The resultant passes through O.

y_R = vertical height of resultant on the gate.

$= R \cos \theta = 1.5 \cos (57.173°) = \mathbf{0.813 \text{ m}}$

2.29 The lower corner of a water tank has the shape of a quadrant of a circle of radius 1.2 m. The water surface is 2.4 m above the centre of curvature. The water tank is 3.0, long. Find the magnitude, direction and location of the total force exerted by the water surface on this curved surface.

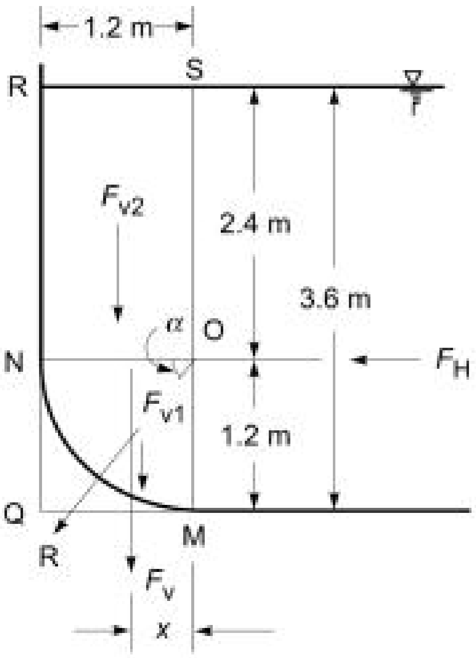


Fig. 2.32

Solution:

Vertical component:

F_V = volume contained in (OMN + ONRS)

$= F_{V1} + F_{V2}$

$$F_V = \left[\frac{\pi}{4} \times (1.2)^2 \times 3.0 \times 9.79\right] + (2.4 \times 1.2 \times 3.0 \times 9.79)$$

$= 33.22 + 84.59 = 117.81$ kN

F_{V1} acts at $\left(\frac{4}{3\pi} \times 1.2\right) = 0.5093$ m from OM

F_{V2} acts at 0.6 m from OM.

By taking moments of vertical forces about O,

$$F_V \cdot x = (33.22 \times 0.5093) + (84.59) \times 0.6$$

$$x = \frac{67.67}{117.81} = 0.574 \text{ m}$$

Horizontal component F_H = Force on the projection of curved face MN on a vertical plane

$= \gamma A \bar{h}$

$= 9.79 \times (1.2 \times 3) (2.4 + 0.6)$

$= 105.73$ kN

Depth of centre of pressure of $F_H = h_c = \bar{h} + \frac{I_{gg}}{A\bar{h}}$

$$= 3.0 + \frac{\frac{1}{12} \times [3 \times (1.2)^3]}{(3 \times 1.2) \times 3}$$

$= 3.04$ m

Resultant force on the curved face MN

$$R = \sqrt{F_V^2 + F_H^2} = \sqrt{(117.81)^2 + (105.73)^2}$$

$= \mathbf{158.3 \text{ kN}}$

If α is the inclination of R to the horizontal

$$\tan \alpha = \frac{F_V}{F_H} = \frac{117.81}{105.73} = 1.114$$

$$\boldsymbol{\alpha = 48° \, 5' \, 36''}$$

The resultant passes through the centre of curvature O.

2.30 A hemispherical projection of diameter 1.2 m exists on one of the vertical sides of a tank. If the tank contains water to an elevation of 3.0 m above the centre of the hemisphere, calculate the horizontal and vertical forces acting on the projection.

Solution: Referring to Fig. 2.33

Vertical force $F_V = F_{V1} - F_{V2}$

= weight of volume of water NSTQ – weight of volume of water MSTQ.

= weight of water contained by the hemisphere MSN

$$= \gamma \cdot \frac{1}{2} \left(\frac{4}{3} \pi R^3\right)$$

$$= 9.79 \times \frac{1}{2} \times \frac{4}{3} \times \pi (1.2)^3 = 35.44 \text{ kN}$$

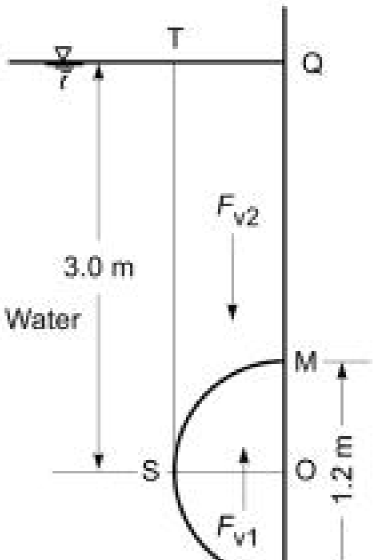


Fig. 2.33

Vertical force F_{V1} = [weight of volume of oil ACC_1B – weight of volume of oil BCC_1]

= weight of volume ACB

$$= \gamma_o \times \frac{1}{2} \times \frac{\pi D^2}{4} \times 1.0$$

$$= 0.9 \times 9.79 \times \frac{1}{2} \times \frac{\pi}{4} \times (2.5)^2 \times 1.0$$

= 21.63 kN

This force acts at a distance of $\frac{4(D/2)}{3\pi}$

$$= \frac{4 \times 2.5/2)}{3\pi}$$

= 0.531 m from vertical at A.

Force due to liquid M:

Horizontal force F_{H2} (Right to left)

$= \gamma_L$ (projected area) × $\bar{h}$

$$= 0.80 \times 9.79 \times (1.25 \times 1) \times \frac{1.25}{2}$$

= 6.12 kN

This force acts at $\frac{1.25}{3}$ = 0.417 m above level of A.

Vertical force F_{V2}

$= \gamma_o \times$ [volume AOD]

$$= 0.8 \times 9.79 \times \left[\frac{\pi}{4} \times (2.5)^2 \times \frac{1}{4}\right] \times 1.0$$

= 9.61 kN

This force acts at a distance of $\frac{4}{3\pi}\left(\frac{D}{2}\right) = \frac{4 \times 2.5}{3\pi \times 2}$

= 0.531 m from the vertical at A.

Resultant force:

R_H = horizontal component = $F_{H1} - F_{H2}$

= 27.53 – 6.12

= 21.41 kN

R_V = vertical component = $F_{V1} + F_{V2}$

= 21.63 + 9.61

= 31.24 kN

Resultant force $F = \sqrt{R_H^2 + R_V^2}$

$$= \sqrt{(21.41)^2 + (31.24)^2}$$

= 37.87 kN

Let α = inclination of the resultant to the horizontal. Then

$$\tan \alpha = \frac{R_V}{R_H} = \frac{31.24}{21.41} = 1.459$$

$$\boldsymbol{\alpha = 55° \, 34' 32''}$$

The resultant acts normal to the cylindrical surface and passes through the centre of curvature O of the cylindrical surface.

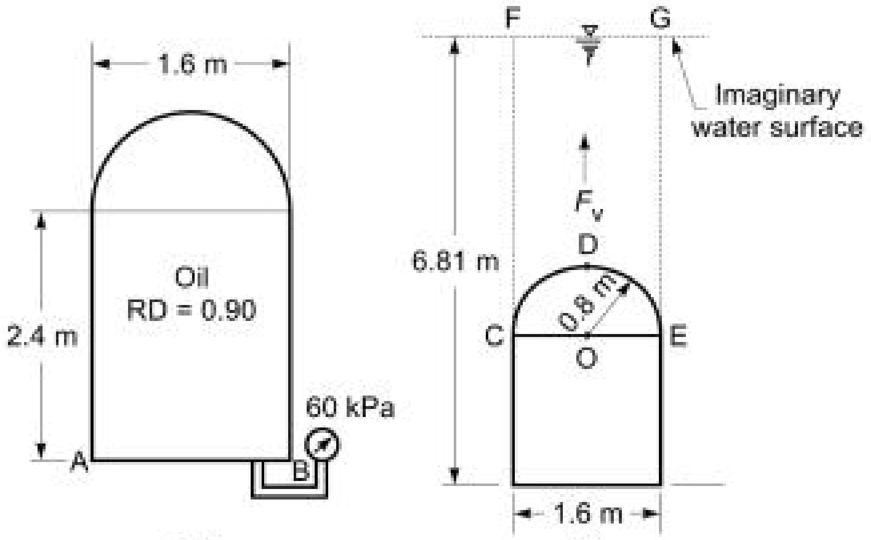
2.34 A 1.6 m wide × 3.0 m long × 2.4 m high rectangular tank has a semi-cylindrical cover of 1.6 m dia × 3.0 m long, as shown in Fig. 2.37(a). The tank contains oil of relative density 0.90 under pressure. If gauge pressure at the bottom of the tank reads 60 kPa find the net force on the curved roof.

Solution: First convert the base pressure of 60 kPa into equivalent column of oil (h_o)

$$60 = \gamma_o \times h_o$$

$$h_o = \frac{60}{(9.79 \times 0.9)} = 6.81 \text{ m}$$

An imaginary free surface of oil at an elevation of 6.81 m above the base AB can be imagined [see Fig. 2.37 (b)].



(a) (b)

Fig. 2.37

By symmetry, horizontal force on the cylindrical cover is zero. The vertical force F_V

$= \gamma_o$ [volume of prism COEGF – curved volume CDE]

$= (0.9 \times 9.79) \times$

$$\left[(1.6 \times 3.0)(6.81 - 2.4) - \left(\frac{\pi}{2} \times \frac{(1.6)^2}{4} \times 3\right)\right]$$

= 8.811[21.168 – 3.016] = **159.9 kN**

This force acts vertically at the centre of the tank along OD.

2.35 A cylindrical tank of 3 m diameter and height 1.5 m has a hemispherical dome. The tank contains oil of relative density 0.83 [Fig. 2.38(a)]. The dome is joined to the cylindrical portion by 6 equally spaced bolts. If the pressure gauge at a point A, 0.6 m above the base of the tank, reads 50 kPa determine the force on each bolt.

Solution: First consider equivalent of pressure P_A in terms of oil column.

$$P_A = \gamma_o h_A$$

$$50 = (0.83 \times 9.79) \cdot h_A$$

$$h_A = 6.153 \text{ m of oil}$$

The imaginary oil surface at an elevation of $h_A = 6.153$ m is now considered. [Fig. 2.38(b)].

Above the base plane MN of the dome, the elevation of the imaginary oil surface is

$$= 6.153 - 0.9 = 5.253 \text{ m}$$

By symmetry, there is no horizontal force on the dome.

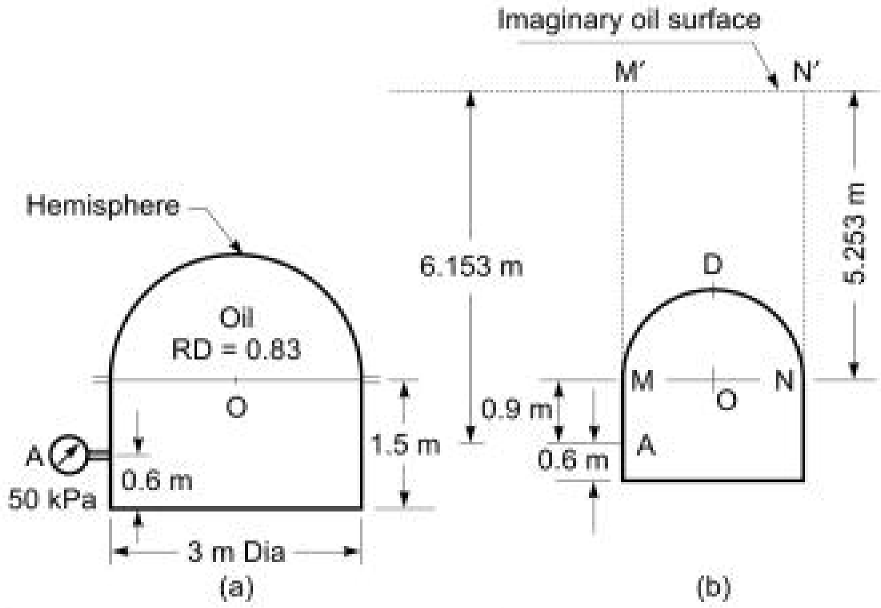


Fig. 2.38 *Example 2.35*

The vertical force F_V = weight of oil above the dome surface up to the imaginary oil surface = weight of volume MDN N′M′.

$$= (0.83 \times 9.79)\left[\left(\frac{\pi(3.0)^2}{4} \times 5.253\right) - \left(\frac{1}{2} \times \frac{4}{3}\pi\left(\frac{3.0}{2}\right)^3\right)\right]$$

$$= 8.126\,(37.131 - 7.069) = 244.28 \text{ kN}$$

This force is shared by six bolts.

$$\text{Tensile force on each bolt} = \frac{244.28}{6} \text{ kN}$$

$$= \mathbf{40.71\ kN}$$

2.36 A masonry weir is of trapezoidal cross section with a top width of 2.0 and of height 5 m with upstream slope of 1 vertical in 0.1 horizontal and a downstream slope of 1 vertical in 0.75 horizontal. If the weir has water stored up to its crest on the upstream side and has a tailwater of 2 m depth on the downstream, calculate, per unit length of the weir,

(i) the resultant force on the base of the weir and

(ii) the minimum and maximum stresses on the base of the weir.

(Assume specific weight of masonry as 22 kN/m^3 and neglect uplift forces).

Solution: Refer to Fig. 2.39

consider 1 m length of the weir.

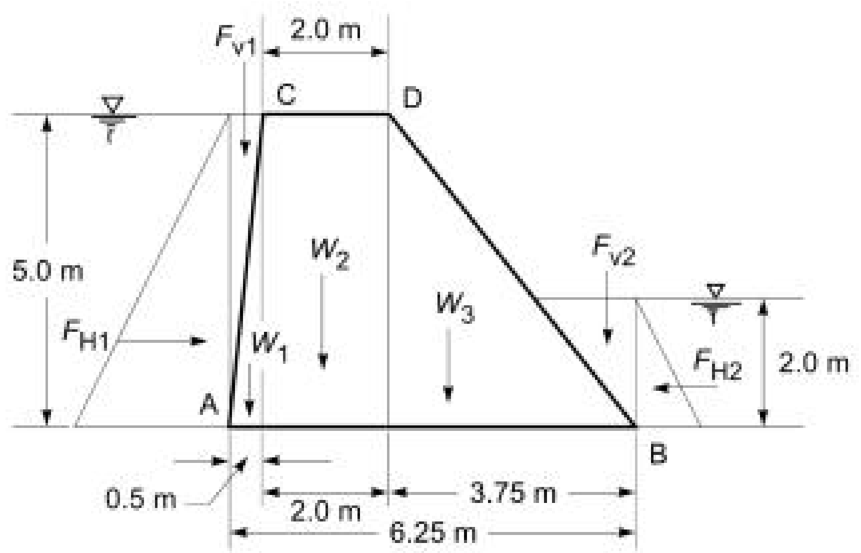


Fig. 2.39

The force acting on the weir are:

(i) weight of masonry $W = W_1 + W_2 + W_3$

(ii) vertical force due to water on the upstream slope, F_{V1}.

(iii) vertical force due to tailwater on the downstream slope, F_{V2}.

(iv) horizontal water force on the upstream side, F_{H1}.

(v) horizontal water force on the downstream side, F_{H2}.

The magnitudes of these forces, their distance from the toe of the weir (edge B) and the moments of these forces about B are tabulated as in Table 2.1.

ΣV = Sum of vertical forces = 480.7 kN

ΣH = Sum of horizontal forces = 102.8 kN

$$R = \text{Resultant} = \sqrt{\Sigma V^2 + \Sigma H^2}$$

$$= \sqrt{(480.7)^2 + (102.8)^2}$$

$$= \mathbf{491.6\ kN}$$

If θ is the inclination of the resultant to horizontal

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{480.7}{102.8} = 4.676, \text{ and } \theta = \mathbf{77.93^\circ}$$

Σ M = 1818.2 – 204 = 1614.2 kN.m

x = distance of point of action of the resultant

$$\text{from } B = \frac{\Sigma M}{\Sigma V} = \frac{1614.2}{480.7} = 3.358$$

As b = base width = 6.25 m, the Eccentricity $e = x - \dfrac{b}{2}$

$$= 3.358 - \frac{6.25}{2} = 0.233 \text{ m}$$

Maximum and minimum stresses

Table 2.1 *Magnitude of the forces and moments and of the lever arm about B-Example 2.36*

Force	Description	Horiz. force (kN)	Vert. force (kN)	Lever arm about B (m)	Moment (Clockwise) (kN.m)	Moment (Anticlockwise) (kN.m)
W_1	$(0.5 \times 5) \times \frac{1}{2} \times 1 \times 22$		27.5	5.917		162.7
W_2	$(2.0 \times 5.0) \times 1.0 \times 22$		220.0	4.75		1045.0
W_3	$(3.75 \times 5) \times \frac{1}{2} \times 1 \times 22$		206.25	2.50		515.6
F_{V1}	$(0.5 \times 5) \times \frac{1}{2} \times 1 \times 9.79$		12.24	6.083		74.5
F_{V2}	$(1.5 \times 2.0) \times \frac{1}{2} \times 1 \times 9.79$		14.69	0.50		7.3
F_{H1}	$(5 \times 1) \times \frac{5}{2} \times 9.79$	122.38		1.667	204.0	
F_{H2}	$(2 \times 1) \times \frac{2}{2} \times 9.79$	– 19.58		0.667		13.1
	Sum	**102.8**	**480.7**		**204.0**	**1818.2**

$$\sigma_{1,2} = \frac{\Sigma V}{b}\left(1 \pm \frac{6e}{b}\right)$$

$$= \frac{480.7}{6.25}\left(1 \pm \frac{6 \times 0.233}{6.25}\right)$$

$\boldsymbol{\sigma_1}$ **= maximum stress = 94.1 kPa**

$\boldsymbol{\sigma_2}$ **= minimum stress = 59.7 kPa**

2.37 A 2.5 m diameter pipe carries oil (relative density = 0.85) at a pressure of 1000 kPA. If the maximum allowable stress of the pipe material is 120 MPa, what thickness of the pipe wall is required?

Solution: Hoop stress (circumferential tensile stress) in a cylinder

$$\sigma = \frac{pD}{2t}$$

If $\sigma = f_a$ = allowable stress in the material.
Minimum thickness t required is

$$t = \frac{pD}{2 f_a}$$

$$= \frac{1000 \times 2.5}{2 \times (120 \times 1000)} = 0.01042 \text{ m}$$

$$= \mathbf{10.42\ mm}$$

A thickness of 10.5 mm can therefore be used.

[***Note:*** The relative density of the fluid does not have any role when once the pressure inside the pipe is stipulated.]

2.38 The gates of a 8 m wide lock include an angle of 120° in the closed position. Each gate is held on by two hinges one placed at 0.8 m and another at 6.0 m from the bottom of the lock. If the water levels are 9.0 m and 3.0 m on the upstream and downstream respectively, determine (i) the resultant force due to water pressure and (ii) magnitudes of reaction at the hinges.

Solution:

Let F = resultant force due to water on each gate acting at the centre of each gate, point O

R = resultant reaction at the hinges

P = reaction between two gates at the common contact area, acting perpendicular to contact area, in this case normal to the centre line

The reaction R must pass through C (see Fig. 2.40), the point of intersection of F and P, due to equilibrium of the system. In triangle ACB, because CO is the perpendicular bisector of AB,

$$\angle \text{CBA} = \angle \text{CAB} = \theta$$

$$\therefore \quad R = P$$

Also $$P = R = \frac{F}{2 \sin \theta}$$

[***Note*** that it is assumed that F, P and R are in the same plane.]

$$B = \text{width of each gate} = \text{AB}$$

$$= \frac{4}{\cos 30°} = 4.62 \text{ m}$$

On the upstream:

$$F_1 = 9.79 \times (4.62 \times 9) \times \frac{9}{2} = 1831.8 \text{ kN}$$

Solution: Refer to Fig. 2.57

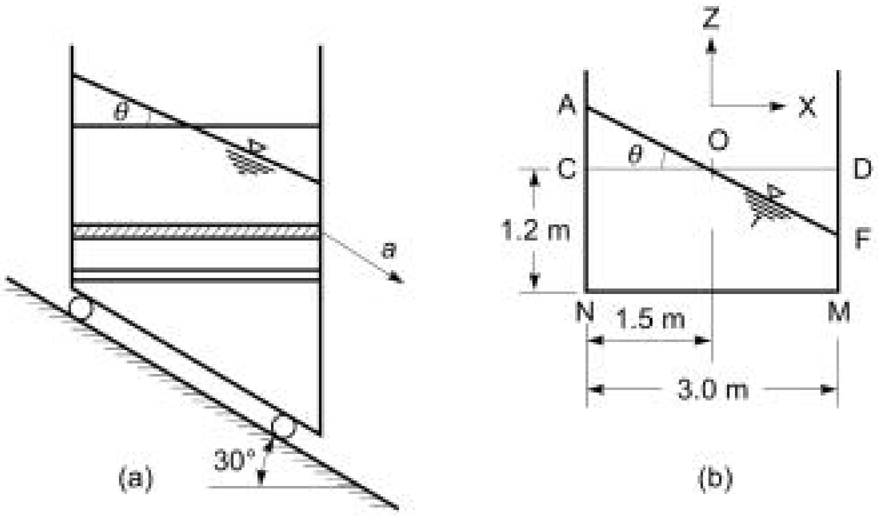


Fig. 2.57

Resolving the acceleration a_s into x- and z-components:

$$a_x = 2\cos 30° = 1.732 \text{ m/s}^2$$

$$a_z = -2\sin 30° = -1.00 \text{ m/s}^2$$

Water surface slope (θ measured clockwise from x-direction)

$$\tan\theta = \frac{a_x}{(a_z + g)} = \frac{1.732}{(-1 + 9.81)} = 0.1966$$

or $\theta = \mathbf{11.12°}$

Piezometric head at the rear

$$h_N = 1.2 + \frac{3.0}{2} \times \tan\theta$$

$$= 1.2 + 0.2949 = 1.495 \text{ m}$$

Piezometric head at the front

$$h_M = 1.2 - \frac{3.0}{2}\tan\theta = 0.905 \text{ m}$$

Pressures at N and M:

$$p_N = 1.495 \times 9.79 = \mathbf{14.64\ kPa}$$

$$p_M = 0.905 \times 9.79 = \mathbf{8.86\ kPa}$$

2.58 An open tank contains water to a depth of 1.5 m. The tank is put in an elevator which accelerates at 2.5 m/s^2. Calculate the pressure force per metre length on one of the sides of the tank when the (a) acceleration is upwards and (b) acceleration is vertically downwards.

Solution:

(a) When the acceleration is upwards

$$a_z = +2.5 \text{ m/s}^2$$

Pressure at any depth h below the free surface

$$p = \gamma h\left(1 + \frac{a_z}{g}\right)$$

$$= \gamma h\left(1 + \frac{2.5}{9.81}\right) = 1.2548\ \gamma h$$

Pressure force on a side wall per metre width

$$F_H = 1.2548 \times 9.79 \times 1.5 \times \frac{1.5}{2}$$

$$= \mathbf{13.82\ kN}$$

(b) When the acceleration is downwards

$$a_z = -2.5 \text{ m/s}^2$$

Pressure at any depth h below free surface

$$p = \gamma h\left(1 + \frac{a_z}{g}\right) = 0.7452\ \gamma h$$

Pressure force on a side wall per metre width

$$F_H = 0.7452 \times 9.79 \times 1.5 \times \frac{1.5}{2}$$

$$= \mathbf{8.207\ kN}$$

2.59 An open cylindrical tank 1 m in diameter and 2 m high contains water up to 1.5 m depth. If the cylinder rotates about its vertical axis what maximum angular velocity can be attained without spilling any water?

Solution: At the maximum permissible angular velocity the water surface will just touch the top rim of the cylinder.

Referring to Fig. 2.58, let C represent the original water level at the axis. As the water surface is a paraboloid of revolution, rise of water surface at the edge above C

= distance OC = $y/2$

where y = water surface elevation at the outer edge above the vertex 0.

Hence $y = 2(2.0 - 1.5) = 1.0$ m

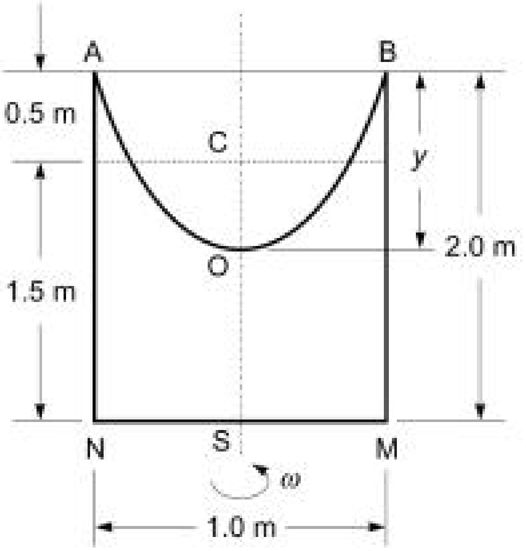


Fig. 2.58

In the present case

$$y = \frac{\omega^2 r^2}{2g}$$

$$1.0 = \frac{\omega^2 \times (0.5)^2}{2 \times 9.81}$$

$$\omega^2 = \frac{2\times 9.81}{(0.5)^2} = 78.48$$

$$\omega = 8.858$$

But $\omega = \dfrac{2\pi N}{60}$ when N = rotations/min

$$N = \frac{60\times 8.858}{2\pi} = \mathbf{84.6\ rpm}$$

2.60 An open cylindrical vessel of 20 cm diameter and 50 cm high contains water to a height of 30 cm (a) If the cylinder is rotated at 180 rpm about its axis, calculate the pressures on the bottom at the centre and at the edge of the vessel; (b) What speed of rotation would cause the water surface to just touch the top rim of the vessel?

Solution: Referring to Fig. 2.59

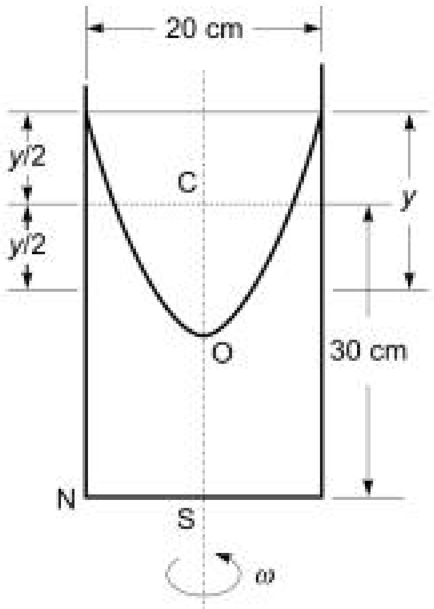


Fig. 2.59

(a) If N = rotations/min

Angular velocity $\omega = \dfrac{2\pi N}{60}$

In the present case

$$\omega = \frac{2\pi\times 180}{60} = 18.85 \text{ rad./s}$$

If y = water surface elevation above the vertex at a radial distance r, at the radial distance r = 10 cm

$$y = \frac{\omega^2 r^2}{2g} = \frac{(18.85)^2\,(0.10)^2}{2\times 9.81}$$

$$= 0.181 \text{ m} = 18.1 \text{ cm}$$

Rise at ends = $y/2$ above original level C.

h_N = piezometric head at a radial distance 10 cm on the base, i.e. point N.

= original depth + $y/2$

$$= 30.00 + \frac{18.10}{2} = 39.05 \text{ cm}$$

h_s = piezometric head at the centre of the base

$= h_N - y = 39.05 - 18.10 = 20.95$ cm

Pressure at N = $p_N = \gamma h_N$

$= 9.79 \times 0.3905 =$ **3.823 kPa**

Pressure at S = $p_s = \gamma h_s$

$= 9.79 \times 0.2095 =$ **2.051 kPa**

(b) In this case

$$\frac{y}{2} = 20 \text{ cm, or } y = 40 \text{ cm}$$

$$h_s = 10 \text{ cm and } h_N = 50 \text{ cm}$$

$$y = \frac{\omega^2 r^2}{2g}$$

or $$\omega = \frac{\sqrt{2gy}}{r} = \frac{\sqrt{2\times 9.81\times 0.4}}{0.10}$$

$$= 28.014 \text{ rad./s}$$

$$N = \text{revolutions/min} = \frac{60\omega}{2\pi}$$

$$= \frac{60\times 28.014}{2\pi}$$

$$= \mathbf{267.5\ rpm}$$

2.61 An open cylinder 30 cm in diameter and 50 cm high is filled with water and rotated about its axis. Calculate the amount of water spilled when the speed of rotation is (a) 180 rpm and (b) 240 rpm.

Solution:

(a) When N = 180 rpm

$$\omega = \frac{2\pi\times 180}{60} = 18.85 \text{ rad./s}$$

At the wall of the cylinder, r = 15 cm = 0.15 m

$$y_{\max} = \frac{\omega^2 r^2}{2g} = \frac{(18.85)^2}{2\times 9.81}\times (0.15)^2$$

$$= 0.4075 \text{ m} = 4.075 \text{ cm}$$

The position of the water surface in the cylinder will be as shown in Fig. 2.60 (a). The water surface will start from the rim of the cylinder and extend downwards as a paraboloid to its vertex at O.

OS = The depth at the vertex

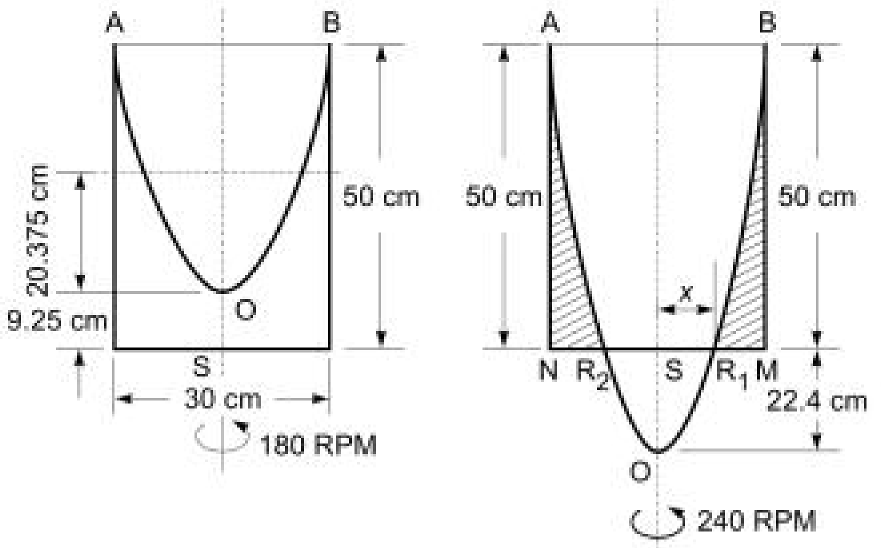
$= h_s = 50 - 40.75$

$= 9.25$ cm

Volume of the paraboloid AOB of height 40.75 cm

$= \dfrac{1}{2} \times$ (volume of enclosing cylinder)

= Volume of cylinder of height 20.375 cm



(a) (b)

Fig. 2.60

Hence total volume of water inside the cylinder

= Volume corresponding to a depth of (9.25 + 20.375)

= 29.625 cm

$= \frac{\pi}{4} \times (0.3)^2 \times 0.29625$

$= 20.9 \times 10^{-3}$ m^3 = 20.9 L

Original volume $= \frac{\pi}{4} \times (0.3)^2 \times 0.50$

= 0.0353 m^3 = 35.3 L

Volume of water spilled = 35.3 – 20.9 = **14.4 L**

(b) When $N = 240$ rpm

$$\omega = \frac{2\pi \times 240}{60} = 25.13 \text{ rad./s}$$

At $r = 0.15$ m,

$$y_{\max} = \frac{\omega^2 r^2}{2g} = \frac{(25.13 \times 0.15)^2}{2 \times 9.81}$$

= 0.724 m = 72.4 cm

As this value is larger than 50 cm it means that the theoretical paraboloid will extend below the base thereby leaving a part of the bottom uncovered by water. The paraboloid will start from the rim and extend up to its vertex O which is 72.4 cm below it as shown in Fig. 2.60(b).

Let x = radius of the exposed portion of the bottom of the cylinder

$$x = SR_1 = SR_2$$

$$\frac{y_{\max}}{OS} = \frac{r^2}{x^2}$$

$$\therefore \quad x = r\sqrt{OS/y_{\max}}$$

$$= 15 \times \sqrt{\frac{22.4}{72.4}}$$

= 8.34 cm

Volume of water spilled = Volume of the paraboloid AOB – Volume of the paraboloid R_1OR_2

$$= \frac{\pi r^2}{2} \times y_{\max} - \frac{\pi x^2}{2} \times (OS)$$

$$= \left[\frac{\pi}{2} \times (0.15)^2 \times 0.724 - \frac{\pi}{2} \times (0.0834)^2 \times (0.224)\right]$$

$$= \frac{1}{2}[0.05118 - 0.004895]$$

= 0.0232 m^3 = **23.2 L**

2.62 A 90° conical vessel of height 2 m carries water whose volume is one half of the volume of the cone. Find the speed at which the vessel may be rotated to make the water just spill out.

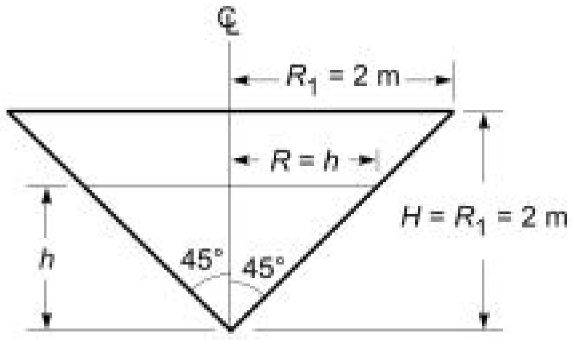


Fig. 2.61

Solution: Refer to Fig. 2.61,

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 H = \frac{1}{3}\pi R_1^2$$

If h = height of water before rotation

$$\frac{1}{3}\pi R_1^3 \,(1/2) = \frac{1}{3}\pi h^3$$

$\therefore \quad h = R/(2^{1/3}) = 0.7937$

Since $\quad R = 2.0$ m, $h = 1.5874$ m.

On rotation, $(\omega^2 R_1^2)/2g = (H - h) = (2.0 - 1.5874) = 0.4126$ m

$$\omega = \frac{2 \times 9.81 \times 0.4126}{(2.0)^2} = 2.0238$$

and $\quad \omega = \frac{2\pi N}{60}$

Hence, $\quad$ RPM $= N = \frac{60 \times 2.0238}{2\pi}$ = **19.326 rpm**

2.63 A closed cylinder of radius 10 cm height 30 cm is filled with water. If the cylinder is rotated about its vertical axis at a speed of 240 rpm, calculate the force exerted at the top and bottom covers of the cylinder.

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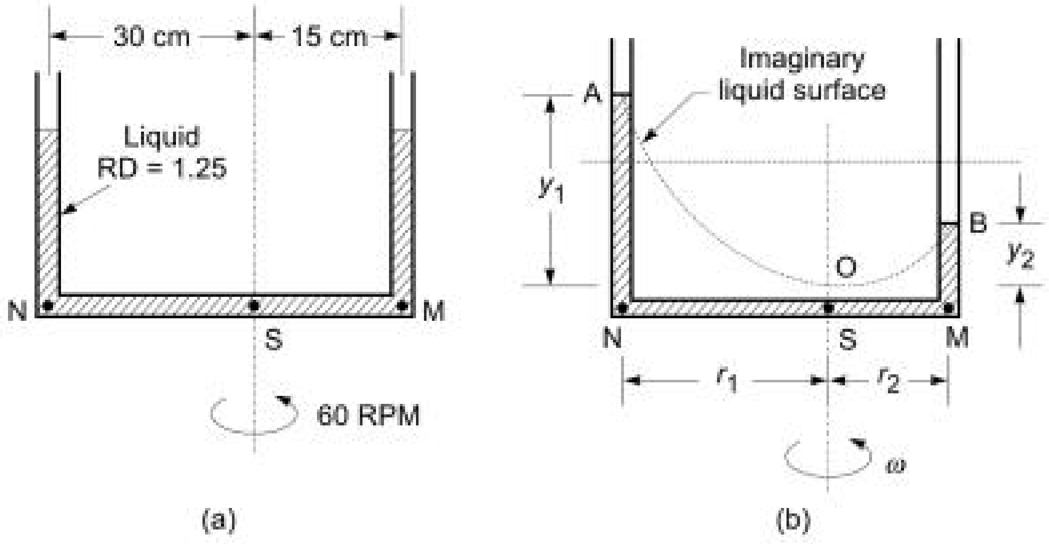


Fig. 2.64

common. The sum of two vertical columns of liquid before rotation

$$= (15 + 15) = 30 \text{ cm}$$

Also $\quad y_1 + y_2 = 18.1 + 4.5 = 22.6$ cm

The difference (30.0 – 22.6) = 7.4 cm is equally distributed in the two limbs to get the column heights in the right and left limbs as:

$$\text{Pressure head } h_S = \frac{7.4}{2} = 3.7 \text{ cm}$$

$$\text{Column NA} = h_N = 18.1 + \frac{7.4}{2} = 21.8 \text{ cm}$$

$$\text{Column MB} = h_M = 4.5 + \frac{7.4}{2} = 8.2 \text{ cm}$$

$$p_N = \text{pressure at N} = 1.25 \times 9790 \times 0.218$$
$$= \mathbf{2668 \text{ Pa}}$$

$$p_M = \text{pressure at M} = 1.25 \times 9790 \times 0.082$$
$$= \mathbf{1003 \text{ Pa}}$$

OBJECTIVE QUESTIONS

2.1 For a fluid at rest
- (a) the shear stress depends upon the coefficient of viscosity
- (b) the shear stress is maximum on a plane inclined at 45° to horizontal
- (c) the shear stress is zero
- (d) the shear stress is zero only on horizontal planes

2.2 If a Mohr circle is drawn for a fluid element inside a fluid body at rest, it would be
- (a) a circle not touching the origin
- (b) a circle touching the origin
- (c) a point on the normal stress axis
- (d) a point on the shear stress axis

2.3 Indicate the *incorrect* answer:
Hydrostatic pressure variation implies that
- (a) the pressure varies linearly with depth
- (b) the piezometric head $[p/\gamma + Z]$ in constant
- (c) the density of the fluid in constant
- (d) pressure varies linearly with distance

2.4 Normal stresses are of the same magnitude in all directions at a point in a fluid
- (a) only when the fluid is frictionless
- (b) only when the fluid is at rest
- (c) only when there is no shear stress
- (d) in all cases of fluid motion

2.5 The basic differential equation for the variation of pressure p in a static fluid with vertical distance y (measured upwards) is
- (a) $dp = -\gamma dy$ (b) $dy = -\gamma dp$
- (c) $dp = -\rho dy$ (d) $dp = -dy$

2.6 In an isothermal atmosphere the pressure
- (a) decreases linearly with elevation
- (b) decreases exponentially with elevation
- (c) increases logarithmically with elevation
- (d) varies inversely as the density

2.7 The piezometric head in a static liquid
- (a) remains constant only on a horizontal plane
- (b) increases linearly with depth below a free surface
- (c) remains constant at all points in the fluid
- (d) decreases linearly with depth below the free surface

2.8 Local atmospheric pressure is measured by
(a) Hydrometer (b) Barometer
(c) Hygrometer (d) Altimeter

2.9 Identify the correct statement:
(a) Local atmospheric pressure is always less than the standard atmospheric pressure
(b) Local atmospheric pressure depends only on the elevation of the place
(c) A barometer reads the difference between the local and standard atmospheric pressure
(d) Standard atmospheric pressure is 760 mm of mercury

2.10 Aneroid barometer measures
(a) local atmospheric pressure
(b) standard atmospheric pressure
(c) gauge pressure
(d) difference between the standard and local atmospheric pressures

2.11 Bourdon gauge measures
(a) absolute pressure
(b) gauge pressure
(c) local atmospheric pressure
(d) standard atmospheric pressure

2.12 In a barometer mercury is preferred over water because
(a) it is a better conductor of heat
(b) its surface is easier to read
(c) it has a higher density and lower vapour pressure
(d) it has a higher vapour pressure and bulk modulus

2.13 A barometer at a given location
(a) shows the local atmospheric pressure which is invariant with time
(b) always shows the local atmospheric pressure which may change with time
(c) shows the standard atmospheric pressure, if it is of aneroid type
(d) shows the local temperature if it is of mercury column type

2.14 In a mercury column-type barometer the correct local atmospheric pressure is obtained by adding correction due to vapour pressure of mercury as follows: H_a =
(a) $H_0 - h_v$ (b) $H_0 + h_v$
(c) H_0/h_v (d) $h_v - H_0$
where H_a = correct local pressure in mm of mercury, H_0 = observed barometer reading in mm of mercury and h_v = vapour pressure of mercury in mm.

2.15 When the barometer reads 740.0 mm of mercury, a pressure of 10 kPa suction at that location is equivalent to
(a) 10.02 m of water (abs)
(b) 9.87 m of water (abs)
(c) 88.53 kPa (abs)
(d) 0.043 kPa (abs)

2.16 The standard sea-level atmospheric pressure is equivalent to
(a) 10.0 m of fresh water of $\rho = 998$ kg/m^3
(b) 10.1 m of salt water of $\rho = 1025$ kg/m^3
(c) 12.5 m of kerosene of $\rho = 800$ kg/m^3
(d) 6.4 m of carbon tetrachloride of $\rho = 1590$ kg/m^3

2.17 The standard atmospheric pressure is 760 mm of mercury. At a certain location the barometer reads 710 mm of mercury. At this place an absolute pressure of 360 mm of mercury corresponds to a gauge pressure in mm of mercury.
(a) 400 mm of vacuum
(b) 350 mm of vacuum
(c) 360 mm of vacuum
(d) 710 mm

2.18 The standard atmospheric pressure is 101.32 kPa. The local atmospheric pressure at a location was 91.52 kPa. If a pressure is recorded as 22.48 kPa (gauge), it is equivalent to
(a) 123.80 kPa (abs) (b) 88.84 kPa (abs)
(c) 114.00 kPa (abs) (d) 69.04 kPa (abs)

2.19 A U-tube manometer measures
(a) absolute pressure at a point
(b) local atmospheric pressure
(c) difference in total energy between two points
(d) difference in pressure between two points

2.20 In the setup shown in Fig. 2.65 assuming the specific weight of water as 10 kN/m^3, the pressure difference between the two points A and B will be
(a) 10 N/m^3 (b) –10 N/m^3
(c) 20 N/m^3 (d) –20 N/m^3

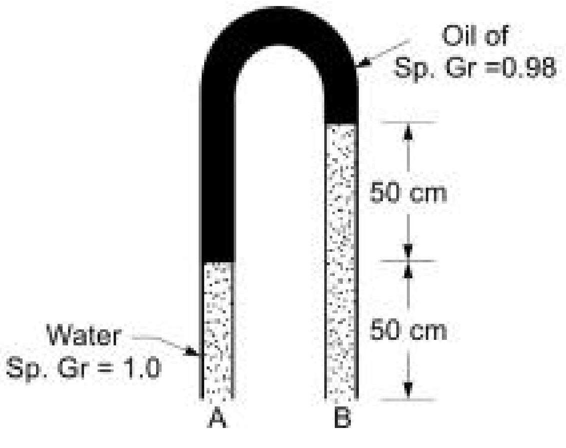


Fig. 2.65

2.21 An inclined manometer contains a liquid of relative density 0.8 and has an inclination of 30° to the horizontal. For a certain pressure the column length was 10 cm. If there is an uncertainty of 1° in the measurement of the angle of inclination, the calculated pressure would have an uncertainty of

(a) 1% (b) 0.28%
(c) 1.75% (d) 3.33%

2.22 A U-tube open at both ends and made of 8 mm diameter glass tube has mercury in the bottom to a height of 10 cm above the horizontal limb. If 19 cc of water is added to one of the limbs, the difference in mercury levels at equilibrium is

(a) 3.0 cm (b) 2.8 cm
(c) 1.0 cm (d) zero

2.23 For a submerged plane in a liquid, the resultant hydrostatic force F on one side of the plane is related to area A, centroidal depth $\bar{h}$, depth of the centre of pressure h_{cp} and depth of bottom edge h_b as $F =$

(a) $\gamma A h_{cp}$ (b) $\gamma A \bar{h}$
(c) $\gamma A h_b$ (d) $A \bar{h} / \gamma$

2.24 For an inclined plane submerged in a liquid the centre of fluid pressure on one side of the plane will be

(a) above the top edge of the area
(b) vertically below the centre of gravity
(c) below the centre of gravity
(d) in the same horizontal plane as the centre of gravity

2.25 An equilateral triangular plate is immersed in water as shown in Fig. 2.66. The centre of pressure below the water surface is at a depth of

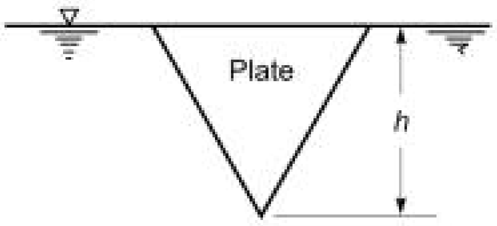


Fig. 2.66

(a) $3h/4$ (b) $h/3$
(c) $2h/3$ (d) $h/2$

2.26 When the water surface coincides with the top edge of a rectangular gate 2.5 m wide × 3 m deep, the depth of centre of pressure is

(a) 1.0 m (b) 1.5 m
(c) 2.0 m (d) 2.5 m

2.27 A circular plate of diameter D is submerged in water vertically so that the top most point is just at the water surface. The centre of pressure of the plate will be below the water surface at a depth of

(a) 5D/8 (b) 11D/16
(c) 2D/3 (d) 3D/4

2.28 The tank in Fig. 2.67 discharges water at constant rate for all water levels above the air inlet R. The height above datum to which water would rise in the manometer tubes M and N respectively, are

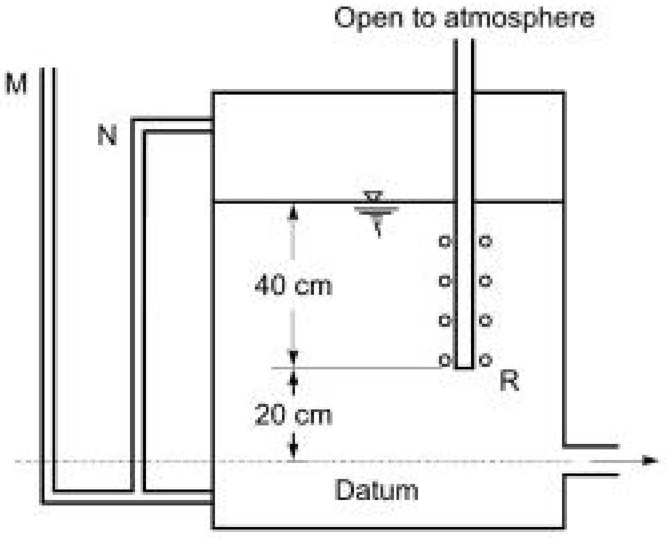


Fig. 2.67

(a) (60 cm, 20 cm) (b) (40 cm, 40 cm)
(c) (20 cm, 20 cm) (d) (20 cm, 60 cm)

2.29 A cylindrical tank of 2 m diameter is laid with its axis horizontal and is filled with water just to its top. The force on one of its end plate in kN, is

(a) 123.0 (b) 61.51
(c) 30.76 (d) 19.58

2.30 A rectangular plate 0.75 m × 2.4 m is immersed in a liquid of relative density 0.85 with its 0.75 m side horizontal and just at the water surface. If the plane of the plate makes an angle of 60° with the horizontal, the pressure force on one side of the plate, in kN, is

(a) 15.6 (b) 7.8
(c) 24.0 (d) 18.0

2.31 A circular annular plate bounded by two concentric circles of diameter 1.2 m and 0.8 m is immersed in water with its plane making an angle of 45° with the horizontal. If the centre of the circles is 1.625 m below the free surface, the total pressure force on one side of the plate in kN, is

(a) 7.07 (b) 10.0
(c) 14.14 (d) 18.0

2.32 A rectangular plate 30 cm × 50 cm is immersed vertically, in water with its longer side vertical. The total force on one side of the plate is estimated as 17.6 kN. If the plate is turned in the vertical plane at its centre of gravity by 90° and if all other factors remain the same, the total force on one side of the plate would now be

(a) 8.8 kN (b) 15.6 kN
(c) 17.6 kN (d) 19.6 kN

2.33 A stationary liquid is so stratified that its density is $\rho_0(1 + h)$ at a depth h below the free surface. At a depth h in this liquid the pressure p in excess of $\rho_0 gh$ is

(a) $\rho_0 gh$ (b) $\rho_0 gh^2$
(c) $\rho_0 gh^3/3$ (d) $\rho_0 gh^2/2$

2.34 A curved surface is submerged in a static liquid. The horizontal component of pressure force on it is equal to

(a) the pressure force on a horizontal projection of the surface
(b) product of the surface are and pressure at the centre of gravity
(c) pressure force on a vertical projection of the surface
(d) weight of the liquid contained between the curved surface and the liquid surface

2.35 A hollow hemispherical object of diameter D was immersed in water with its plane surface coinciding with the free surface. The vertical component of force on the curved surface is given by F_v =

(a) $\frac{3}{8}\gamma\pi D^3$ (b) $\frac{1}{12}\gamma\pi D^3$
(c) $\frac{1}{24}\gamma\pi D^3$ (d) zero

2.36 A 2.0 m diameter penstock pipe carries water under a pressure head of 100 m. If the wall thickness is 7.5 mm, the tensile stress in the pipe wall, in MPa, is

(a) 65.3 (b) 130.5
(c) 231.0 (d) 1305.0

2.37 The centre of buoyancy of a submerged body

(a) coincides with the centre of gravity of the body
(b) coincides with the centroid of the displaced volume of the fluid
(c) is always below the centre of gravity of the body
(d) is always above the centroid of the displaced volume of liquid

2.38 An object weighing 100 N in air was found to weigh 75 N when fully submerged in water. The relative density of the object is

(a) 4.0 (b) 4.5
(c) 2.5 (d) 1.25

2.39 An iceberg has 12% of its volume projecting above the surface of the sea. If the density of sea water is 1025 kg/m^3, the density of the iceberg is

(a) 878 kg/m^3 (b) 1000 kg/m^3
(c) 1148 kg/m^3 (d) 902 kg/m^3

2.40 An object weighs 50 N in water and 80 N in an oil of relative density 0.80. Its volume in litres is

(a) 15.3 (b) 60.0
(c) 30.6 (d) 50.0

2.41 A floating body displaces

(a) a volume of liquid equal in magnitude to its own volume
(b) a volume of liquid equal to its own submerged weight
(c) a weight of liquid equal in magnitude to its own weight
(d) a weight of liquid which depends upon the volume of the container

2.42 A metal block is thrown into a deep lake. As it sinks deeper in water, the buoyant force acting on it

(a) increases
(b) remains the same
(c) decreases
(d) first increases and then decreases

2.43 When a block of ice floating on water in a container melts, the level of water in the container

(a) rises
(b) first falls and then rises
(c) remains the same
(d) falls

2.44 When a ship enters sea from a river one can expect it

(a) to rise a little
(b) to sink a little
(c) to remain at the same level of draft
(d) to rise or fall depending on whether it is of wood or steel

2.45 Buoyant force is the

(a) lateral force acting on a submerged body
(b) resultant force acting on a submerged body
(c) resultant hydrostatic force on a body due to fluid surrounding it
(d) the resultant force due to water on a body

2.46 A right circular wooden cone (Specific gravity = 0.8) with a base diameter of 0.6 m and hight of 0.8 m floats in water such that its axis is vertical and apex is downward. The immersed depth of cone is

(a) 0.480 m (b) 0.533 m
(c) 0.600 m (d) 0.743 m

2.47 A 12 cm steel cube (RD = 7.6) is submerged in a two layered fluid system. The bottom layer is mercury (rel. den = 13.6) and the top layer is kerosene. The height of the top surface of the cube above the interface of the two liquids is

(a) 4.76 cm (b) 2.95 cm
(c) 6.0 cm (d) zero

2.48 Two cubes of size 1.0 m sides, one of relative density = 0.60 and another of relative density = 1.15, are connected by a weightless wire and placed in a large tank of water. Under equilibrium the lighter cube will project above the water surface to a height of

(a) 10 cm (b) zero
(c) 50 cm (d) 25 cm

2.49 If B = centre of buoyancy, G = is the centre of gravity and M = metacentre, of a floating body, the body will be in stable equilibrium if

(a) MG = 0 (b) M is below G
(c) BG = 0 (d) M is above G

2.50 In a floating body I = moment of inertia of water line area about the longitudinal axis, V = volume of displaced fluid, B = centre of pressure, G = centre of gravity and M = metacentre. For stable equilibrium of this body

(a) $BG = \frac{I}{V} + MG$ (b) $MG = \frac{I}{V} + BG$

(c) $MG = \left(\frac{I}{V}\right)/BG$ (d) $BG + MG = \frac{I}{V}$

2.51 A body is floating in a liquid as shown in Fig. 2.68. The centre of buoyancy, centre of gravity and metacentre are labeled as B, G and M respectively. The body is

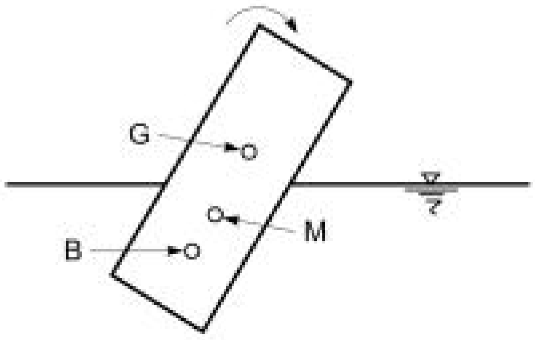


Fig. 2.68

(a) vertically stable
(b) vertically unstable
(c) rotationally stable
(d) rotationally unstable

2.52 An open box of base 2 m × 2 m contains a liquid of specific gravity 0.80 to a height of 2.5 m. When it is given an acceleration vertically upwards of 4.9 m/s^2, the pressure on the base of the tank, in kPa, is

(a) 9.8 (b) 36.8
(c) 19.6 (d) 29.4

2.53 A tank, open at top, contains a liquid with a relative density of 0.85 to a depth of 1.2 m. The acceleration which should be given to the tank to make the pressure at the bottom atmospheric is

(a) 9.8 m/s^2 vertically upward
(b) 9.8 m/s^2 vertically downward
(c) 8.33 m/s^2 vertically upward
(d) 8.33 m/s^2 vertically downward

2.54 An open tank contains a liquid and is made to slide down an inclined plane with uniform velocity. The free surface of the liquid

(a) will be parallel to the plane of the inclined plane
(b) will be horizontal
(c) will be inclined to the horizontal at an angle of $\tan^{-1} 1/g$
(d) will be inclined to the horizontal at an angle which depends upon the slope of the inclined plane

2.55 An open tank containing a liquid slides down a frictionless inclined plane. The free surface of the liquid will be

(a) inclined to the horizontal at an angle which depends upon the slope of the inclined plane
(b) horizontal
(c) parallel to the plane of the inclined plane
(d) inclined at an angle of ($\tan^{-1} 1/g$) to the horizontal

2.56 A U-tube shown in Fig. 2.69 is closed at A and open at B. It is filled with water.
The acceleration to the right needed to make the pressure at C atmospheric is

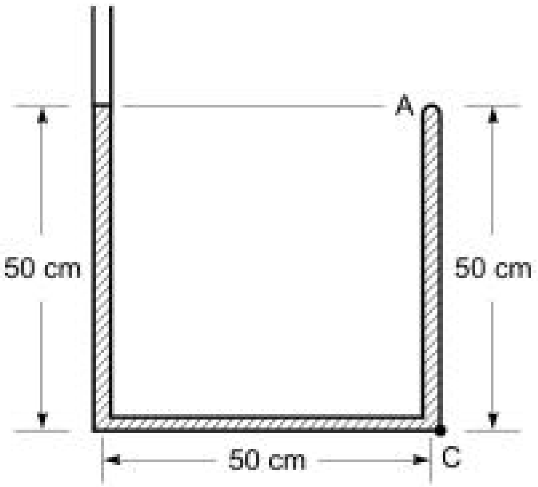


Fig. 2.69

(a) $\frac{1}{2}$g (b) g

(c) 2g (d) $\frac{3}{2}$g

2.57 An open cubical tank was initially filled with water. When the tank was accelerated on a horizontal plane along one of its sides it was found that one third of the volume of water spilled out. The acceleration was

(a) $\frac{1}{3}$g (b) $\frac{1}{2}$g

(c) $\frac{2}{3}$g (d) g

2.58 An open cylindrical vessel filled with a liquid is falling freely with an acceleration *g*. The absolute pressure at any point in the liquid is

(a) zero

(b) above atmospheric pressure

(c) below atmospheric pressure

(d) equal to atmospheric pressure

2.59 Oil of density 800 kg/m^3 stands within 0.8 m from the top of an open tank 2.2 m high, 3.0 m wide and 8 m long. The greatest acceleration that this tank can have without any spilling over is

(a) 9.81 m/s^2 (b) 0.98 m/s^2

(c) 1.96 m/s^2 (d) 5.23 m/s^2

2.60 A liquid in a circular container is given a rigid body rotation about the axis of the cylinder. The piezometric line in a cross section is

(a) a horizontal line (b) a circular arc

(c) a parabola (d) a vertical line

2.61 An open cylindrical tank with its axis vertical is 0.8 m high and is 0.8 m in diameter. It is filled with an oil of density 800 kg/m^3 and is rotated at 120 rpm about the axis of the cylinder. The gauge pressure at the centre of the bottom of the tank is

(a) –0.49 m of oil (b) zero

(c) 1.29 m of oil (d) 6.27 kPa

2.62 A liquid in an open right circular cylinder is given rigid body rotation about the axis of the cylinder. The pressure distribution

(a) in any horizontal plane is uniform

(b) in any vertical plane is uniform

(c) on the bottom of the tank is uniform

(d) in any vertical plane is hydrostatic

2.63 An open circular cylinder 1.2 m high is filled with a liquid to its top. The liquid is given a rigid body rotation about the axis of the cylinder and the pressure at the centre of the bottom is found to be 0.3 m of liquid. The ratio of the volume of liquid spilled out of the cylinder to the original volume is

(a) 3/8 (b) 3/4

(c) 1/2 (d) 1/4

2.64 A liquid undergoing a rigid body rotation in a container is said to have

(a) circulatory flow

(b) circulation

(c) forced vortex motion

(d) free vortex motion

2.65 A 20 cm diameter open cylindrical container contains kerosene (relative density = 0.80) to a height of 20 cm. It is rotated about a vertical axis coinciding with the axis of the cylinder. If the bottom of the cylinder at the axis is just exposed, the speed of rotation is

(a) 94.6 rpm (b) 267.5 rpm

(c) 133.8 rpm (d) 535.0 rpm

2.66 If a cylinder containing a liquid is rotated about a vertical axis coinciding with the axis of the cylinder, the pressure in a vertical

(a) decreases as depth

(b) increases as depth

(c) decreases as square root of depth

(d) increases as square root of depth

2.67 A right circular cylinder is open at the top and filled with a liquid to its top level. It is rotated about its vertical axis at such a speed that half of the liquid spills out. The pressure at the point of intersection of the axis and the bottom is

(a) same as before rotation

(b) 1/2 the value before rotation

(c) 1/4 the value before rotation

(d) equal to the atmospheric pressure

2.68 For a U-tube containing water and rotating about an axis as in Fig. 2.70, the difference in water surface elevations ($h_a - h_b$) is

(a) 6.04 cm (b) 1.51 cm

(c) 3.02 cm (d) 24.15 cm

2.69 The U-tube shown in Fig. 2.71 contains water and is rotated at 120 rpm, about a vertical axis passing through the midpoint O of the horizontal limb. The pressure head at O is

(a) 8.05 cm (b) 11.95 cm

(c) 20.0 cm (d) 6.35 cm

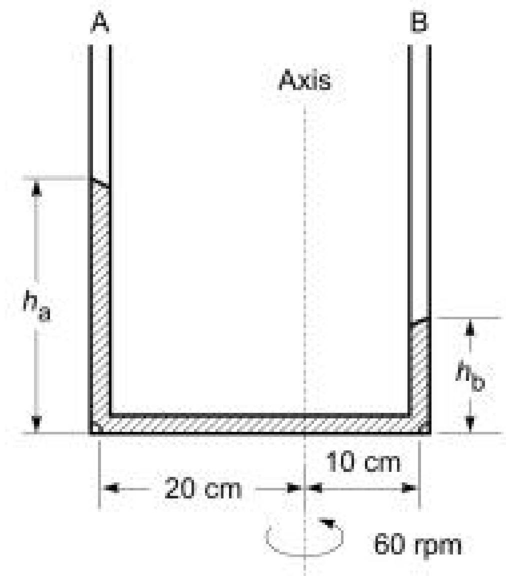


Fig. 2.70 *Question 2.68*

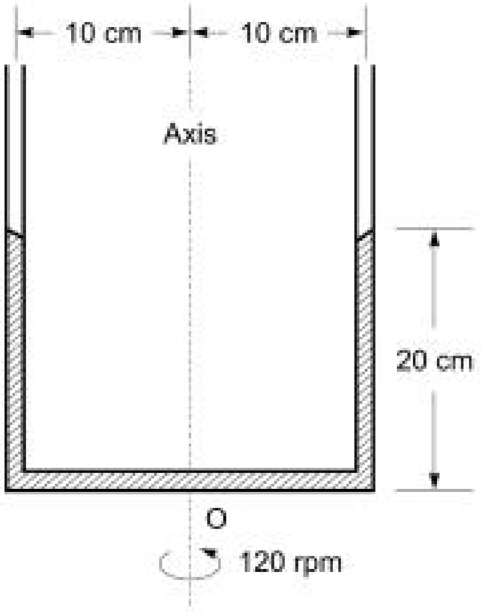


Fig. 2.71 *Question 2.69*

2.70 A 25 cm long pipe closed at one end (A) is filled with water and then the other end (B) is capped. It is then placed in a horizontal position and rotated at 30 rad./s about a vertical axis passing through the capped end, (Fig. 2.72). The difference of pressure between the two ends is

(a) 28.1 kPa
(b) 5.3 kPa
(c) 65.1 kPa
(d) 113.2 kPa

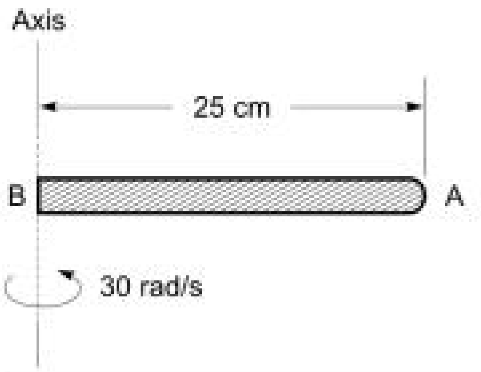


Fig. 2.72 *Question 2.70*

PROBLEMS

A. Measurement of Pressure

2.1 Calculate the pressure in pascals corresponding to

(a) 8 cm column of a liquid of relative density 0.80.
(b) 6 cm column of mercury.
(c) 2.0 m column of water.

(Ans. (a) 626.6 Pa, (b) 7988.6 Pa, (c) 19.58 kPa)

2.2 A bourdon gauge connected to a closed tank indicates 35 kPa at a place where the atmospheric pressure is 95.480 kPa. If the tank is now carried to a higher elevation where the atmospheric pressure is 70.11 kPa, what reading will be indicated by the gauge?

(Ans. p (gauge) = 60.37 kPa)

2.3 A hydraulic press has a ram of 150 mm and a plunger of 20 mm diameter. Find the force required on the plunger to lift a weight of 40 kN. If the plunger has a stroke of 0.40 m and makes 30 strokes per minute, determine the rate at which the weight is lifted per minute and the power required by the plunger.

(Ans. $F = 0.711$ kN, rate = 0.2133 m/min, $\mathbf{P} = 142$ W)

2.4 An open tank contains water to a depth of 2.5 m and an oil of relative density 1.25 to a depth of 1.5 m. Determine the pressure at (a) the water surface (b) at the oil-water interface (c) at a depth of 3.5 m below the free surface and (d) at the bottom of the tank.

(Ans. (a) $p_1 = 0$, (b) $p_2 = 24.475$ kPa (c) $p_3 = 36.713$ kPa, (d) $p_4 = 41.831$ kPa)

2.5 Calculate the pressure and density of air at an elevation of 4500 m above sea level if the atmospheric pressure and density at sea level are 101.35 kPa and 1.2255 kg/m^3. Assume isothermal process.

(Ans. $p_2 = 59.429$ kPa, $\rho_2 = 0.7186$ kg/m^3)

2.6 Assuming adiabatic process exists, determine the pressure and density of air at an elevation of 500 m above sea level given that the atmospheric pressure and density at an elevation of 3000 m are 70.107 kPa and 0.9092 kg/m^3 respectively. Assume $k = 1.4$.

(Ans. $p_2 = 95.054$ kPa, $\rho_2 = 1.1187$ kg/m^3)

2.7 For the system shown in Fig. 2.73 calculate the air pressure p_A to make the pressure at N one third of that at M.

(Ans. $p_A = 0.2937$ kPa)

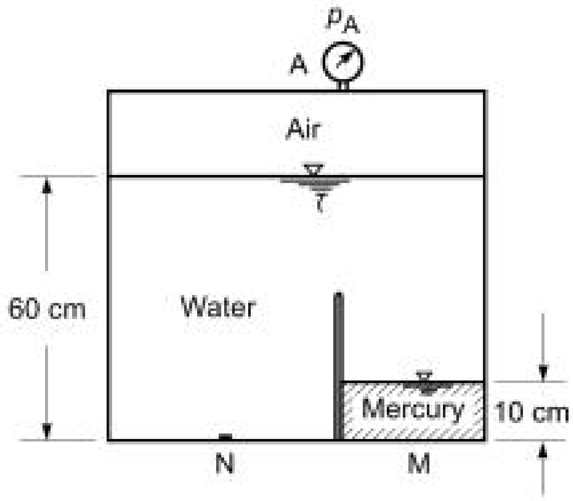

p_A
A
Air
60 cm
Water
Mercury
10 cm
N
M

Fig. 2.73

2.8 For the system shown in Fig. 2.74 determine the pressures at A and B.

(Ans. $p_A = 626.6$ Pa (gauge), $p_B = -4072.6$ Pa (gauge))

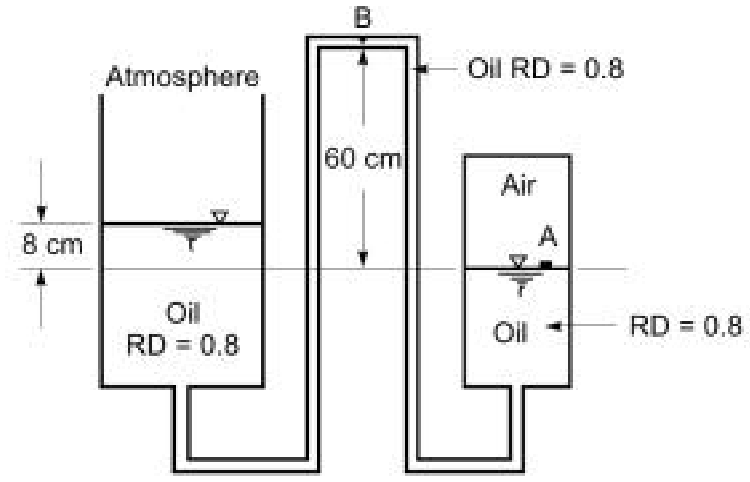

B
Atmosphere
Oil RD = 0.8
60 cm
Air
A
8 cm
Oil
RD = 0.8
Oil
RD = 0.8

Fig. 2.74 *Problem 2.8*

2.9 If the pipe in Fig. 2.75 contains water and there is no flow, calculate the value of the manometer reading h.

(Ans. $h = 0$)

2.10 For the manometer arrangement of Problem 2.9 (Fig. 2.75) there is a flow of water from A towards B. If the manometer reading is $h = 5$ cm, calculate the pressure difference $(p_A - p_B)$.

(Ans. $(p_A - p_B) = 30.643$ kPa))

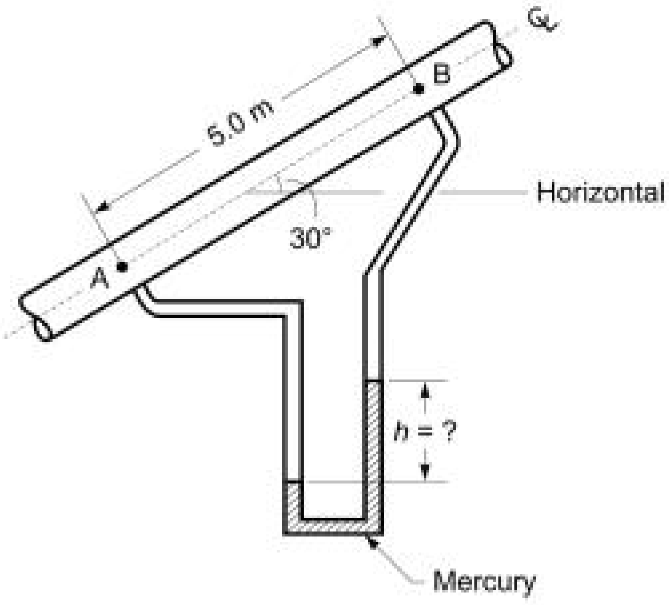

℄
B
5.0 m
Horizontal
30°
A
h = ?
Mercury

Fig. 2.75 *Problem 2.10*

2.11 Two points M and N in a pipe system (Fig. 2.76) are connected a mercury differential manometer. The connecting tube is filled with oil of relative density 0.85. The high pressure point M is 0.80 m above point N. If the mercury column reading is 8 cm (as shown in the figure), what is the pressure difference between M and N in (i) kPa and (ii) metres of water.

(Ans. (i) $(p_M - p_N) = 3.33$ kPa; (ii) $h_w = 0.34$ m))

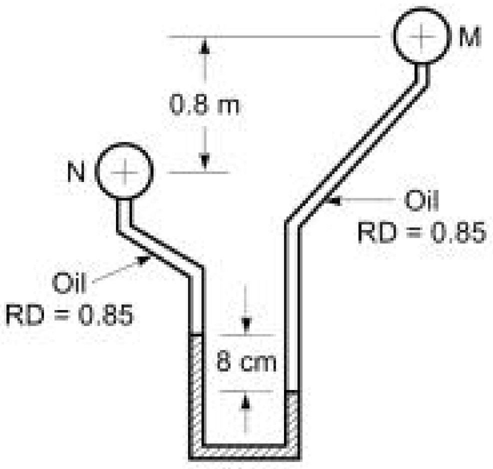

M
0.8 m
N
Oil
RD = 0.85
Oil
RD = 0.85
8 cm

Fig. 2.76

2.12 A tube filled with mercury is placed in a bowl of mercury as in Fig. 2.77. The

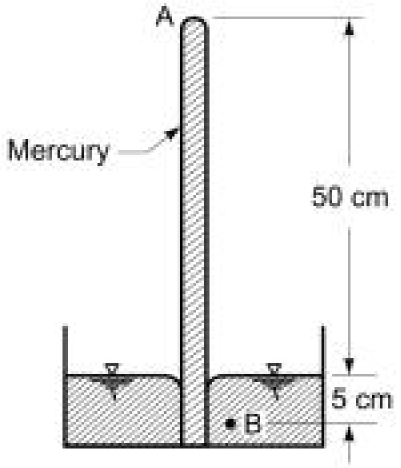

A
Mercury
50 cm
5 cm
B

Fig. 2.77

tube is closed at point A and the other end is open. Calculate the absolute pressures at points A and B if the atmospheric pressure is 95.48 kPa.

(Ans. p_A (abs) = 28.91 kPa (abs), p_B (abs) = 102.137 kPa (abs))

2.13 The barometric pressure at sea level is 760 mm of mercury while on a mountain top it is found to be 685 mm. If the specific weight of air is assumed constant at 11.8 N/m^3, calculate the height of the mountain.

(Ans. $H = 846.3$ m)

2.14 Calculate the pressure and density of air at an elevation of 5000 m above mean sea level. The atmospheric pressure and temperature at sea level are 101.35 kPa and 15°C respectively. The temperature lapse rate is 0.007 K/m. The density of air at sea level is 1.2255 kg/m^3.

(Ans. $p_2 = 52.854$ kPa (abs), $\rho_2 = 0.7413$ kg/m^3)

B. Forces of Plane Surfaces

2.15 A vertical rectangular gate 2.0 m wide and 2.5 m high is subjected to water pressure on one side, the water surface being at the top of the gate. The gate is hinged at the bottom and is held by a horizontal chain at the top. Calculate the tension in the chain.

(Ans. $F = 20.39$ kN)

2.16 An annular ring cut in a sheet metal has 1.5 m outer diameter and 1.0 m inner diameter. It is inserted vertically in a liquid of relative density 0.90 with its centre 1.75 m below the surface. Calculate the total force on one side of this ring and the location of the centre of pressure.

(Ans. $F = 15.138$ kN; $h_p = 1.866$ m)

2.17 A stone masonry wall 6 m high and 2.5 m wide retains water to a height of 5 m on one side. If the specific weight of masonry is 25 kN/m^3, find the direction and magnitude of the resultant force per unit width of wall. Also find the point of action of the resultant force on the base of the wall. What are the maximum and minimum stresses at the base of the wall?

(Ans. $R = 394.5$ kN, $\theta = 71.9°$ to the horizontal, $x = 1.794$ m from the upstream, $\sigma_{\max} = 3.46$ Pa, $\sigma_{\min} = -46$ kPa)

2.18 A right angled triangular plate of base b and height h is immersed in water vertically with its base horizontal and coincident with the water surface. Calculate the total pressure force on one side of the plate and the coordinates of the centre of pressure.

$$\left(Ans.\ F = \frac{\gamma b h^2}{6}, x_p = \frac{b}{4}, y_p = \frac{h}{2}\right)$$

2.19 A vertical gate of width 2.0 m and height 2.5 m controls a sluice opening in a dam. The top of the gate is 10 m below the water surface. If the gate weights 80 kN, find the vertical force required to raise the gate. The coefficient of friction between, the gate and the guides can be assumed to be 0.25. (Neglect buoyancy effect on the gate)

(Ans. $F = 218$ kN)

2.20 Some regular geometrical laminae are shown in Fig. 2.78. These laminae are immersed vertically with their upper edges or top most points on the water surface as indicated in the figures. Derive the expression for the centre of pressure as indicated against each lamina in Table 2.1.

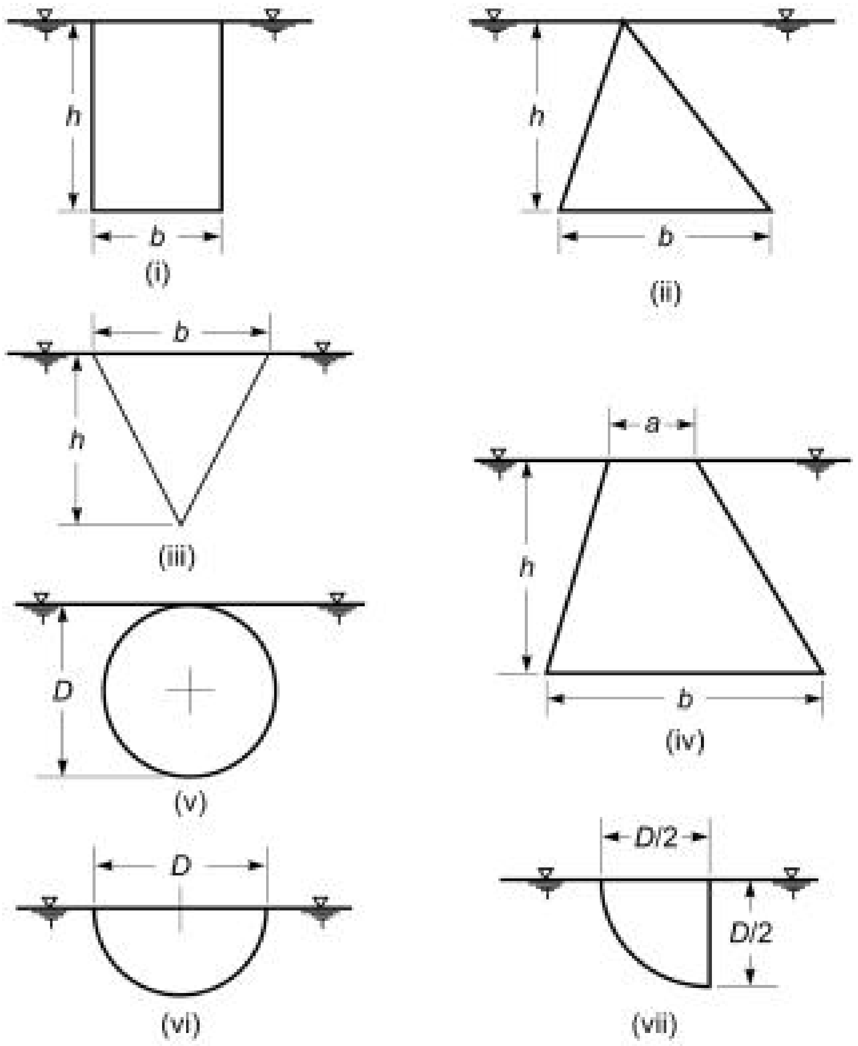


Fig. 2.78 *Problem 2.20*

2.21 A thin plate in the form of a rhombus is immersed in water with a vertex on the free surface and the diagonal through that vertex vertical. Show that the centre of pressure is located at a depth of $h_p = \frac{5}{7}h$ where h = length of the vertical diagonal.

Table 2.1 Location of centre of pressure — Problem 2.20

Lamina	*Description*	*Depth of centre of pressure* h_p
Fig. 2.78(i)	Rectangle	$\frac{2}{3}h$
Fig. 2.78(ii)	Trapezoid	$\frac{h}{2}\frac{(a+3b)}{(a+2b)}$
Fig. 2.78(iii)	Triangle with base on the free surface	$\frac{h}{2}$
Fig. 2.78(iv)	Triangle with vertex on the surface; base is horizontal	$\frac{3}{4}h$
Fig. 2.78(v)	Circle	$\frac{5}{8}D$
Fig. 2.78(vi)	Semicircle with diameter on the free surface	$\frac{3}{32}\pi D$
Fig. 2.78(vii)	Quadrant of circle with one of the bounding radii on the free surface	$\frac{3}{32}\pi D$

2.22 Calculate the force F required to hold the hinged door in Fig. 2.79 in closed position. The door is a 0.5 m square. An air pressure of 30 kPa acts over the water surface.

(Ans. F = 7.217 kN)

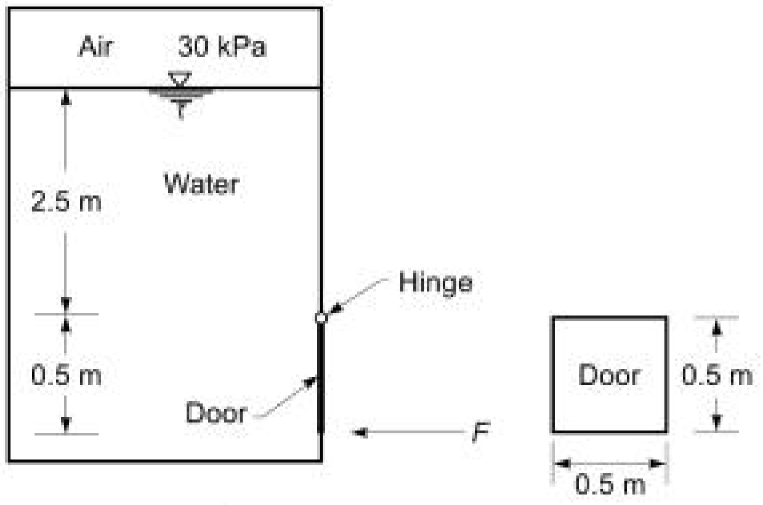


Fig. 2.79

2.23 A vertical gate 5 m × 2.5 m in size and weighing 0.5 tonnes slides along guides fitted on the side walls of an overflow spillway at its crest. The coefficient of friction between the gate and the guide is 0.25. What force will be exerted at the hoisting mechanism to lift the gate when the head of water over the crest is 2.0 m ?

(Ans: F = 3 tonnes)

2.24 The counterweight pivot gate shown in Fig. 2.80 controls the flow from a tank. The gate is rectangular and is 3 m × 2 m. Determine the value of the counterweight W such that the upstream water can be 1.5 m deep.

(Ans. W = 84.8 kN)

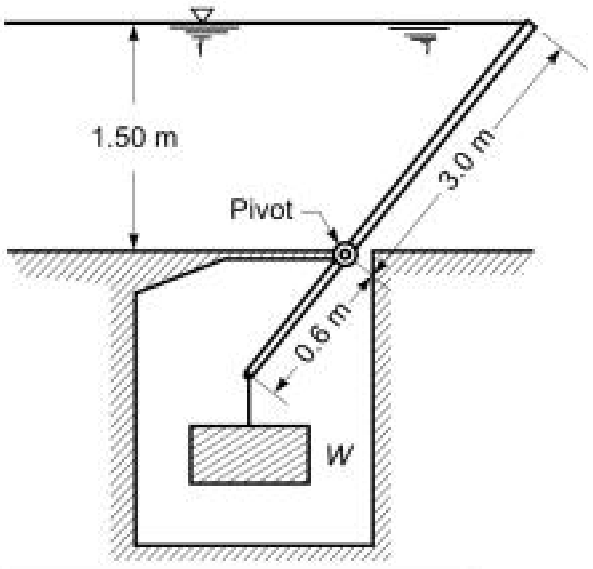


Fig. 2.80 *Problem 2.24*

2.25 The gate shown in Fig. 2.81 weighs 10 kN per metre width. Its centre of gravity is 0.3 m from the vertical face and 0.6 m above the lower horizontal face. Find the height H at which the gate will just topple about the hinge.

(Ans. H = 3.38 m)

2.26 A tank shown in Fig. 2.82 has an isosceles triangular gate hinged at the edge M. Find the horizontal force at M required to keep the gate closed. The weight of the gate is too small and can be neglected.

(Ans. F = 13.06 kN)

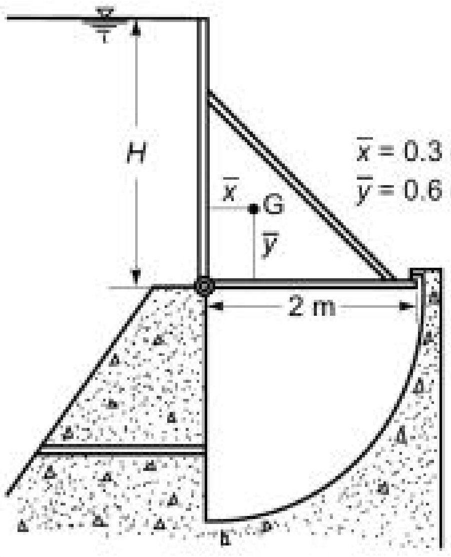


Fig. 2.81 *Problem 2.25*

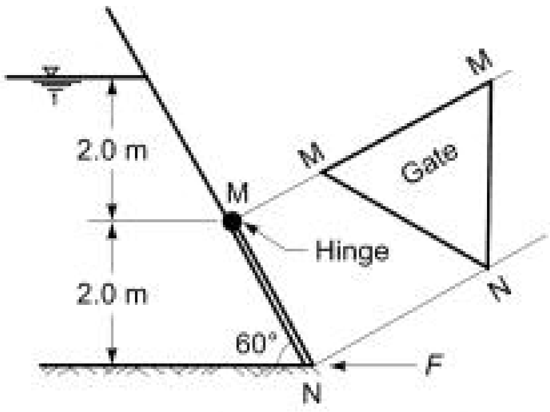


Fig. 2.82 *Problem 2.26*

2.27 The rectangular gate AB in Fig. 2.83 is 2.0 m high and 1.5 m wide. Calculate the net hydrostatic force on the gate.

(Ans. $F_1 - F_2 = 37.83$ kN)

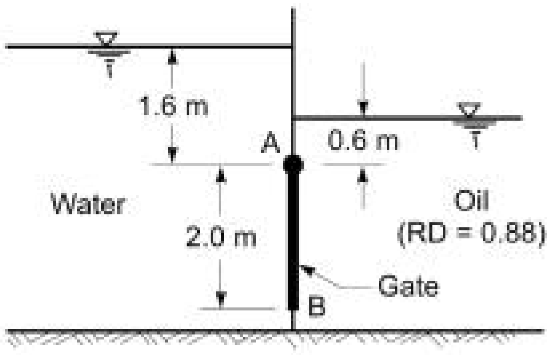


Fig. 2.83

2.28 A circular plate 3 m in diameter is submerged in water so that the greatest and least depths below the free surface are 2.0 m and 1.0 m respectively. Find (a) the total pressure force on one side of the plate and (b) the position of the centre of pressure.

(Ans. $F = 103.77$ kN, $h_p = 1.5416$ m)

2.29 For the flash board shown in Fig. 2.84 find the depth of water H and the compressive force on the strut per metre length of the flash board when the board is about to trip.

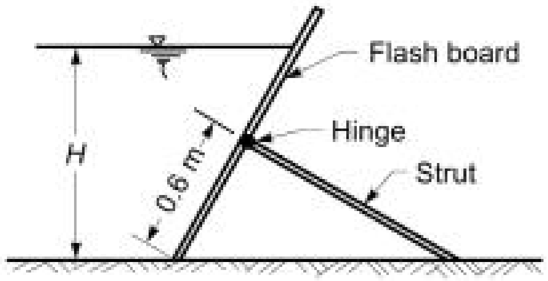


Fig. 2.84

(***Hint:*** At the time of tipping the centre of pressure coincides with the hinge.)

(Ans. $H = 1.559$ m; $F = 13.73$ kN)

2.30 A square aperture in the vertical side of a tank has one diagonal vertical and is completely covered by a plane plate hinged along one of the upper sides of the aperture. The diagonals of the aperture are 2 m long and the tank contains a liquid of relative density 1.15. The centre of aperture is 1.5 m below the free surface. Calculate the thrust exerted on the plate by the liquid and the position of the centre of pressure.

(Ans. $F = 33.78$ kN, $h_p = 1.611$ m)

2.31 A concrete gravity dam is in the form of a right angled triangle in section with the vertical side facing the water. The height of the dam is 10 m and the base width is 8 m. Assuming a specific weight of 25 kN/m^3 for the concrete, calculate the resultant force per unit length of dam and its point of action on the floor of the dam when it retains 10 m of water. What are the minimum and maximum vertical stresses on the base of the dam?
(Ans. $R = 1113.3$ kN, $\theta = 26.08°$, $\sigma_1 = 153.1$ kPa, $\sigma_2 = 96.9$ kPa)

2.32 A tank contains a suspension to a height H. The suspension can be considered to be a liquid whose density increases linearly, from ρ_0 at the surface, towards the bottom. Show that the pressure at any depth y is given by

$$p = \rho_0 g y + \frac{K}{2} g y^2 + p_0$$

where p_0 = pressure at the free surface and K = rate of increase of the density with depth. Show that the force per unit width on a vertical side of the tank is given by

$$F = \left[\frac{\rho_0 g H^2}{2} + \frac{K}{6} g H^3 + p_0 H\right]$$

C. Forces On Curved Surfaces

2.33 A circular cylinder of 1.8 m diameter and 2.0 m long is used for water level control in a tank. If at a given instance it retains water as shown in Fig. 2.85 determine the reaction at the joint A.

(Ans. R_H = 31.72 kN, R = 61.96 kN R_v = 53.23 kN, tan θ = 0.596 where θ = inclination of R to the vertical. M_A = 36.0 kN.m (clockwise))

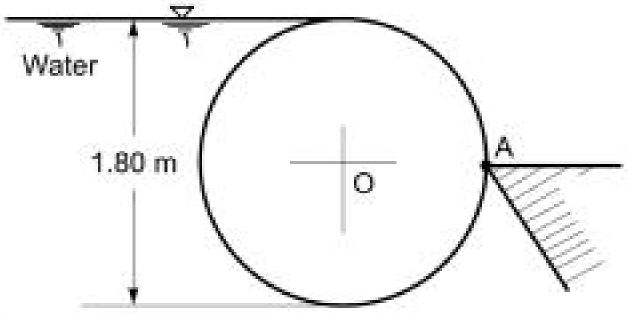


Fig. 2.85 *Problem 2.33*

2.34 A cylinder of diameter 0.6 m is located in water as shown in Fig. 2.86. The cylinder and the wall are smooth. If the length of the cylinder is 1.5 m, find (i) its weight, (ii) the resultant force exerted by the wall on the cylinder and (iii) the resultant moment about the centre of the cylinder due to water forces on the cylinder.

(Ans. 4410 kN, 661.5 kN, zero)

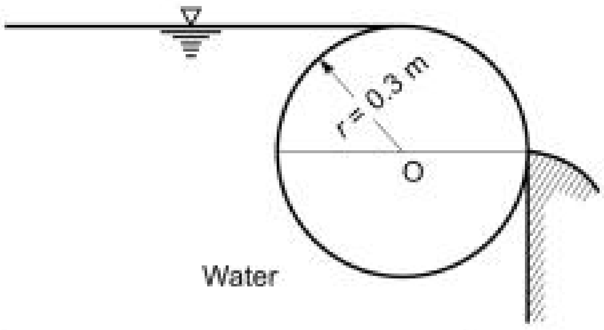


Fig. 2.86 *Problem 2.34*

2.35 A radial gate retains 5 m of water above the crest of a dam as shown in Fig. 2.87. Find the resultant water force on the gate per metre length.

(Ans. R = 124.4 kN, α = inclination of R to the horizontal = 10.235°. The resultant passes through the centre O.)

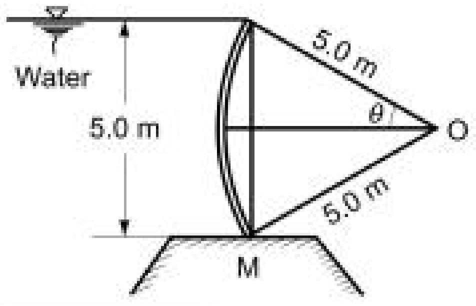


Fig. 2.87

2.36 For a cylindrical gate 4 m long shown in Fig. 2.88 calculate the resultant force due to fluid pressure.

(Ans. R = 219.64 kN, α = inclination of R to the horizontal = 40.34°, the resultant passes through the centre of the gate.)

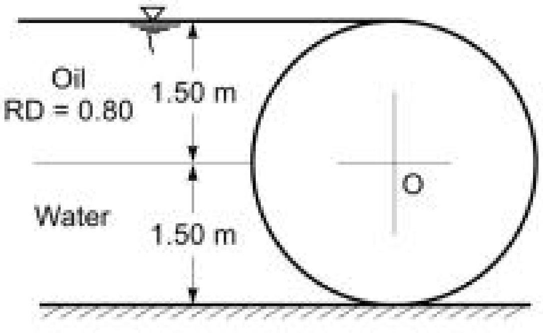


Fig. 2.88 *Problem 2.36*

2.37 For the container shown in Fig. 2.89 estimate the resultant force on the hemispherical bottom.

(Ans. R_V = 52.46 kN, R_H = 0 (due to symmetry))

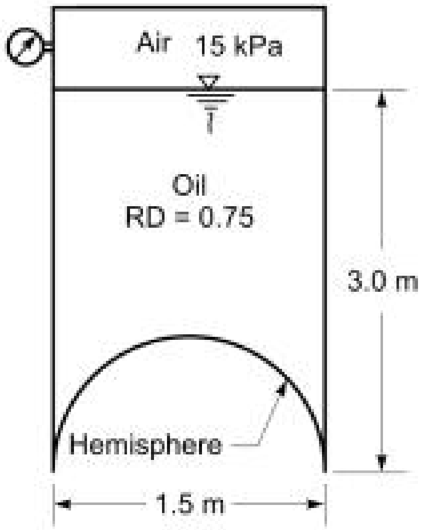


Fig. 2.89

2.38 A cylindrical barrier of diameter 2.0 m and width 3.0 m retains water on both sides with water surface elevations of 1.5 m and 0.5 m above the lowest point of the barrier, respectively. Calculate the resultant hydrostatic force on the barrier.

(Ans. R = 54.69 kN; α = inclinations of R to the horizontal = 57.5°)

2.39 A 1.8 m diameter cylindrical tank is laid with its axis horizontal on a level ground. Each of its ends are closed by a hemispherical dome. The tank contains oil of relative density 0.9 under pressure. If a pressure gauge on the top of the tank reads 22 kPa, calculate the resultant force on the hemispherical end.

(Ans. R = 54.69; α = inclination of R to the horizontal = 10°, the resultant passes through the centre of the hemisphere)

2.40 A cylindrical tank of diameter 2.5 m is founded with its axis horizontal. The tank contains water in the bottom half and an oil of relative density 0.82 on the top half with a common interface. The fluids are under pressure and a pressure gauge on the top of the cylinder reads 15 kPa. Calculate the force per metre length on the upper half of the cylindrical tank.

(Ans. F_V = 42.9 kN/m length (upward))

2.41 A conical valve of weight 2.5 kN deeps water from flowing out of a tank as shown in Fig. 2.90. It is held in position by a counter weight of 10 kN connected by a string passing over frictionless pullies as in Fig. 2.90. Find the maximum height H of water in the tank, which will make the device to function without any leak.

(Ans. H = 1.064 m)

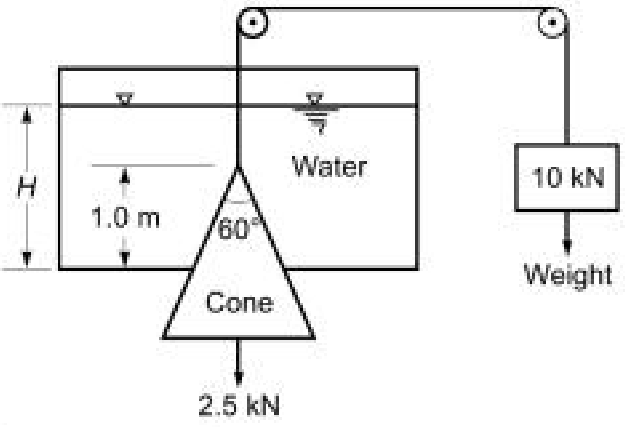


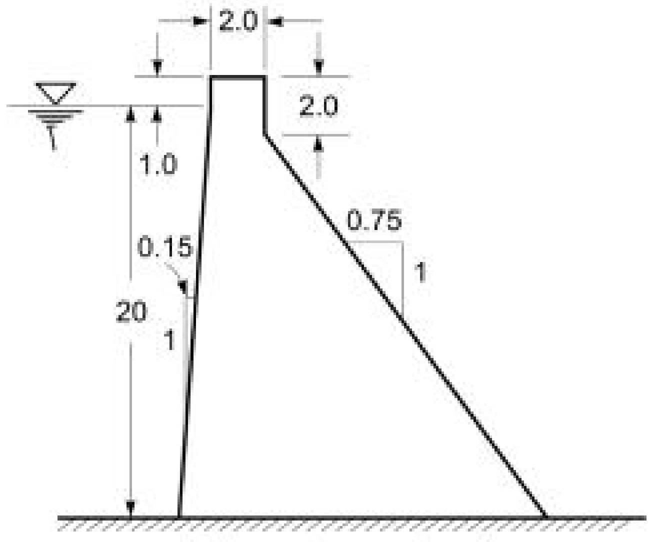
Fig. 2.90

2.42 A gravity dam has a cross section as shown in Fig. 2.91. It retains water to a depth of 20 m on the upstream. Calculate (i) the resultant force at the base per unit length of the dam, (ii) maximum and minimum stresses at the base. Assume specific weight of dam material to be 25 kN/m^3 and neglect all uplift pressures.

(Ans. (i) R = 5817 kN, α = inclination of R to the horizontal = 70.33°, (ii) σ_1 = 295.6 kPa on the upstream end of the base, σ_2 = 273.6 kPa on the downstream end of the base)

2.43 A rectangular water tank has its vertical sides joined to the horizontal bottom through a smooth curve which can be approximated to a quadrant of a circle of radius 1.6 m. If the tank contains 2.0 m of water and 2.5 m of oil relative density 0.8 over it, calculate the resultant force per metre length of the curved surface.

(Ans. R = 76.1 kN; α = inclination of R to the horizontal = 48.8°, the resultant passes through the centre of the arc.)



All dimensions are in metres

Fig. 2.91 *Problem 2.42*

2.44 A cylinder of 0.8 m diameter is made of 5 mm thick plates. If the maximum permissible stress in the plates is 100 MPa, calculate the maximum permissible pressure inside the cylinder.

(Ans. σ = 1.25 MPa)

2.45 A pressure vessel in the form of cylinder of 1.5 m diameter is to have a fluid under a pressure of 1.2 MPa. If the allowable stress in the material of the walls is 140 MPa find the minimum thickness of the walls.

(Ans. t = 6.5 mm)

2.46 A spherical vessel is made up of a material of permissible stress of 150 MPa. The diameter of the vessel is 15 m and the thickness of the sides is 5 mm. Find the maximum permissible pressure in this vessel.

(Ans. p = 2.0 MPa)

2.47 A pair of lock gates each of 3 m width make an angle of 120° when closed. The gates are supported by two hinges at 0.5 m and 4.5 m above the bottom. Determine the reaction force on each hinge when the lock has 4.8 m and 1.5 m of water on upstream and downstream respectively.

(Ans. R_{top} = 93.0 kN, R_{bottom} = 212.3 kN)

D. Buoyancy

2.48 A closed cylindrical tank of diameter 2.0 m, height 1.2 m and weighing 20 kN is floating with its axis vertical in sea water (relative density = 1.025).

(a) Find the depth of the cylinder below the water surface.

(b) What would be the depth of immersion if an additional load of 5.0 kN is added at the top?

(Ans. (a) h_1 = 0.6344 m,
(b) h_2 = 0.793 m)

2.49 Two spheres each of 1.5 m diameter weigh 4.0 and 20.0 kN respectively. They are connected by a short rope and placed in a water reservoir. Find the tension in the rope and the percentage of volume of the lighter sphere that will be above water.

(Ans. T = 2.7 kN; percentage volume = 61.27%)

2.50 A metal sphere of volume $V_m = 1\ \text{m}^3$, relative density $S_m = 2$ and fully immersed in water is attached by a flexible wire to a buoy of volume $V_b = 1\ \text{m}^3$ and relative density $S_b = 0.1$ [See Fig. 2.92]. Calculate the tension T in the wire and volume of the buoy that is submerged.

(Ans. T = 1.96 kN, $V_b = 0.2\ \text{m}^3$)

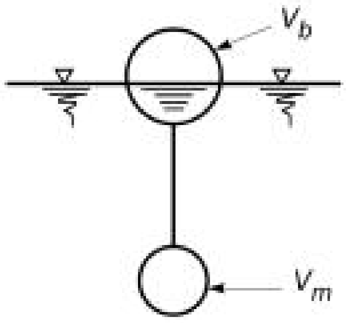


Fig. 2.92

2.51 An iceberg has a specific weight of 9.0 kN/m^3 and floats in sea water of specific weight 10.05 kN/m^3. What percentage of the total volume of the iceberg will be above the sea water surface?

(Ans. 10.5%)

2.52 A hydrometer is to be so built that the mark corresponding to relative density of 1.0 and 1.5 are 8 cm apart on a 5 mm diameter stem. How far from the mark of 1.0 will the relative density mark of 1.25 be located?

(Ans. h = 4.8 cm)

2.53 A hydrometer has a 6 mm diameter stem. The distance between markings of relative density 1.0 and 0.90 is 10 cm. Determine the weight of the hydrometer.

(Ans. W = 0.249 N)

2.54 A 10 cm cube of steel (relative density = 7.85) is to float on mercury in a container of square cross section, with a clearance of 1 cm all round. Find the weight of mercury (relative density = 13.6) required.

(Ans. W = 53 N)

2.55 An open cylindrical bucket 30 cm diameter and 50 cm long whose wall thickness and weight can be considered negligible is forced open end first into water until its lower edge is 10 m below the water surface. What force will be required to maintain this position? Assume that the trapped air undergoes any change under isothermal conditions. Atmospheric pressure = 100 kPa, temperature of water = 20°C.

(Ans. F = 177 N)

2.56 A pontoon of rectangular cross sectional area is 7.0 m long, 3.0 m wide and 1.5 m high. The depth of submergence of the pontoon is 0.9 m and its centre of gravity is 0.7 m above its bottom. Determine its metacentric height.

(Ans. MG = 0.583 m)

2.57 A solid cylinder of diameter 30 cm and height 1.20 m is to float in water with its axis vertical. Determine its stability if the relative density of the cylinder is 0.5.

(Ans. MG = – 0.2906 m; unstable)

2.58 A solid cylinder of diameter 30 cm and height 15 cm floats with its axis vertical in sea water (rel. den. = 1.03). If the relative density of the cylinder material is 0.9, examine the stability of the cylinder.

(Ans. MG = 3.34 cm; stable)

2.59 A cube of side a floats with one of its axes vertical in a liquid of relative density S_L. If the relative density of the cube material is S_c, find the condition for the metacentric height to be zero.

$$\left(\textit{Ans.}\ \frac{S_L}{S_c} = 1.268 \text{ or } 4.732 \right)$$

2.60 A solid cube of sides 0.5 m is made of a material of relative density 0.5. The cube floats in a liquid of relative density 0.95 with two of its faces horizontal. Examine its stability.

(Ans. MG = – 0.0393 m; unstable)

2.61 A rectangular barge of width b and depth of submergence H has its centre of gravity at the waterline. Find the metacentric height and the value of the ratio b/H for which the barge is stable.

$$\left(\textit{Ans.}\ \text{MG} = \frac{b^2}{12H} - \frac{H}{2}, \text{ for stability } b/H \geq \sqrt{6} \right)$$

2.62 A solid cone with an apex angle of 60° and relative density S_c is floating in mercury (rel. den. = 13.6) with its vertex downwards and axis vertical. Determine the range of S_c over which the cone is in stable equilibrium.

(Ans. $S_c > 5.738$)

2.63 A cone of relative density 0.8 is to float in water with its axis vertical and vertex downwards. Find the least apex angle of the cone for stable equilibrium.

(Ans. $\theta = 31° \, 03' \, 33''$)

2.64 Show that, when a solid hemisphere of density ρ_s and radius r floats with its plane base immersed in a liquid of density ρ_f, the equilibrium is stable and the metacentric height is $\frac{3}{8} r \left(\frac{\rho_f}{\rho_s} - 1 \right)$.

E. Rigid Body Motion

2.65 An open tank is 7 m long, 2 m wide and 1.5 m deep. It contains oil of relative density 0.8 to a depth of 1.0 m. If the tank is given a horizontal acceleration at a constant value of 2.5 m/s^2 along its length, calculate the amount of oil spill. What are the pressures on the bottom of the tank at its front and rear end?

(Ans. Spill = 5.171 m^3, $p_{\text{front}} = 0$ (atmospheric), $p_{\text{rear}} = 14.685$ kPa)

2.66 An open tank 5 m long, 3 m wide and 2 m deep contains 1.5 m water. What is the maximum horizontal acceleration that can be given to the tank without causing spill over, and what is the alignment of the tank with respect to the movement?

(Ans. $a_x = 3.27$ along the width)

2.67 An open tank 3.0 m long, 2.0 m wide and 2.0 m deep contains water to a depth of 0.9 m. What minimum horizontal acceleration along the length should be given to have zero depth of the water along the front edge of the tank?

(Ans. $a_x = 5.886$ m/s^2)

2.68 A 2.5 m long open tank is mounted on a carriage which moves up a plane inclined at 35° to the horizontal at an acceleration of 2.0 m/s^2. What is the slope of the water surface? If the tank is 1.2 m deep and initially contains water to a height of 0.6 m, what would be the depths at the forward and rear edges of the tank?

(Ans. $\theta = 8.5°$ with lower depth in the front, $y_{\text{front}} = 0.413$ m, $y_{\text{rear}} = 0.787$ m)

2.69 A 30 cm diameter cylinder contains oil of specific weight 7.5 kN/m^3 to a height of 120 cm. Calculate the force on the bottom of the tank when the tank undergoes an acceleration of 3.5 m/s^2 (a) vertically downwards and (b) vertically upwards.

(Ans. (a) $F_V = 0.863$ kN, (b) $F_V = 0.409$ kN)

2.70 An open tank containing water slides down an inclined plane without friction. Show that the water surface will be (a) parallel to the plane if the acceleration is equal to the component of g along the inclined plane, (b) horizontal if the velocity of slide is constant.

2.71 A closed tank shown in Fig. 2.93 is given an acceleration of 3.6 m/s^2, the accelerations vector being inclined at an angle of 35° to the horizontal. Calculate the pressures at point M, N and R.

Ans. ($P_M = 10.406$ kPa, $P_N = 25.73$ kPa; $P_R = 3.703$ kPa)

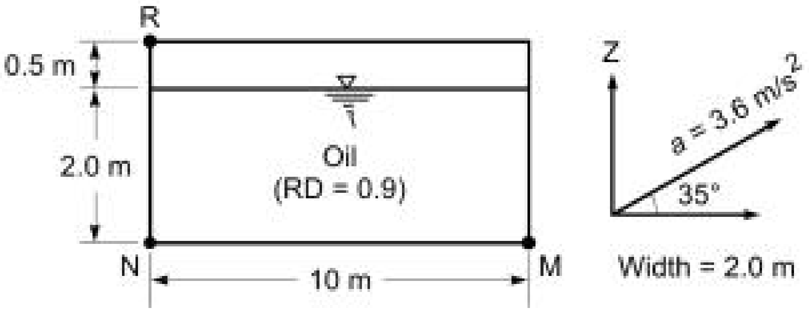


Fig. 2.93

2.72 An open cylindrical tank of 30 cm diameter is 40 cm high. The tank is filled with water to a depth of 30 cm. If the tank is rotated about the vertical axis of the cylinder, find the maximum speed of rotation that does not cause any spill of the liquid.

(Ans. $N = 126$ rpm)

2.73 A 40 cm diameter cylindrical tank is 35 cm high and is open at the top. Initially it contains water to a depth of 20 cm. If the tank is rotated about its vertical axis at 120 rpm, calculate the amount of liquid spilled out.

(Ans. $V_s = 0.34$ L)

2.74 If the tank in Problem 2.73 is rotated at 150 rpm, calculate the volume of water spilled out.

(Ans. $V_s = 7.01$ L)

2.75 A closed cylinder 40 cm in diameter and 40 cm in height is filled with oil of relative density 0.80. If the cylinder is rotated about its vertical axis at a speed of 200 rpm, calculate the thrust of oil on top as well as bottom of the cylinder.

(Ans. $F_T = 440$ N, $F_B = 834$ N)

2.76 A hemispherical bowl of radius 1.0 m is full of water and is to be rotated about its vertical axis at 30 rpm. Estimate the amount of water that will overflow.

(Ans. 0.790 m^3/s)

2.77 A U-tube has a liquid of relative density 0.85 in its limbs to a height of 50 cm above the horizontal limb of 30 cm length (Fig. 2.94). What will be the difference in elevation of the two free surfaces when the tube is rotated about a vertical axis 10 cm from one leg and 20 cm from the other, at 180 rpm?

(Ans. $(y_2 - y_1) = 54.3$ cm)

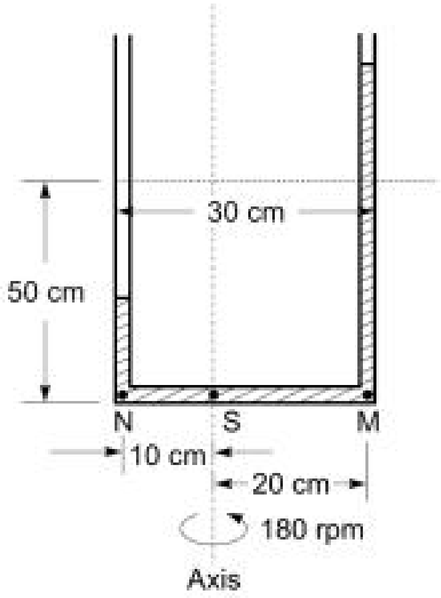


Fig. 2.94

2.78 For the U-tube of Problem 2.77, under the given conditions, determine the pressures at points N, S and M.

(Ans. $p_N = 1.79$ kPa, $p_S = 0.372$ kPa, $p_M = 6.042$ kPa)

2.79 A 20 cm high cylinder open at one end contains 10 cm of water and its top is connected to a 1.0 m lever arm. If the cylinder is rotated in the vertical plane at an angular velocity of 5 rad./s, calculate the pressure at the bottom of the cylinder when (a) it is at its highest point and (b) at its lowest point in the rotation. (Use an average radial distance of 1.15 m in calculating accelerations).

(Ans. (a) $p_b = 1890$ Pa, (b) $p_b = 3848$ Pa)

2.80 At what angular velocity must a U-tube, of 30 cm horizontal limb length and filled with water to a depth of 30 cm in the vertical limbs, be rotated about a vertical axis at mid distance from the vertical limbs, to cause cavitation at the point of intersection of the axis with the horizontal limb? The vapour pressure of water can be taken as 2.50 kPa (abs) and the atmospheric pressure as 95.5 kPa (abs). The density of water is 998 kg/m^3.

(Ans. $N = 869$ rpm)

(***Hint:*** Cavitation in water occurs when the local pressure reaches the vapour pressure)

2.81 For the U-tube containing mercury, shown in Fig. 2.95 what speed of rotation causes the differences in limb heights as indicated?

(Ans. $N = 48.8$ rpm)

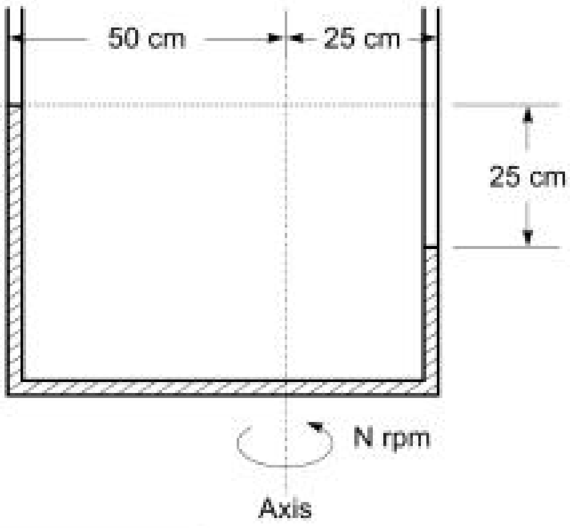


Fig. 2.95

3

Fluid Flow Kinematics

CONCEPT REVIEW

3.1 INTRODUCTION

The kinematics of fluid flow deals with the velocity, acceleration and other related aspects of space-time relations without specifically considering the associated forces. Thus the basic definitions and classification, aspects dealing with velocities and accelerations, continuity equation, stream function and potential functions to describe the fluid flow are dealt with in this chapter. The ideas of kinematics are essential in the successful application of energy and momentum equations (described in subsequent chapters) to solve various fluid flow problems. It is to be remembered that velocity and acceleration are vectors and hence have magnitude as well as direction.

To visualize the flow of fluids the following patterns generated by lines are used:

- A *stream line* as a line tangent to the velocity vector at every point in a given instant.
- A *path line* is the actual path traversed by a given fluid particle
- A *streak line* is a locus of particles which had earlier passed through a chosen point
- A *time line* is a set of fluid particles that form a line at a given instant.

Only stream lines are convenient for mathematical calculations. Other lines are used for flow visualizations.

3.2 CLASSIFICATION OF FLOW

(A) *Steady flow:* Fluid flow conditions at any point do not change with time. For example

$$\boxed{\frac{\partial \mathbf{V}}{\partial t}=0,\ \frac{\partial p}{\partial t}=0,\ \frac{\partial \rho}{\partial t}=0}$$

In a steady flow steam line, path line and streak line are identical.

Unsteady flow: Flow parameters at any point change with time, e.g. $\frac{\partial \mathbf{V}}{\partial t} \neq 0$.

(B) *Uniform flow*: The velocity vector **V** is identically same at all points at a given instant. Thus

$$\boxed{\frac{\partial \mathbf{V}}{\partial t}=0}$$

Non-uniform flow: The velocity vector **V** at any instant varies from point to point.

3.3 STREAMLINE

In a fluid flow, a continuous line so drawn that it is tangential to the velocity vector at every point is known as a streamline. If the velocity vector $\mathbf{V} = \mathbf{i}u + \mathbf{j}v + \mathbf{k}w$ then the differential equation of a streamline is given by

$$\boxed{\frac{dx}{u}=\frac{dy}{v}=\frac{dz}{w}} \tag{3.1}$$

3.4 ACCELERATION

Acceleration is a vector.

(i) In the natural co-ordinate system, viz. along and across a streamline (Fig. 3.1).

$$a = \frac{\mathrm{d}V}{\mathrm{d}t} \quad \text{and} \quad a = \sqrt{a_s^2 + a_n^2} \tag{3.2}$$

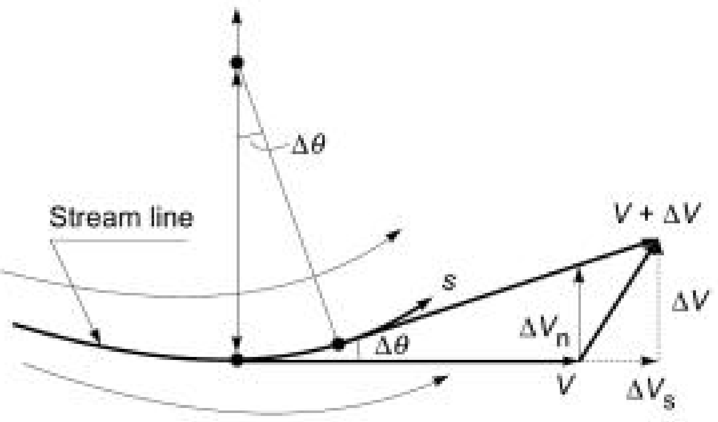


Fig. 3.1

In the tangential direction:

$$a_s = \frac{\partial V_s}{\partial t} + V\frac{\partial V_s}{\partial s} \tag{3.3}$$

In the normal direction

$$a_n = \frac{\partial V_n}{\partial t} + \frac{V_s^2}{r} \tag{3.4}$$

where r = radius of curvature of the streamline at the point, V_s = tangential component of the velocity V and V_n = normal component of velocity generated due to change in direction. The terms $\partial V_s/\partial t$ and $\partial V_n/\partial t$ are called *local accelerations*. Also $V(\partial V_s/\partial_s)$ = *tangential convective acceleration* and V^2/r = *normal convective acceleration*.

(ii) In Cartesian co-ordinates:

$$\mathbf{V} = \mathbf{i}u + \mathbf{j}v + \mathbf{k}w$$

Acceleration a_x, a_y and a_z in the x, y, z directions respectively are:

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \tag{3.5}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \tag{3.6}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \tag{3.7}$$

(iii) In two-dimensional polar co-ordinates (Fig. 3.2)

$$a_r = \frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v^2}{r} \tag{3.8}$$

$$a_\theta = \frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} - \frac{v_r v_\theta}{r} \tag{3.9}$$

3.5 CONTINUITY EQUATION

3.5.1 In One-dimensional Analysis

In steady flow, mass rate of flow into stream tube is equal to mass rate of flow out of the tube

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \tag{3.10}$$

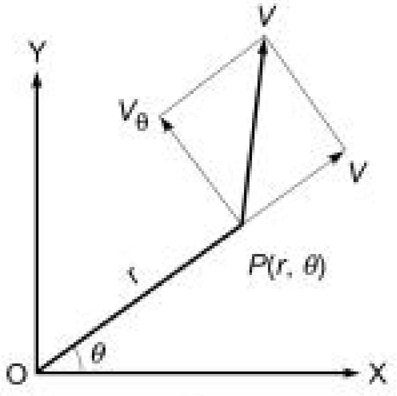


Fig. 3.2

(i) For incompressible fluid, under steady flow (Fig. 3.3).

$$\boxed{A_1V_1 = A_2V_2} \tag{3.11}$$

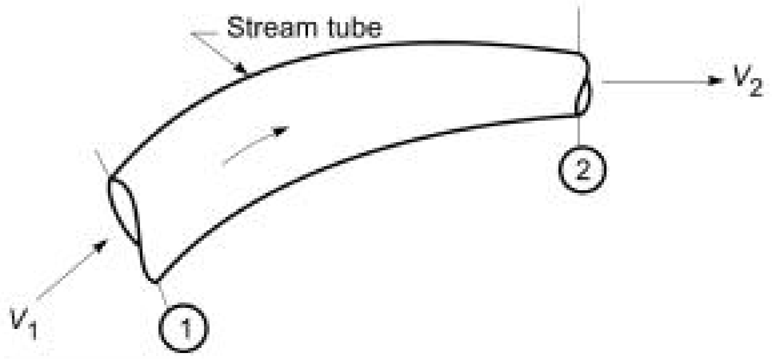


Fig. 3.3

(ii) When there is a variation of velocity across the cross section of a tube, for an incompressible fluid discharge, (Fig. 3.4)

$$\boxed{Q = \int_{A_1} v\,\mathrm{d}A = \int_{A_2} v\,\mathrm{d}A} \tag{3.12}$$

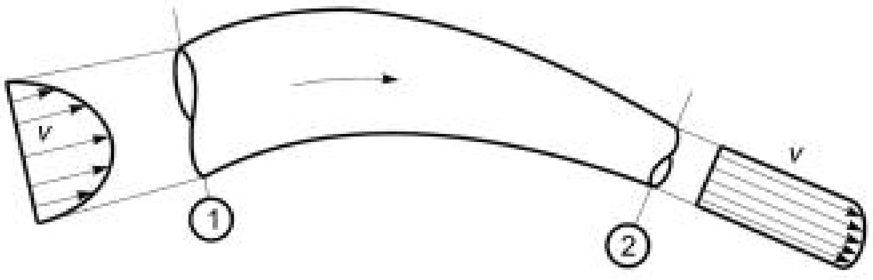


Fig. 3.4

3.5.2 In Differential Form

Cartesian co-ordinates:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \tag{3.13}$$

For incompressible fluid ($\mathrm{d}\rho/\mathrm{d}t$) = 0) and hence Eq. 3.13 is simplified as

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0} \tag{3.14}$$

3.6 ROTATIONAL AND IRROTATIONAL MOTION

Consider a rectangular fluid element of sides dx and dy [(Fig. 3.5(a)]. Under the action of velocities acting on it let it undergo deformation as shown in Fig. 3.5(b) in a time dt.

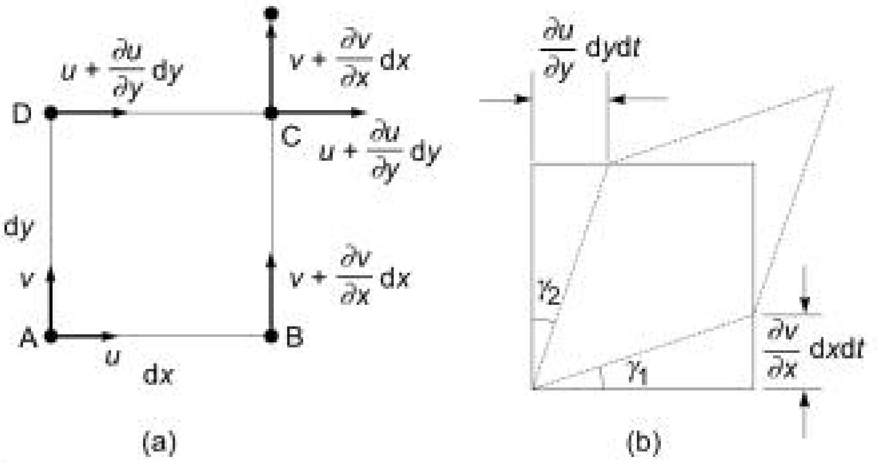

$u + \frac{\partial u}{\partial y}$ dy
$v + \frac{\partial v}{\partial x}$ dx
D
C
$u + \frac{\partial u}{\partial y}$ dy
dy
v
$v + \frac{\partial v}{\partial x}$ dx
A
B
u
dx
(a)
$\frac{\partial u}{\partial y}$ dydt
γ_2
γ_1
$\frac{\partial v}{\partial x}$ dxdt
(b)

Fig. 3.5

$$\gamma_1 = \text{angular velocity of element AB} = \frac{\partial v}{\partial x}$$

$$\gamma_2 = \text{angular velocity of element AD} = \frac{\partial u}{\partial y}$$

Considering the anticlockwise rotation as positive, the average of angular velocities of two mutually perpendicular elements is defined as the rate of rotation.

Thus rotation about z-axis

$$\boxed{\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)} \tag{3.15}$$

Thus for a three-dimensional fluid element, three rotational components as given in the following are possible:

$$\left.\begin{aligned} &\text{About z axis, } \omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \\ &\text{About y axis, } \omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \\ &\text{About x axis, } \omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \end{aligned}\right\} \tag{3.16}$$

Fluid motion with one or more of the terms ω_x, ω_y or ω_2 different from zero is termed *rotational motion*.

Twice the value of rotation about any axis is called as *vorticity* along that axis. Thus the equation for vorticity along z-axis is $\zeta_z = 2w_z = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$.

A flow is said to be *irrotational* if all the components of rotation are zero,

viz. $\omega_x = \omega_y = \omega_z = 0,$

i.e. $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0,\ \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0;$

and $\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0$

Thus for a two-dimensional irrotational flow

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

or

$$\boxed{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0} \tag{3.17}$$

3.6.1 Circulation

In rotational fluid motion, circulation is very useful concept. Circulation is defined as the line integral of the tangential component of the velocity taken around a closed contour (Fig. 3.6). The limiting value of circulation divided by the area of the closed contour, as the area tends to zero, is the vorticity along an axis normal to the area.

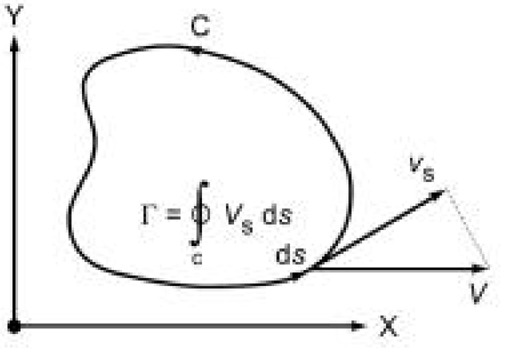


Fig. 3.6 *Circulation Concept*

3.7 STREAM FUNCTION

In a two-dimensional flow consider two streamlines S_1 and S_2. The flow rate (per unit depth) of an incompressible fluid across the two streamlines is constant and is independent of the path, (path *a* or path *b* from A to B in Fig. 3.7). A stream function ψ is so defined that it is constant along a streamline and the difference of ψs for the two streamlines is equal to the flow rate between them. Thus $\psi_B - \psi_A$ = flow rate between S_1 and S_2. The flow left to right is taken as positive, in the sign convention. The velocities u and v in x and y directions are given by

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{3.18}$$

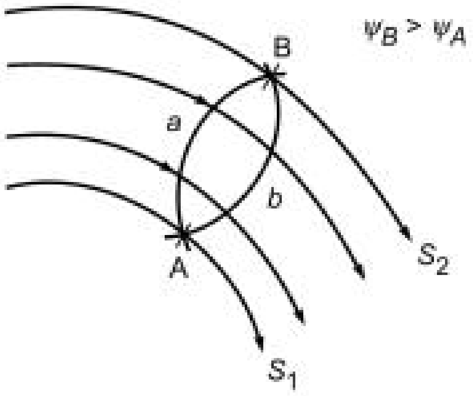


Fig. 3.7

The stream function ψ is defined as above for two dimensional flows only.

For an irrotational flow, $\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0$

and hence, $-\dfrac{\partial^2 \psi}{\partial x^2} - \dfrac{\partial^2 \psi}{\partial y^2} = 0$

That is, the Laplace equation $\dfrac{\partial^2 \psi}{\partial x^2} + \dfrac{\partial^2 \psi}{\partial y^2} = 0$ (3.19)

is satisfied by the stream function in irrotational flow. Conversely, if ψ does not satisfy $\nabla^2 \psi = 0$, then the flow is rotational.

In polar coordinates $v_r = \dfrac{1}{r}\dfrac{\partial \psi}{\partial \theta}$ and $v_\theta = -\dfrac{\partial \psi}{\partial r}$

3.8 POTENTIAL FUNCTION

In irrotational flows, the velocity can be written as a gradient of a scalar function ϕ called velocity potential.

$$u = \frac{\partial \phi}{\partial x},\ v = \frac{\partial \phi}{\partial y} \text{ and } w = \frac{\partial \phi}{\partial z} \qquad (3.20)$$

Considering the equation of continuity (Eq. 3.14) for an incompressible fluid,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

and substituting the expressions for u, v and w in terms of ϕ,

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \qquad (3.21)$$

Thus the velocity potential satisfies the Laplace equation. Conversely, any function ϕ which satisfies the Laplace equation is a possible irrotational fluid flow case.
Lines of constant ϕ are called *equipotential lines* and it can be shown that these lines will form orthogonal grids with ψ = constant lines. This fact is used in the construction of flow nets for fluid flow analysis.

[**Note:** Some authors define ϕ such that

$$u = -\frac{\partial \phi}{\partial x},\ v = -\frac{\partial \phi}{\partial y} \text{ and } w = -\frac{\partial \phi}{\partial z}$$]

3.9 RELATION BETWEEN Ψ AND φ FOR 2-DIMENSIONAL FLOW

- ϕ exists for irrotational flow only.
- $\boxed{u = \dfrac{\partial \phi}{\partial x} = \dfrac{\partial \psi}{\partial y}}$ (3.22)
- $\boxed{v = \dfrac{\partial \phi}{\partial y} = -\dfrac{\partial \psi}{\partial x}}$ (3.23)
- By continuity equation,
- $\boxed{\dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial y^2} = 0}$ (3.21(a))
- By irrotational flow condition,
- $\boxed{\dfrac{\partial^2 \psi}{\partial x^2} + \dfrac{\partial^2 \psi}{\partial y^2} = 0}$ (3.19)
- ψ = constant along a streamline.
- ϕ = constant along an equipotential line which is normal to streamlines.

3.10 SOME COMMON FORMULAE IN CYLINDRICAL CO-ORDINATES

1. Equation of continuity:

$$\frac{1}{r}(\rho V_r) + \frac{\partial}{\partial r}(\rho V_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho V_\theta) = 0 \qquad (3.24)$$

For incompressible fluid flow:

$$\frac{V_r}{r} + \frac{\partial V_r}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} = 0 \qquad (3.25)$$

2. Stream function ψ:

$$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad (3.26)$$

$$V_\theta = -\frac{\partial \psi}{\partial r} \qquad (3.27)$$

3. Potential function ϕ:

$$V_r = \frac{\partial \phi}{\partial r} \qquad (3.28)$$

$$V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} \tag{3.29}$$

4. Laplace equation $\frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0$ (3.30)

3.11 ELEMENTARY INVISCID PLANE FLOWS

Since the Laplace equation is linear, several interesting potential flow situations can be constructed by using elementary solutions and method of superposition. The basic flow types are Uniform flow, Source, Sink and Vortex. These are briefly described below.

3.11.1 Uniform Flow

A stream of constant velocity U in x-direction is shown in Fig. 3.8 and has

$$\psi = \mathrm{U}y \quad \text{and} \quad \phi = \mathrm{U}x \tag{3.31}$$

In polar coordinates $\psi = \mathrm{U}\ r \sin\theta$ and $\phi = \mathrm{U}\ r \cos\theta$ (3.32)

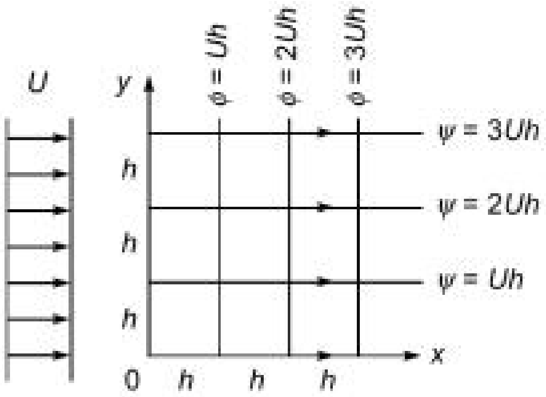


Fig. 3.8 *Uniform Flow*

3.11.2 Line Source and Sink

A two-dimensional flow emanating from a point in the x-y plane and imagined to flow uniformly in all directions is called a *source*. Since the two-dimensional source is a line in the z-direction, it is known as a *line source*.

The total flow per unit time per unit length of the line source is called the *strength m* of the source. The velocity at a radial distance r from the source is

$$v_r = \frac{m}{r} \tag{3.33}$$

The stream function ψ and the potential function ϕ for such line source is given by

$$\psi = m\theta \quad \text{and} \quad \phi = m \ln r \tag{3.34}$$

Figure 3.9 shows a source flow.

A sink is a negative source. It is a line in z-direction into which fluid flows radially in x-y plane (Fig. 3.10).

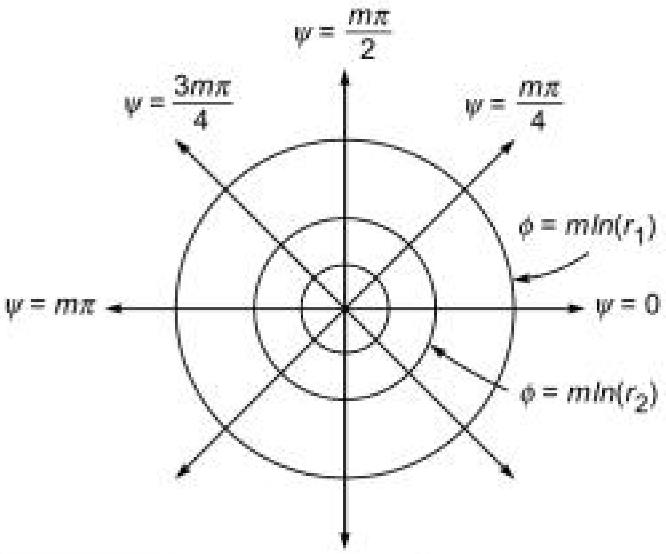


Fig. 3.9 *Line Source*

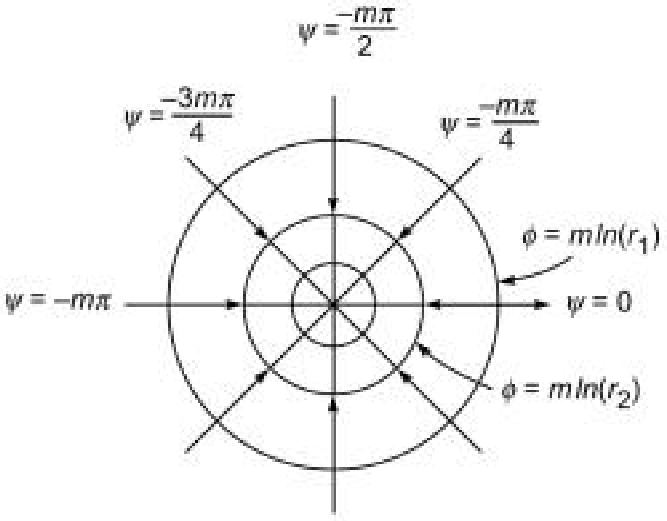


Fig. 3.10 *Line Sink*

3.11.3 Line Vortex

Suppose we reverse the role of ψ and ϕ in Fig. 3.9 yielding

$$\psi = -K \ln r \quad \text{and} \quad \phi = K\theta \tag{3.35}$$

from which we get $v_r = 0$ and $v_0 = K/r$ representing a circulating flow (Fig. 3.11). Such a flow is known as line *vortex* and K in Eq. 3.35 is known as *Vortex strength*. The centre of the vortex is a singular point and the circulation Γ of the vortex around a circular path about the centre is given by

$$\Gamma = 2\pi K \tag{3.36}$$

3.11.4 Two Dimensional Doublet

The limiting case of a line source approaching a line sink of equal strength while keeping constant the product of their strength and the distance between them (λ) is known as a two dimensional *doublet*. For a doublet.

$$\psi_{\text{doublet}} = \frac{(\lambda y)}{(x^2 + y^2)} = -\frac{\lambda \sin\theta}{r} \tag{3.37}$$

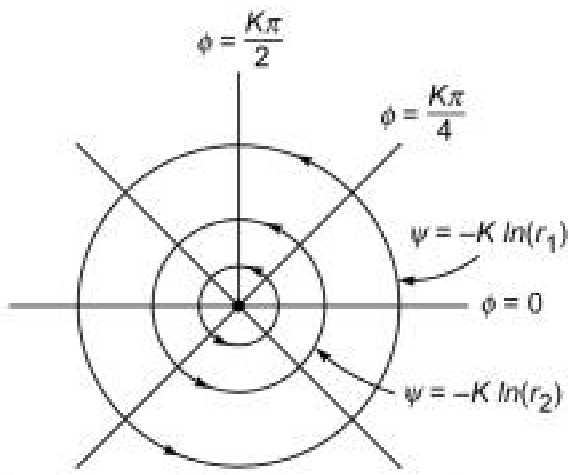

$\phi = \frac{K\pi}{2}$
$\phi = \frac{K\pi}{4}$
$\psi = -K\ln(r_1)$
$\phi = 0$
$\psi = -K\ln(r_2)$

Fig. 3.11 *Line Vortex*

$$\phi_{\text{doublet}} = \frac{\lambda x}{(x^2 + y^2)} = \frac{\lambda \cos\theta}{r} \tag{3.38}$$

Figure 3.12 shows the streamlines and equipotential lines in a doublet.

3.11.5 Other Inviscid Flows

Using the basic flow elements described above various flow situations can be created by the method of superposition. A few examples are given below in Table 3.1.

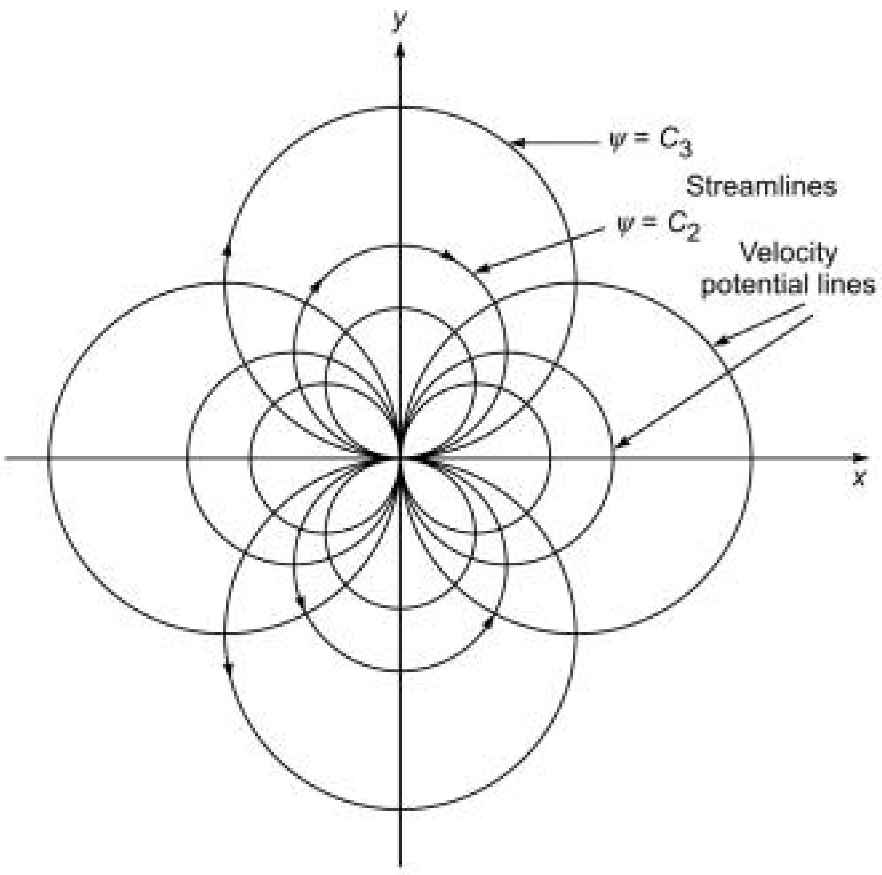

y
$\psi = C_3$
Streamlines
$\psi = C_2$
Velocity potential lines
x

Fig. 3.12 *Doublet*

Table 3.1 *Some Ideal Fluid Flow Simulations*

Sl. No.	*Name*	*Combination (and Flow description)*	*Equation of Stream function*
1.	Rankine Half body	Source + uniform flow [curved, roughly elliptical half body]	$\psi = Ur\sin\theta + m\theta$
2.	Rankine oval	Source + sink + Uniform flow [cylindrical oval shaped body]	$\psi = Ur\sin\theta + m(\theta_1 - \theta_2)$
3.	Circular cylinder	Uniform flow + doublet [circular cylinder]	$\psi = Ur\sin\theta - \frac{\lambda\sin\theta}{r}$
4.	Rotating circular cylinder	Uniform flow + doublet + vortex [rotating circular cylinder]	$\psi = Ur\sin\theta - \frac{\lambda\sin\theta}{r} - K\ln r$

WORKED EXAMPLES

3.1 A circular pipe 10 cm in diameter has a 2 m length which is porous. In this porous section the velocity of exit is known to be constant (Fig. 3.13). If the velocities at inlet and outlet of the porous section are 2.0 m/s and 1.2 m/s respectively, estimate (i) the discharge emitted out through the walls of the porous pipe and (ii) the average velocity of this emitted discharge.

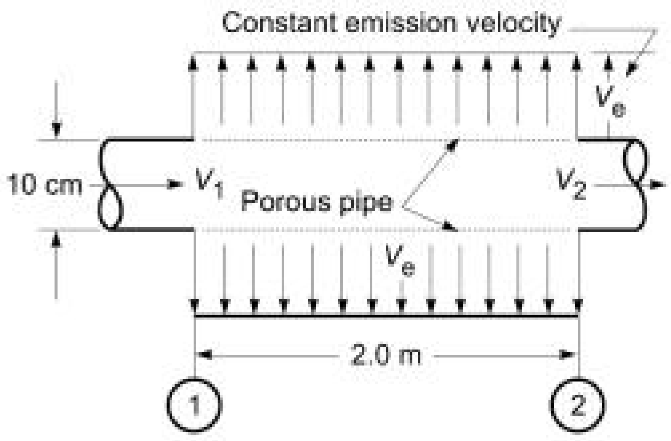

Constant emission velocity
V_e
10 cm
V_1
Porous pipe
V_2
V_e
2.0 m
1
2

Fig. 3.13

Solution: A = area of pipe cross section

$$= \frac{\pi}{4} \times (0.1)^2 = 7.854 \times 10^{-3}\ \text{m}^2$$

$$Q_1 = \text{inlet discharge} = V_1 A$$
$$= 2.0 \times 7.854 \times 10^{-3} = 0.01571\ \text{m}^3/\text{s}$$
$$Q_2 = \text{outlet discharge} = V_2 A$$
$$= 1.2 \times 7.854 \times 10^{-3} = 0.0094248\ \text{m}^3/\text{s}$$

(i) Q_e = discharge emitted through walls of the porous pipe

$$= Q_1 - Q_2 = 0.01571 - 0.0094248$$
$$= \mathbf{0.0052852\ m^3/s}$$

(ii) Surface area of emission $A_e = \pi DL$

$$= \pi \times 0.1 \times 2.0 = 0.6283\ \text{m}^2$$

$$V_e = \text{Velocity of emission} = \frac{Q_e}{A_e} = \frac{6.283 \times 10^{-3}}{0.6283}$$
$$= \mathbf{0.01\ m/s}$$

3.2 The axial velocity profile in a pipe is given by

$$u = u_m \left(1 - \frac{r}{R}\right)^n$$

where R = radius of the pipe and u_m = maximum velocity at $r = R$. Calculate the average velocity for $n = 1/5$ and $n = 1/2$.

Solution: Refer to Fig. 3.14

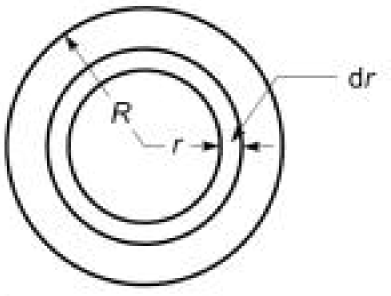


Fig. 3.14

If average velocity = V
Then

$$V.\pi R^2 = Q = \int_0^R u.2\pi r \cdot \mathrm{d}r$$

$$V = \int_0^R \frac{u_m \cdot 2\pi r}{\pi R^2}\left(1 - \frac{r}{R}\right)^n \mathrm{d}r$$

$$= 2u_m \int_{r/R=0}^{r/R=1} \frac{r}{R}\left(1 - \frac{r}{R}\right)^n d\left(\frac{r}{R}\right) = 2u_m$$

$$\left[-\frac{1}{(n+1)(n+2)}\left(1-\frac{r}{R}\right)^{n+2} - \frac{1}{(n+1)}\frac{r}{R}\left(1-\frac{r}{R}\right)^{n+1}\right]_0^1$$

$$= 2u_m \left[\frac{1}{(n+1)} - \frac{1}{(n+2)}\right]$$

when $n = 1/5,\ V = 2u_m\left(\frac{1}{1/5+1} - \frac{1}{1/5+2}\right)$

$$= 2u_m \times \frac{25}{66} = \mathbf{\frac{25}{33}\,u_m}$$

when $n = 1/2,\ V = 2u_m\left(\frac{1}{1/2+1} - \frac{1}{1/2+2}\right)$

$$= 2u_m \times 4/15 = \mathbf{8/15\ u_m}$$

3.3 A centrifugal pump has an axial inlet of 12 cm diameter and an impeller of 30 cm diameter. The width of the impeller at the outlet is 25 mm (Fig. 3.15). 5% of the outlet area can be assumed to be occupied by the blades. For a flow of 80 L/s, estimate the radial component of velocity at the outlet of the impeller. What is the axial velocity in the inlet pipe?

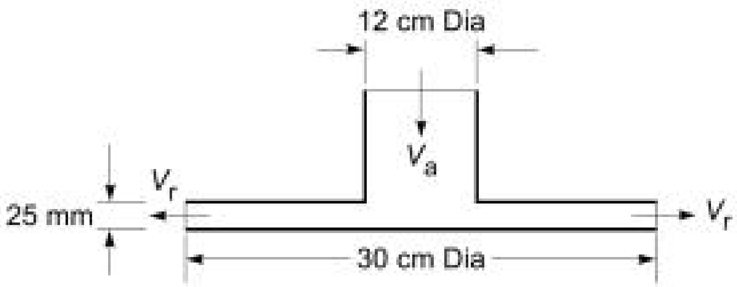


Fig. 3.15

Solution:

Discharge $Q = 0.08\ \text{m}^3/\text{s}$

At the outlet

Area $A = \pi \times 0.30 \times 0.025 \times 0.95$
$= 0.02238\ \text{m}^2/\text{s}$

Radial velocity at the edge of the impeller
$= Q/A = 0.08/0.02238 =$ **3.575 m/s**

At the inlet, $V_a \cdot A_a = 0.08$

where V_a is the axial velocity, at the inlet pipe and A_a is the area of the inlet.

$$V_a = \frac{0.08}{\frac{\pi}{4}(0.12)^2} = \mathbf{7.074\ m/s}$$

3.4 Two large circular plates are kept at a distance y_0 apart and contain an incompressible flow in between. If the bottom plate is fixed and the top plate is moved downwards at a constant rate of V_0, (Fig. 3.16) estimate the velocity at which the fluid moves at a radial distance of r. What is the corresponding acceleration?

Solution: In time Δt, at a radial distance r, volume displaced by the upper plate

$$= V_0 \times \pi r^2$$
$$= \text{Radial out flow} = V_r \times 2\pi r \times y_0$$

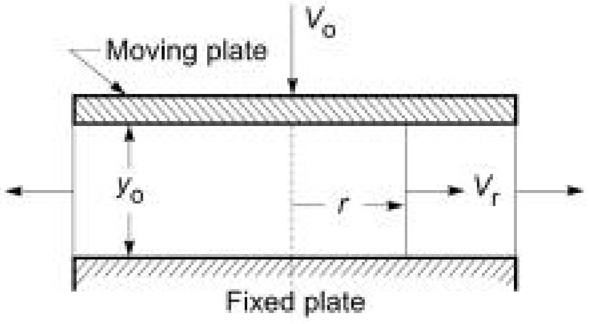


Fig. 3.16 *Example 3.4*

where V_r is the fluid velocity at a radial distance r.

$$V_r = \frac{V_0 \pi r^2}{2\pi r y_0} = \frac{V_0 r}{2y_0}$$

As the flow is radially outwards, acceleration

$$\boldsymbol{a_r = V_r \frac{\partial V_r}{\partial r} = \frac{V_0 r}{2y_0}\left(\frac{V_0}{2y_0}\right) = \frac{V_0^2 r}{4y_0^2}}$$

3.5 In a steady laminar boundary layer over a flat plate the velocity distribution is given by $\frac{u}{U_0} = 2\eta - 2\eta^3 + \eta^4$ where $\eta = y/\delta$ and δ = thickness of the boundary layer at a distance x from the leading edge of the plate (Fig. 3.17). Obtain an expression for the mass rate of outflow per unit width across an imaginary plane AB shown in the figure.

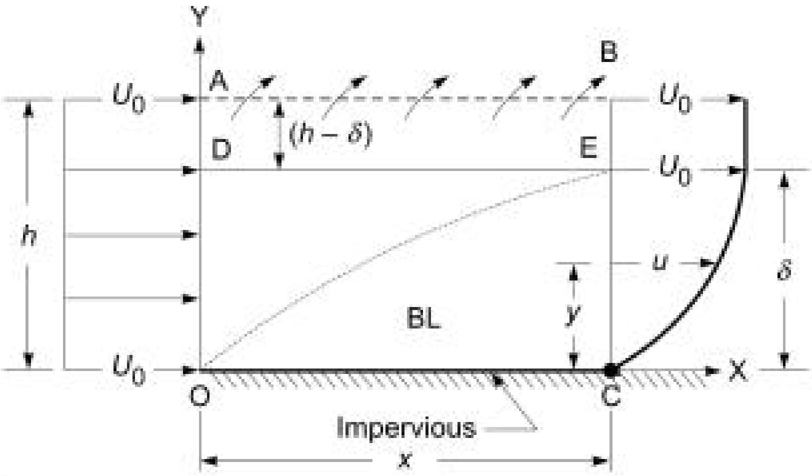


Fig. 3.17

Solution: By continuity equation for a volume ABCO: Mass rate of flow across AO – Mass rate across BC = Mass rate of flow across AB.

Since across section BC, $u = U_0$ for $y \geq \delta$,
the mass rate across AD = $\rho U_0(h - \delta)$ = mass rate across BE = $\rho U_0(h - \delta)$.

Hence, per unit width

$$\rho U_0 \delta - \int_0^\delta \rho u \, dy = \rho Q_{\text{AB}} = \text{mass rate across AB.}$$

$$\rho Q_{\text{AB}} = \rho U_0 \delta - \rho \int_0^\delta U_0 \left(2\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^3 + (y/\delta)^4\right) dy$$

$$= \rho U_0 \left[\delta - \frac{2}{\delta}\frac{\delta^2}{2} + \frac{2}{\delta^3}\frac{\delta^4}{4} - \frac{1}{5}\frac{\delta^5}{\delta^4}\right]$$

$$= \rho U_0 \delta \left(1 - 1 + \frac{2}{4} - \frac{1}{5}\right) = \boldsymbol{\frac{3}{10}\rho U_0 \delta}$$

= Mass rate of outflow across AB per unit width.

3.6 Show that in an unsteady flow in a prismatic open channel the continuity equation is given by

$$\frac{\partial Q}{\partial x} + T\frac{\partial y}{\partial t} = 0$$

where T = top width of the channel section and Q = discharge at any instant t.

Solution: Refer to Fig. 3.18.

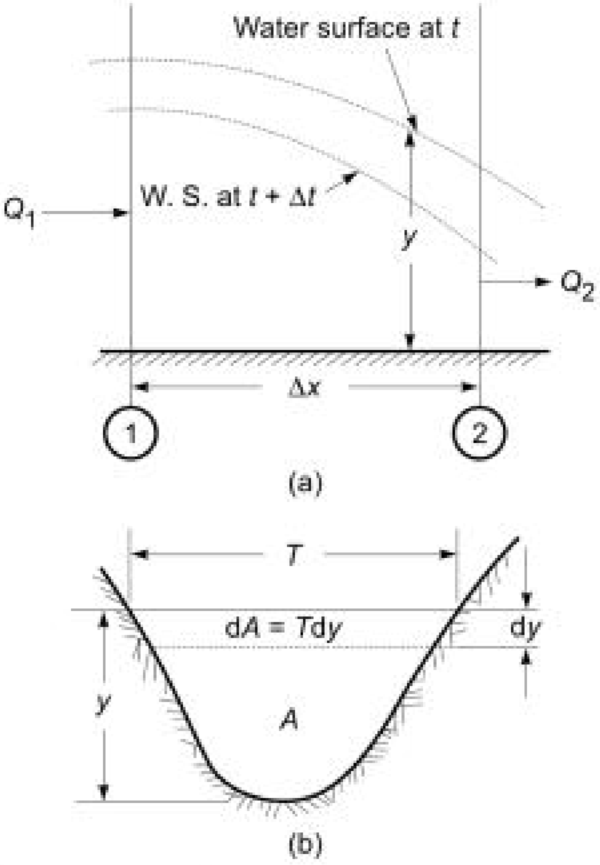


Fig. 3.18

Consider section 1 and 2 Δx apart. Let $Q_2 > Q_1$ at any instant t. Then

$$Q_2 - Q_1 = \frac{\partial Q}{\partial x} \cdot \Delta x$$

In a time interval Δt, volume rate of excess outflow over the inflow = $\frac{\partial Q}{\partial x}\Delta x \, \Delta t$. If the top width of the channel is T at a depth y, the water surface will drop by $\Delta y = \frac{\partial y}{\partial t}\Delta t$ and the decrease in storage between sections 1 and 2 is

$$-\Delta S = -\Delta A \times \Delta x = -T\Delta y \, \Delta x = -T\frac{\partial y}{\partial t}\Delta t \, \Delta x$$

Hence $\frac{dx}{3x} = \frac{dy}{4y} = -\frac{dz}{7z}$

Considering equations involving x and y, on integration

$$\frac{1}{3}\ln x = \frac{1}{4}\ln y + \ln C_1'$$

where C_1' = a constant

or $y = C_1 x^{4/3}$ (1)

where C_1 is another constant.

Similarly, by considering equations with x and z and on integration

$$\frac{1}{3}\ln x = -\frac{1}{7}\ln z + \ln C_2'$$

where C_2' = a constant

$$z = \frac{C_2}{x^{7/3}} \quad (2)$$

where C_2 is another constant.

Putting the coordinates of the point M (1, 4, 5)

from Eq. (1) $C_1 = \frac{4}{(1)^{4/3}} = 4$

from Eq. (2) $C_2 = 5 \times 1^{7/3}$

The streamline passing through M is given by

$$\mathbf{y = 4x^{4/3} \quad and \quad z = 5/x^{7/3}}$$

3.10 For the following velocity vectors determine the magnitude of velocity at A ($x = 2$, $y = -3$, $z = 1$, $t = 2$).

(i) $\mathbf{V} = (10t + xy)\mathbf{i} + (-yz - 10t)\mathbf{j} + (-yz + z^2/2)\mathbf{k}$

(ii) $\mathbf{V} = 4x\mathbf{i} + (-4y + 3t)\mathbf{j}$

Solution:

(i) $\mathbf{V} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$ and A ($x = 2$, $y = -3$, $z = 1$, $t = 2$)

Here $u = 10t + xy \Rightarrow u_a = (10 \times 2) + (2 \times -3) = 14$

$v = -yz - 10t \Rightarrow v_a = -(-3 \times 1) - (10 \times 2) = -17$

$w = -yz + z^2/2 \Rightarrow w_a = -(-3 \times 1) + (1)^2/2 = 3.5$

Magnitude of velocity at A =

$$V_A = \sqrt{u_a^2 + v_a^2 + w_a^2}$$

$$= \sqrt{(14)^2 + (-17)^2 + (3.5)^2}$$

$$= \mathbf{22.3\ units}$$

(ii) $\mathbf{V} = 4x\mathbf{i} + (-4y + 3t)\mathbf{j} + (0)\mathbf{k}$

At A ($x = 2$, $y = -3$, $z = 1$, $t = 2$)

$u = 4x \Rightarrow u_a = 4 \times 2 = 8$

$v = -4y + 3t \Rightarrow v_a = (-4 \times -3) + 3 \times 2 = 18$

$w = 0 \Rightarrow w_a = 0$

Magnitude of velocity at A =

$$V_A = \sqrt{u_a^2 + v_a^2 + w_a^2}$$

$$= \sqrt{8^2 + 18^2} = \mathbf{19.7\ units}$$

3.11 The velocity along the centreline of a nozzle of length L is given by

$$V = 2t\left(1 - \frac{x}{2L}\right)^2$$

where V = velocity in m/s, t = time in seconds from commencement of flow, x = distance from inlet to the nozzle. Find the convective acceleration, local acceleration and the total acceleration when $t = 3$ s, $x = 0.5$ m and $L = 0.8$ m.

Solution:

(i) Local acceleration $= \frac{\partial V}{\partial t} = 2\left(1 - \frac{x}{2L}\right)^2$

at $t = 3$ s and $x = 0.5$ m, $\frac{\partial V}{\partial t} = 2\left(1 - \frac{0.5}{2 \times 0.8}\right)^2$

$$= \mathbf{0.945\ m/s^2}$$

(ii) Convective acceleration $= V\frac{\partial V}{\partial x}$

$$= 2t\left(1 - \frac{x}{2L}\right)^2 \cdot 2t \cdot 2\left(1 - \frac{x}{2L}\right)\left(-\frac{1}{2L}\right)$$

$$= -\frac{4t^2}{L}\left(1 - \frac{x}{2L}\right)^3$$

At $t = 3$ s and $x = 0.5$ m

Convective acceleration

$$= -\frac{4 \times 3^2}{0.8}\left(1 - \frac{0.5}{2 \times 0.8}\right)^3$$

$$= \mathbf{-14.623\ m/s^2}$$

(iii) Total acceleration = (local + convective) acceleration = 0.945 − 14.623 = $\mathbf{-13.68\ m/s^2}$

3.12 A 2.0 m long conical diffuser 20 cm in diameter at the upstream end has 80 cm diameter at the downstream end. At a certain instant the discharge through the diffuser is observed to be 200 L/s of water and is found to increase uniformly at a rate of 50 L/s per second. Estimate the local, convective and total acceleration at a section 1.5 m from the upstream end.

Solution: Refer to Fig. 3.20.

Diameter at section XX

$$D_x = \left(\frac{D_2 - D_1}{L}\right) \cdot x + D_1$$

$$= \left(\frac{0.8 - 0.2}{2}\right) \cdot x + 0.20$$

$$= 0.3x + 0.20$$

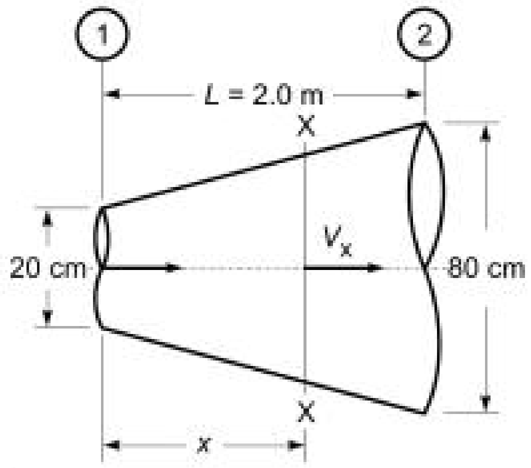


Fig. 3.20

At $x = 1.5$ m, $D = 0.65$ m

$$A = \text{area at section XX}$$

$$= \frac{\pi}{4} \times (0.65)^2 = 0.3318 \text{ m}^2$$

$$V = \text{Velocity} = \frac{Q}{A} = \frac{0.200}{0.3318} = 0.6027 \text{ m/s}$$

(i) $\dfrac{\partial V}{\partial t}$ = rate of increase of velocity

$$= \frac{\partial (Q/A)}{\partial t} = \frac{1}{A_x}\frac{\partial Q}{\partial t}$$

$$= \frac{1}{0.3318}(0.050) = \mathbf{0.1507 \text{ m/s}^2}$$

= local acceleration at XX

(ii) Convective acceleration $a_s = V_x \dfrac{\partial V_x}{\partial x}$

$$V_x = \frac{Q}{A_x} = \frac{Q}{\frac{\pi}{4}(0.3x+0.2)^2} = \frac{0.20}{\frac{\pi}{4}(0.3x+0.2)^2}$$

$$= 0.2546/(0.3x + 0.20)^2$$

$$\frac{\partial V_x}{\partial x} = (0.2546)(-2)(0.3x + 0.20)^{-3}(0.3)$$

$$= -0.15276/(0.3x + 0.20)^3$$

Hence convective acceleration

$$a_s = V_x \frac{\partial V_x}{\partial x} = \frac{0.2546}{(0.3x+0.2)^2} \times \left(\frac{-0.15276}{(0.3x+0.20)^3}\right)$$

$$= -\frac{0.03889}{(0.3x+0.2)^5}$$

At $x = 1.5$ m, convective acceleration

$$a_s = -\frac{0.03889}{(0.3 \times 1.5 + 0.2)^5} = \mathbf{-0.3352 \text{ m/s}^2}$$

(iii) Total acceleration = local acceleration + convective acceleration

$$= \frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial t}$$

At $x = 1.5$ m, total acceleration = 0.1507 – 0.3352
$= \mathbf{-0.1845 \text{ m/s}^2}$

3.13 The velocity vector in an incompressible flow is given by

$$\mathbf{V} = (6xt + yz^2)\mathbf{i} + (3t + xy^2)\mathbf{j} + (xy - 2xyz - 6tz)\mathbf{k}$$

(i) Verify whether the continuity equation is satisfied.
(ii) Determine the acceleration vector at point A (1, 1, 1) at $t = 1.0$.

Solution:

(i) $\mathbf{V} = (6xt + yz^2)\mathbf{i} + (3t + xy^2)\mathbf{j} + (xy - 2xyz - 6tz)\mathbf{k}$
$= u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$

$$u = 6xt + yz^2; \quad \frac{\partial u}{\partial x} = 6t$$

$$v = 3t + xy^2; \quad \frac{\partial v}{\partial y} = 2xy$$

$$w = xy - 2xyz - 6tz;$$

$$\frac{\partial w}{\partial z} = -2xy - 6t$$

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 6t + 2xy - 2xy - 6t = 0$$

Hence the continuity equation is satisfied.

(ii) Acceleration $\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$= 6x + (6xt + yz^2)(6t) + (3t + xy^2)(z^2) + (xy - 2xyz - 6tz)(2yz)$$

At point A (1, 1, 1) and at $t = 1$,

$$a_x = 6 + (6 + 1)(6) + (3 + 1)(1) + (1 - 2 - 6)(2)$$

= 38 units

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$= 3 + (6xt + yz^2)(y^2) + (3t + xy^2)(2xy) + (xy - 2xyz - 6tz)(0)$$

At point A (1, 1, 1) and at $t = 1$

$$a_y = 3 + (6 + 1)(1) + (3 + 1)(2) = 18 \text{ units}$$

Similarly

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$= -6z + (6xt + yz^2)(y - 2yz) + (3t + xy^2)(x - 2xz) + (xy - 2xyz - 6tz)(-2xy - 6t)$$

At point A (1, 1, 1) and at $t = 1$

$$a_z = -6 + (6 + 1)(1 - 2) + (3 + 1)(1 - 2) + (1 - 2 - 6)(-2 - 6)$$

$$= -6 - 7 - 4 + 56 = 39 \text{ units}$$

Hence at A (1, 1, 1) and at $t = 1$,

$$\mathbf{a} = 38\mathbf{i} + 18\mathbf{j} + 39\mathbf{k}$$

3.14 For steady, incompressible flow, verify whether the following values of u and v are possible:

(i) $u = 4xy + y^2$, $v = 6xy + 3x$

(ii) $u = 2x^2 + y^2$, $v = -4xy$ and

(iii) $u = -x/(x^2 + y^2)$, $v = -y/(x^2 + y^2)$

Solution: For steady, incompressible flow the following continuity equation must be satisfied:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(i) $u = 4xy + y^2$; $v = 6xy + 3x$

$$\frac{\partial u}{\partial x} = 4y \; ; \; \frac{\partial v}{\partial y} = 6x$$

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 4y + 6x \neq 0$$

Hence the flow is not possible.

(ii) $u = 2x^2 + y^2$; $v = -4xy$

$$\frac{\partial u}{\partial x} = 4x \; ; \quad \frac{\partial v}{\partial y} = -4x$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 4x - 4x = 0$$

Hence the flow is possible.

(iii) $u = -x/(x^2 + y^2)$; $v = -y/(x^2 + y^2)$

$$\frac{\partial u}{\partial x} = \frac{-(x^2 + y^2) + 2x^2}{(x^2 + y^2)^2} \; ;$$

$$\frac{\partial v}{\partial y} = \frac{-(x^2 + y^2) + 2y^2}{(x^2 + y^2)^2}$$

$$\frac{\partial u}{\partial x} = \frac{(x^2 - y^2)}{(x^2 + y^2)^2} \; ; \quad \frac{\partial v}{\partial y} = \frac{(-x^2 + y^2)}{(x^2 + y^2)^2}$$

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{x^2 - y^2 - x^2 + y^2}{(x^2 + y^2)^2} = 0$$

Hence, the flow is possible.

3.15 Determine which of the velocity component sets given below satisfy the equation of continuity:

(a) $u = cx$, $v = -cy$ (b) $u = -cx/y$, $v = c \ln xy$

(c) $u = A \sin xy$, $v = -A \sin xy$ (d) $u = x + y$, $v = x - y$

(e) $u = 2x^2 + zy$

$v = -2xy + 3y^3 + 3zy$

$w = -\frac{3}{2} z^2 - 2xy - 6yz$

Solution:

(a) $u = cx$; $v = -cy$

$$\frac{\partial u}{\partial x} = c \; ; \; \frac{\partial v}{\partial y} = -c$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = c - c = 0$$

Hence, the continuity equation is satisfied.

(b) $u = -cx/y$; $v = c \ln xy$

$$\frac{\partial u}{\partial x} = -c/y; \; \frac{\partial v}{\partial y} = \frac{c}{xy} \cdot x = c/y$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -c/y + c/y = 0$$

The continuity equation is satisfied.

(c) $u = A \sin xy$; $v = -A \sin xy$

$$\frac{\partial u}{\partial x} = Ay \cos xy; \; \frac{\partial v}{\partial y} = -Ax \cos xy$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \neq 0$$

Hence the continuity equation is not satisfied.

(d) $u = x + y$; $v = x - y$

$$\frac{\partial u}{\partial x} = 1; \; \frac{\partial v}{\partial y} = -1$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 - 1 = 0$$

Hence the continuity equation is satisfied.

(e) $u = 2x^2 + zy; \; \frac{\partial u}{\partial x} = 4x$

$v = -2xy + 3y^2 + 3zy;$

$$\frac{\partial v}{\partial y} = -2x + 9y^2 + 3z$$

$$w = -\frac{3}{2} z^2 - 2xz - 6yz;$$

$$\frac{\partial w}{\partial z} = -3z - 2x - 6y$$

Hence $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

$$= 4x - 2x + 9y^2 + 3z - 3z - 2x - 6y$$

$$\neq 0$$

The continuity equation is thus not satisfied.

3.16 Calculate the unknown velocity component in the following, so that the equation of continuity is satisfied.

(a) $u = A(x^2 + y^2)$, $v = ?$ (b) $u = Ae^x$, $v = ?$

(e) $u = -A \ln \frac{x}{L}$, $v = ?$ (d) $u = ?$, $v = Axy$

3.22 A flow is described by the stream function $\psi = 2\sqrt{3}xy$. Locate the point at which the velocity vector has a magnitude of 4 units and makes an angle of 150° with the x-axis.

Solution: The velocity vector is diagrammatically shown in Fig. 3.21

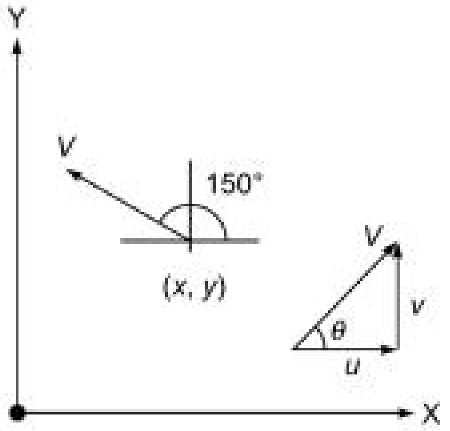


Fig. 3.21

$$\psi = 2\sqrt{3}\,xy$$

$$u = \frac{\partial\psi}{\partial y} = 2\sqrt{3}\,x$$

$$v = -\frac{\partial\psi}{\partial x} = -2\sqrt{3}\,y$$

$$V = \sqrt{u^2+v^2}$$

$$4 = \sqrt{4(3)(x^2+y^2)} \qquad (1)$$

Also, $\tan\theta = \frac{v}{u}$

$$\tan 150° = -\frac{2\sqrt{3}}{2\sqrt{3}}\cdot\frac{y}{x}$$

$$-0.5774 = -y/x$$

$$y = 0.5774\,x \qquad (2)$$

Substituting in Eq. (1)

$$4.0 = \sqrt{4\times 3\times[(x^2+(0.5774)^2x^2)]}$$

$$= \sqrt{16x^2} = 4x$$

$$x = 1.00$$

$$y = 0.5774$$

The required point (x, y) is **(1.00, 0.5774).**

3.23 The stream function ψ for uniform flow past a circular cylinder of radius a in polar co-ordinates is given by $\psi = U_\infty\left(1-\frac{a^2}{r^2}\right) r\sin\theta$ where U_∞ is the free stream velocity parallel to the horizontal axis. Determine the velocity on the cylinder at $\theta = 90°$.

Solution:

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}U_\infty\left(1-\frac{a^2}{r^2}\right) r\cdot\cos\theta$$

$$= U_\infty\left(1-\frac{a^2}{r^2}\right)\cos\theta$$

At $r = a$ and $\theta = 90°$, $\mathbf{V_r = 0}$

$$V_\theta = -\frac{\partial\psi}{\partial r} = -U_\infty\sin\theta\left(1-a^2\left(-\frac{1}{r^2}\right)\right)$$

$$= -U_\infty\sin\theta\left(1+\frac{a^2}{r^2}\right)$$

At $r = a$ and $\theta = 90°$, $\mathbf{V_\theta = -2\,U_\infty}$

3.24 Values of ϕ for various flows are given. Determine the corresponding values of ψ.

(i) $\phi = 3xy$ (ii) $\phi = 4(x^2 - y^2)$

(iii) $\phi = x + y + 3$

Solution:

(i) $\phi = 3xy$

$$u = \frac{\partial\phi}{\partial x} = 3y = \frac{\partial\psi}{\partial y}$$

Hence $\psi = \frac{3}{2}y^2 + f(x)$

$$v = \frac{\partial\phi}{\partial y} = 3x = -\frac{\partial\psi}{\partial x}$$

$$= -f'(x)$$

$f'(x) = -3x$ and hence $f(x) = -3/2x^2 + c$

$\therefore$ $\boldsymbol{\psi = -3/2(x^2 - y^2) + c}$ **where c is a constant**

(ii) $\phi = 4(x^2 - y^2)$

$$u = \frac{\partial\phi}{\partial x} = 8x = \frac{\partial\psi}{\partial y}$$

Hence $\psi = 8xy + f(x)$

$$v = \frac{\partial\phi}{\partial y} = -8y = -\frac{\partial\psi}{\partial x}$$

$$= -8y - f'(x)$$

Hence $f'(x) = 0$ and $f(x)$ = constant = c

$\therefore$ $\boldsymbol{\psi = 8xy + c}$

(iii) $\phi = x + y + 3$

$$u = \frac{\partial\phi}{\partial x} = 1 = \frac{\partial\psi}{\partial y}$$

Hence $\psi = y + f(x)$

$$v = \frac{\partial\phi}{\partial y} = 1 = -\frac{\partial\psi}{\partial x}$$

$$= -f'(x)$$

$f'(x) = -1$ and hence $f(x) = -x$

Hence $\boldsymbol{\psi = -x + y + c}$ **where c is a constant**

3.25 Verify whether the following functions are valid potential functions:

(i) $\phi = \text{A}\,xy$ (ii) $\phi = m \ln x$

(iii) $\phi = \text{A}(x^2 - y^2)$ (iv) $\phi = \text{A}\cos x$

Solution: A valid potential function satisfies the Laplace equation.

(i) $\phi = \text{A}\,xy$

$$\frac{\partial\phi}{\partial x} = \text{A}y\,;\ \frac{\partial\phi}{\partial y} = \text{A}x$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 + 0 = 0$$

Hence $\phi = Axy$ is a valid potential function.

(ii) $\phi = m \ln x$

$$\frac{\partial\phi}{\partial x} = \frac{m}{x}\,;\ \frac{\partial\phi}{\partial y} = 0$$

$$\frac{\partial^2\phi}{\partial x^2} = -\frac{m}{x^2}\,;\ \frac{\partial^2\phi}{\partial y^2} = 0$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial x^2} = -m/x^2 \neq 0$$

Hence $\phi = m \ln x$ is not a valid potential function.

(iii) $\phi = \text{A}(x^2 - y^2)$

$$\frac{\partial\phi}{\partial x} = 2\text{A}x\,;\ \frac{\partial\phi}{\partial y} = -2\text{A}y$$

$$\frac{\partial^2\phi}{\partial x^2} = 2\text{A}\,;\ \frac{\partial^2\phi}{\partial y^2} = -2\text{A}$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 2\text{A} - 2\text{A} = 0$$

Hence $\phi = \text{A}(x^2 - y^2)$ is a valid potential function.

(iv) $\phi = \text{A}\cos x$

$$\frac{\partial\phi}{\partial x} = -\text{A}\sin x\,;\ \frac{\partial\phi}{\partial y} = 0$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = -\text{A}\cos x \neq 0$$

Hence $\phi = \text{A}\cos x$ is not a valid potential function.

3.26 Which of the following functions represent possible irrotational flow:

(i) $\psi = xy$ (iii) $\phi = Ur\cos\theta + \frac{U}{r}\cos\theta$

(ii) $\psi = \text{A}(x^2 - y^2)$ (iv) $\phi = \left(r - \frac{2}{r}\right)\sin\theta$

Solution: For an irrotational fluid flow phenomenon, ϕ as well as ψ satisfy Laplace equation.

(i) $\psi = xy$

$$\frac{\partial\psi}{\partial x} = y\,;\ \frac{\partial\psi}{\partial y} = x$$

$$\frac{\partial^2\psi}{\partial x^2} = 0\,;\ \frac{\partial^2\psi}{\partial y^2} = 0$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial x^2} = 0$$

Hence $\psi = xy$ represents a possible irrotational flow.

(ii) $\psi = \text{A}(x^2 - y^2)$

$$\frac{\partial\psi}{\partial x} = 2\text{A}x\,;\ \frac{\partial\psi}{\partial y} = -2\text{A}y$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 2A - 2A = 0$$

Hence $\psi = A(x^2 - y^2)$ represents a possible irrotational flow field.

(iii) $\phi = Ur\cos\theta + \frac{U}{r}\cos\theta$

Laplace equation is radial co-ordinates (r, θ)

$$\frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = 0$$

$$\frac{\partial\phi}{\partial r} = U\cos\theta - \frac{U}{r^2}\cos\theta$$

$$\frac{\partial^2\phi}{\partial r^2} = +\frac{2U}{r^3}\cos\theta$$

$$\frac{\partial\phi}{\partial\theta} = -Ur\sin\theta - \frac{U}{r}\sin\theta$$

$$\frac{\partial^2\phi}{\partial\theta^2} = -Ur\cos\theta - \frac{U}{r}\cos\theta$$

L.H.S. of Laplace equation is

$$\frac{1}{r}\left(U\cos\theta - \frac{U}{r^2}\cos\theta\right) + \left(\frac{2U}{r^3}\cos\theta\right) + \frac{1}{r^2}\left(-Ur\cos\theta - \frac{U}{r^2}\cos\theta\right)$$

$$= U\cos\theta\left(\frac{1}{r} - \frac{1}{r^3} + \frac{2}{r^3} - \frac{1}{r} - \frac{1}{r^3}\right)$$

$$= 0$$

The Laplace equation is satisfied and hence **the given function ϕ represents a possible irrotational flow.**

(iv) $\phi = \left(r - \frac{2}{r}\right)\sin\theta$

$$\frac{\partial\phi}{\partial r} = \left(1 + \frac{2}{r^2}\right)\sin\theta$$

$$\frac{\partial^2\phi}{\partial r^2} = -\frac{2}{r^3}\sin\theta$$

$$\frac{\partial\phi}{\partial\theta} = \left(r - \frac{2}{r}\right)\cos\theta$$

3.28 If ψ is a stream function then the velocity u and v are given by

(a) $u = \partial\psi/\partial y$, $v = -\,\partial\psi/\partial x$ (b) $u = -\,\partial\psi/\partial y$, $v = -\,\partial\psi/\partial x$

(c) $u = \partial\psi/\partial y$, $v = \partial\psi/\partial x$ (d) $u = \partial\psi/\partial x$, $v = -\,d\,\psi/\partial y$

3.29 The stream function in a two-dimensional flow field is given by $\psi = x^2 - y^2$. The magnitude of the velocity at point (1, 1) is

(a) 2 (b) $2\sqrt{2}$

(c) 4 (d) 8

3.30 In a two-dimensional, incompressible flow, if the fluid velocity components are $u = x - 4y$ and $v = -y$ then the stream function ψ is given by

(a) $x^2 - xy + 2y^2$ (b) $2x^2 + 2xy + y^2$

(c) $2x^2 + xy - 2y^2$ (d) $2x^2 - xy + 2y^2$

3.31 Stream function is defined for

(a) Flow of a perfect fluid only

(b) All 2–D incompressible flows

(c) All 3–D flows

(d) Irrotational flows only

3.32 Velocity potential exists for

(a) Flow of a perfect fluid only

(b) Stready, irrotational flow only

(c) All irrotational flows

(d) All 3–D flows

3.33 A velocity potential exists

(a) whenever the real fluid flow exists

(b) when the flow is real and rotational

(c) when the flow satisfies the conditions of irrotational motion.

(d) when the flow satisfies the equations of continuity.

3.34 If ϕ is a potential function in two-dimensional flow the velocity components u and v are defined as

(a) $u = \partial\phi/\partial y$, $v = -\,\partial\phi/\partial x$ (b) $u = \partial\phi/\partial x$, $v = \partial\phi/\partial y$

(c) $u = \partial\phi/\partial x$, $v = -\,\partial\phi/\partial y$ (d) $u = \partial\phi/\partial y$, $v = \partial\phi/\partial x$

3.35 The velocity potential function for a line source varies with radial distance r as

(a) $1/r$ (b) $1/r^2$

(c) r (d) $\ln r$

3.36 Lines of constant ϕ

(a) are parallel streamlines

(b) are parallel to the streamlines

(c) are normal to the streamlines

(d) can intersect each other

3.37 In 2-D radial co-ordinates the velocities v_{r} and v_θ are expressed in terms of ϕ as

(a) $v_{\mathrm{r}} = \dfrac{1}{r}\dfrac{\partial\phi}{\partial\theta}$, $v_\theta = \dfrac{\partial\phi}{\partial r}$ (b) $v_{\mathrm{r}} = -\dfrac{\partial\phi}{\partial r}$, $v_\theta = \dfrac{1}{r}\dfrac{\partial\phi}{\partial\theta}$

(c) $v_r = \dfrac{1}{r}\dfrac{\partial\phi}{\partial\theta}$, $v_\theta = -\dfrac{\partial\phi}{\partial r}$ (d) $v_r = \dfrac{\partial\phi}{\partial r}$, $v_\theta = \dfrac{1}{r}\dfrac{\partial\phi}{\partial\theta}$

3.38 If $\phi = 3\,x\,y$, the x and y components of velocity at the point (1, 3) will be

(a) $u = -9$, $v = -3$

(b) $u = -3$, $v = -9$

(c) $u = 9$, $v = -3$

(d) $u = 9$, $v = 9$

3.39 If $u = x^2 + y^2$ and $v = 2\,xy$, the velocity potential ϕ = constant +

(a) $x\left(\dfrac{x^2}{3} + y^2\right)$ (b) $x\left(\dfrac{x^2}{3} + y^3\right)$

(c) $x\left(\dfrac{x^2}{6} + \dfrac{y^3}{2}\right)$ (d) $x + y$

3.40 If for a flow, a stream function ψ exists and satisfies the Laplace equation, then

(a) the flow is rotational

(b) the flow is irrotational but does not necessarily satisfy continuity equation

(c) the flow satisfies continuity equation but does not necessarily satisfy condition for irrotational flow

(d) the continuity equation is satisfied and the flow is irrotational

3.41 If a stream function ψ exists it implies that

(a) the function ψ represents a possible flow field

(b) the flow is irrotational

(c) the flow is steady, incompressible

(d) the potential function also exists

3.42 The potential function exists for

(a) irrotational motion of incompressible fluids only

(b) irrotatinal motion of fluids whether compressible or incompressible

(c) for two-dimensional irrotational flow only

(d) for steady flows only

3.43 Cauchy-Reimann equations relating ϕ and ψ are

(a) $\dfrac{\partial\phi}{\partial x} = \dfrac{\partial\phi}{\partial y}$; $\dfrac{\partial\psi}{\partial x} = -\dfrac{\partial\psi}{\partial y}$

(b) $\frac{\partial \phi}{\partial x} = -\frac{\partial \phi}{\partial y}$; $\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$

(c) $\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial x}$; $\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial y}$

(d) $\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y}$; $\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$

3.44 Indicate the *incorrect* statement:

A flow net

(a) is applicable to irrotational fluid flow

(b) for a given boundary is the same whether the flow is in one direction or the other

(c) for a given boundary is applicable to one chosen direction of flow; if the flow is reversed the flow net will change

(d) will be no constructed that the size of the mesh is inversely proportional to the local velocity

3.45 The velocity potential ϕ at any point for a two-dimensional, steady, irrotational flow in polar coordinates is given by

$$\phi = \frac{\lambda \cos \theta}{r}$$

This equation represents a

(a) vortex	(b) sink
(c) source	(d) doublet

3.46 The velocity potential ϕ at any point for a two-dimensional, steady, irrotational flow is given by $\phi = K\,\theta$.

This equation represents a

(a) vortex	(b) sink
(c) source	(d) doublet

3.47 Inviscid, incompressible flow about a stationary cylinder in uniform flow can be simulated by superposition of uniform flow and

(a) a sink and a vortex

(b) a doublet

(c) a vortex

(d) a doublet and a vortex.

PROBLEMS

3.1 Water is pumped into the tank shown in Fig. 3.22 at 100 L/s. Both kerosene (relative) density = 0.8) and oil relative density = 0.90) are driven out. If 30 L/s oil is driven out estimate the volume of kerosene coming out of tank per second.

(*Ans.* Q = 91.25 L/s)

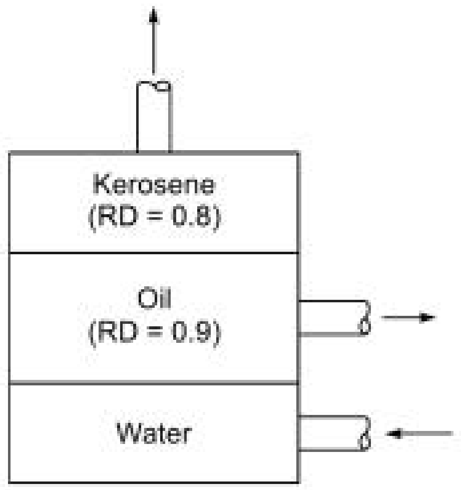


Fig. 3.22

3.2 A water tank has a 3 cm diameter inlet at A, a 4 cm diameter outlet at B and a 3 cm diameter controllable inlet at C, (Fig. 3.23). If the velocity of water at the inlet A is 2.0 m/s and the velocity of flow going out at B = 1.85 m/s, what should be the velocity at the inlet at C to see that the water level in the tank does not change?

(*Ans.* V = 1.285 m/s)

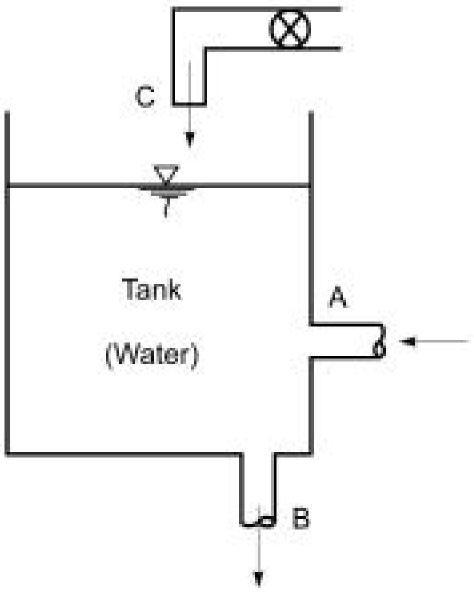


Fig. 3.23

3.3 Figure 3.24 shows a Y-junction of three pipes. The velocity distribution in pipe 1 is uniform while in pipes 2 and 3 it is given by $u = u_m \left(1 - \frac{r}{R}\right)^{1/7}$.

Pipes 1, 2 and 3 have diameters 8 cm, 6 cm and 4

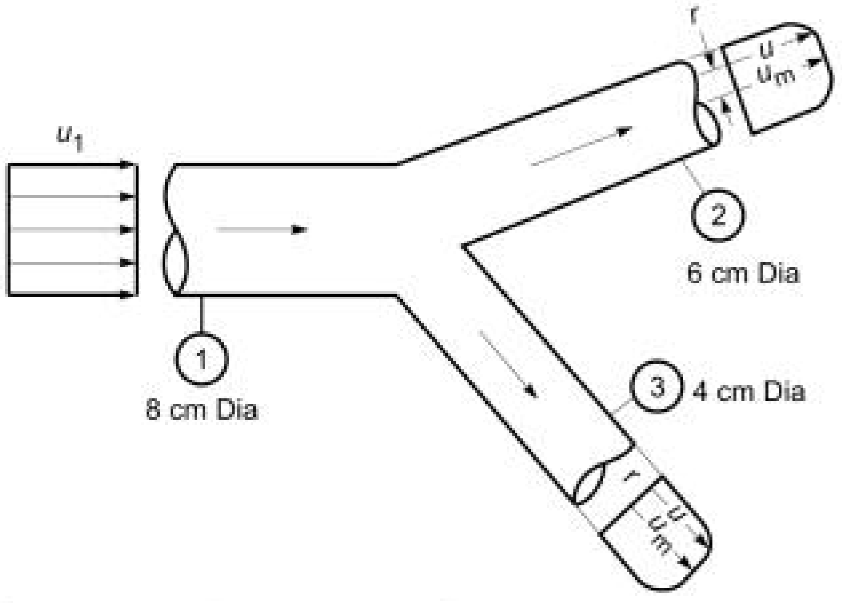


Fig. 3.24 *Problem 3.3*

cm respectively. If the maximum velocity u_m in pipes 2 and 3 are 0.8 m/s and 0.6 m/s respectively, estimate the value of the uniform velocity u_1 in pipe 1.

(*Ans.* $u_1 = 0.49$ m/s)

3.4 A two-dimensional duct 10 cm high carries an incompressible fluid flow. At the entrance section 1 (Fig. 3.25), the velocity can be assumed to be as shown in the figure.

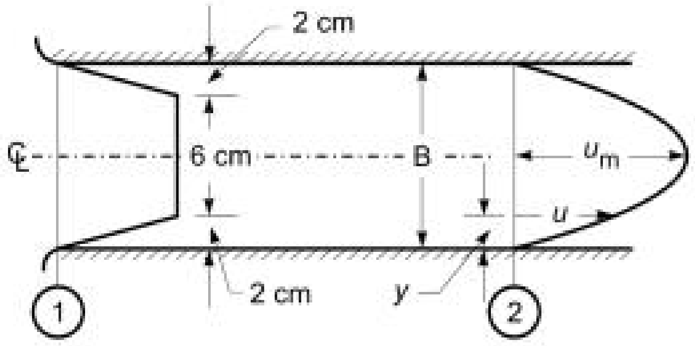


Fig. 3.25

After the flow is fully developed, at a section 2, the velocity u at a distance y from the boundary is given by

$$\frac{u}{u_m} = 4\left(\frac{y}{B} - \frac{y^2}{B^2}\right)$$

where B = height of the duct and u_m = velocity at the centreline of the duct. If the maximum velocity at entrance section 1 = 0.8 m/s, estimate the value of u_m at section 2.

(*Ans.* $u_m = 0.96$ m/s)

3.5 Figure 3.26 shows a flow of incompressible fluid of density ρ flowing past an impervious plate AD. BC is an imaginary plane at a height h above the plate. The flow approaching the plate is of uniform velocity U and the velocity profile at a section DC of plate is $\frac{u}{U} = (y/h)^{1/7}$. Estimate the mass rate of outflow across BC per unit width of the plate.

$$\left(Ans.\ q_{BC} = \frac{1}{8}\rho U h\right)$$

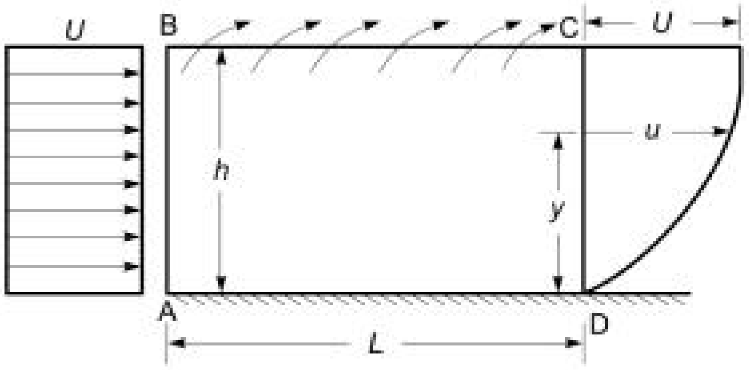


Fig. 3.26 *Problem 3.5*

3.6 Water flows in a pipe network shown in Fig. 3.27. Fill in the missing discharges so that continuity equation is satisfied.

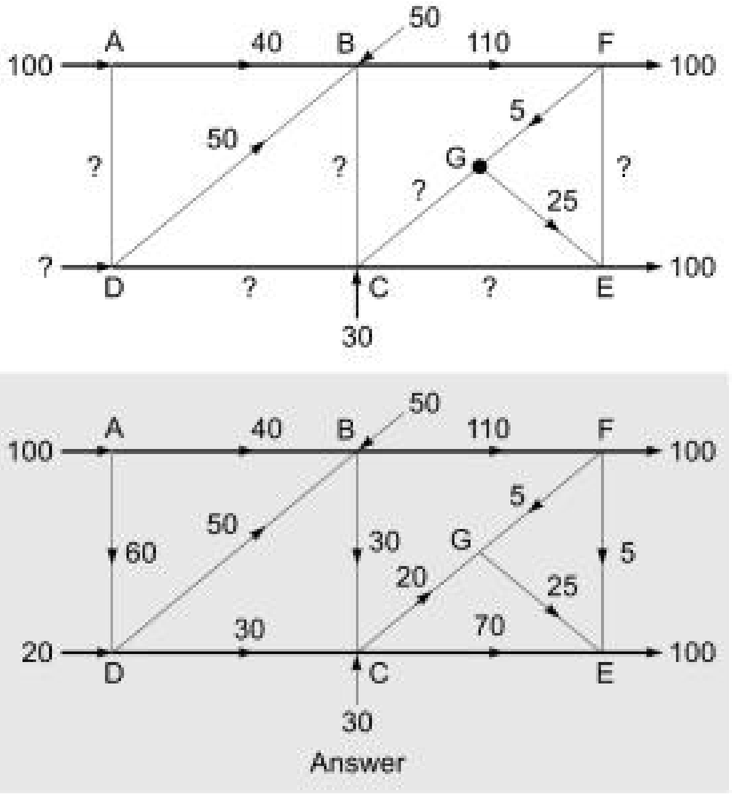


Fig. 3.27

3.7 The velocity in a flow is found to vary as $v_x = V_0/\left(1 - \frac{x}{L}\right)^2$. Determine the acceleration at a point $x = 0.5$ m when $L = 1.5$ m and $V_0 = 3.0$ m/s.

(*Ans.* $a = 71.40$ m/s^2)

3.8 A two-dimensional duct contains a straight sided contraction as in Fig. 3.28. The depth is constant at 30 cm. At a certain time an incompressible fluid flows in the duct at a rate of 0.3 m^3/s and is decreasing at the rate of 0.1 m^3/s per second.

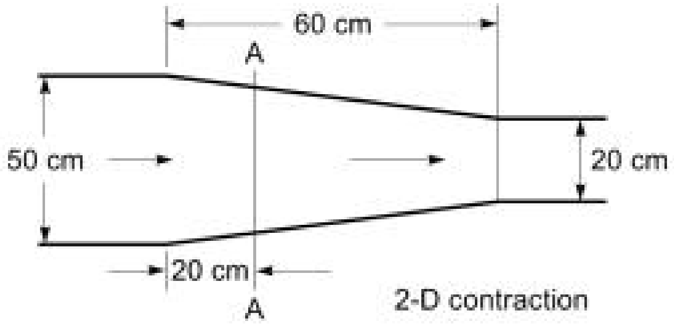


Fig. 3.28

Estimate the acceleration at the section AA 20 cm from the inlet.

(*Ans.* $a = -8.648$ m/s^2)

3.9 A 90 cm diameter pipe is reduced to 30 cm diameter in a length of 1.5 m. Water flows through this pipe at a rate of 280 L/s. If at an instant the discharge is found to decrease at the rate of 60 L/s per second, estimate the total acceleration at a distance of 75 cm from the 90 cm diameter inlet.

(*Ans.* $a = 1.0953$ m/s^2)

3.10 A 40 cm diameter pipe is reduced uniformly to 20 cm diameter in a length of 0.5 m. If the steady discharge of water through this pipe is 100 L/s, determine the acceleration at 10 cm from the 20 cm diameter end (i) when the flow is steady and (ii) when the flow is increasing at a rate of 600 L/s per minute.

(*Ans.* (i) $a = 16.289$ m/s^2, (ii) $a = 16.51$ m/s^2)

3.11 For the following flows find the equation of the streamline passing through the indicated point:

(i) $\mathbf{V} = 2x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$ ..... passing through (1, 1, 1)

(ii) $\mathbf{V} = 4y\mathbf{i} - 3x\mathbf{j}$ ...... passing through (– 1, 2)

(iii) $\mathbf{V} = -x\mathbf{i} + 2y\mathbf{j} + (3 - z)\mathbf{k}$ ...... passing through (1, 1, 2)

(*Ans.* $x^{1/2} y = 1$ and $x^{1/2} z = 1$, (ii) $3x^2 + 4y^2 = 19$, (iii) $xy^{1/2} = 1$ and $x = 3 - z$)

3.12 Calculate the circulation per unit area around a closed contour not including the centre in the following types of vortex motion: (i) $v_s = cr$ and (ii) $v_s = c/r$, where v_s = tangential velocity at a radial distance r and c = a constant.

(*Ans.* (i) $\Gamma/A = 2C$ and (ii) $\Gamma/A = 0$)

313 For the following set of velocity components verify whether the continuity equation is satisfied. If so, determine the (i) vorticity vector and (ii) acceleration vector at point A (1, 1, 1)

$$u = 2x^2 + 3y$$
$$v = -2xy + 3y^3 + 3zy$$
$$w = -\frac{3}{2}z^2 - 2xz - 9y^2 z$$

(*Ans.* (i) $\zeta_z = -(2y + 3)$; $\zeta_x = -(18yz + 3y)$, $\zeta_y = 2z$, (ii) $a_x = 32$, $a_y = -7.5$ and $a_z = 93$)

3.14 Check whether the following sets of velocity components satisfy the continuity equation of steady incompressible flow.

(i) $u = 4x + 2y - 3$, $v = 2x + 4y + 3$ (ii) $u = 4xy + y^2$, $v = 6xy + 3x + 2$

(iii) $u = 2x^2 + y^2$, $v = -4xy$ (iv) $u = x^3 + y^3$, $v = x - 3x^2y$

(v) $u = \dfrac{C(y^2 - x^2)}{(x^2 + y^2)^2}$

$v = \dfrac{-2Cxy}{(x^2 + y^2)^2}$

(*Ans.* (i) not satisfied, (ii) not satisfied, (iii) satisfied (iv) satisfied, (v) not satisfied)

3.15 Calculate the unknown velocity component in the following so that the equation of continuity is satisfied.

(i) $u = \frac{2}{3}xy^3 - x^2y$, $v = ?$ (ii) $u = Aye^x$, $v = ?$

$$\left(Ans.\ (i)\ v = -\frac{1}{6}y^4 + xy^2 + f(x),\quad (ii)\ v = -\frac{Ay^2}{2}e^x + f(x)\right)$$

3.16 For a two-dimensional incompressible flow, the x component of velocity and boundary conditions are as follows:

$u = x^2y^2 + 2xy$ and at $y = 0$, $u = v = 0$

Determine the y component of the velocity.

$$\left(Ans.\ v = -\frac{2}{3}xy^3 - y^2\right)$$

3.17 Given the components of a velocity vector in a three-dimensinal flow, as below, determine the missing component.

(i) $u = 2x^2 + 2xy$, $v = 4yz$, $w = ?$

(ii) $u = 3x^2$, $v = 4yxz$, $w = ?$

(iii) $u = x^3 + y^2 + 2z^2$, $v = -x^2y - yz - xy$, $w = ?$

{(*Ans.* (i) $w = -(4xz + 2yz + 2z^2) + f(x, y)$, (ii) $w = -(6xz + 2xz^2) + f(x, y)$, (iii) $w = 2x^2z - \dfrac{z^2}{2} - xz + f(x, y)$}

3.18 If the velocity u in a two-dimensional turbulent boundary layer over a flat plate is given by $\dfrac{u}{U_0} = (y/\delta)^{1/7}$ where U_0 = constant and δ = boundary layer thickness = $K\,x^{4/5}$, obtain an expression for the velocity component v in the y direction.

$$\left(Ans.\ v = \frac{1}{10}\,\frac{KU_0(y/\delta)^{8/7}}{(\delta/K)^{1/4}}\right)$$

3.19 For the following sets of velocity components obtain the relevant stream functions.

(i) $u = 6y$ (ii) $u = \dfrac{y^3}{3} + 2x - x^2y$

$v = 6x$ $\quad v = xy^2 - 2y - \dfrac{x^2}{3}$

(iii) $u = -\text{A} \ln x$

$v = A\ \dfrac{y}{x} + x^2$

$$\left(Ans.\ \text{(i) } \psi = 3(y^2 - x^2) + C,\right.$$

$$\text{(ii) } \psi = \frac{y^4}{12} - \frac{x^2y^2}{2} + 2xy + \frac{x^3}{9},$$

$$\left.\text{(iii) } \psi = -\text{A}y \ln x - \frac{x^3}{3} + \text{C}\right)$$

3.20 The velocity components of a steady, two-dimensional flow of an ideal fluid, are $u = 2xy$ and $v = a^2 + x^2 - y^2$. Show that the velocity potential exists and determine the same.

($Ans.$ $\phi = a^2y + x^2y - y^3/3$ + constant)

3.21 If the velocity potential $\phi = (x^3/3) - x^2 - xy^2 + y^2$, determine the corresponding stream function.

($Ans.$ $\psi = x^2y - 2xy - (y^3/3)$ + constant)

3.22 Find the velocity potential if the velocity field is

$$u = \frac{x}{x^2 + y^2}\ ;\ v = \frac{y}{x^2 + y^2}$$

$$\left(Ans.\ \phi = \frac{1}{2}\log(x^2 + y^2)\right)$$

3.23 Verify whether the following flow fields are rotational. If so, determine the components of rotation about various axes.

(i) $u = xyz$ (ii) $u = xy$

$v = zx$ $\quad v = \dfrac{1}{2}(x^2 - y^2)$

$w = \dfrac{1}{2}yz^2 - xy$

$$\left(Ans.\ \text{(i) Rotational, } \omega_z = \frac{1}{2}z(1 - x),\right.$$

$$\omega_x = \frac{1}{2}(z^2/2 - 2x),\ \omega_y = \frac{1}{2}y(x + 1)$$

$$\left.\text{(ii) Irrotational, } \omega_x = \omega_y = 0\right)$$

3.24 Calculate the velocity at the point (3, 3) for the following stream function.

(i) $\psi = -x \ln xy + x$

(ii) $\psi = \dfrac{1}{2}(y^2 - x^2) + xy + 6$

($Ans.$ (i) $V = 2.42$ units,
(ii) $V = 6$ units)

3.25 Given the following stream functions, determine the corresponding potential functions. Also, estimate the discharge, per unit depth, in the z direction passing between the streamlines through the points (1, 3) and (3, 3).

(i) $\psi = 3xy$

(ii) $\psi = \dfrac{3}{2}(y^2 - x^2)$

$$\left(Ans.\ \phi = \frac{3}{2}(x^2 - y^2) + c;\ 18 \text{ units},\right.$$

$$\left.\text{(ii) } \phi = 3xy;\ -12 \text{ units}\right)$$

3.26 Verify whether the following functions are valid potential functions.

(i) $\phi = y^3 - 3x^2y$ (ii) $\phi = y^4 - 6x^2y^2$

(iii) $\phi = x^2 - 3x^2y$ (iv) $\phi = x^3 - y^3$

($Ans.$ (i) Yes, (ii) No, (iii) No, (iv) No.)

3.27 Determine the corresponding conjugate functions relating to the following functions representing irrotational fluid flows.

(i) $\phi = Ur\cos\theta$

(ii) $\psi = -m\theta + k\ln r$

($Ans.$ (i) $\psi = Ur\sin\theta + \text{C}$,
(ii) $\phi = -k\theta - m\ln r$)

3.28 The potential flow around a cylinder with circulation is given by

$$\phi = U\left(r + \frac{a^2}{r}\right)\cos\theta - \text{C}\theta$$

where a = radius of the cylinder, U = velocity far away from the cylinder and C = a measure of circulation. Calculate the corresponding stream function and the velocities v_r and v_θ.

$$\left(Ans.\ \psi = U\left(r - \frac{a^2}{r}\right)\sin\theta - \text{C In } r;\right.$$

$$v_r = U\left(1 - \frac{a^2}{r^2}\right)\cos\theta,$$

$$\left.v_\theta = -U\sin\theta\left(1 + \frac{a^2}{r^2}\right) - \frac{\text{C}}{r}\right)$$

$$\frac{p_2}{\gamma} + \frac{(0.7074)^2}{2\times 9.81} + 107.00 - 2.00 = \frac{30}{9.79} + \frac{(2.829)^2}{2\times 9.81} + 100.00$$

$$\frac{p_2}{\gamma} + 0.0255 + 107.00 - 2.00 = 3.064 + 0.4080 + 100.00$$

$$p_2/\gamma = -\ 1.554 \text{ m (gauge)}$$

$$p_2 = -\ 1.554 \times 9.79$$

$$= \mathbf{-15.21\ kPa\ (gauge)}$$

4.2 In a smooth pipe of uniform diameter 25 cm a pressure of 50 kPa was observed at section 1 which was at elevation 10.00 m. At another section 2 at elevation 12.00 m the pressure was 20 kPa and the velocity was 1.25 m/s. Determine the direction of flow and the head loss between these two sections. The fluid in the pipe is water.

Solution: Refer to Fig. 4.4

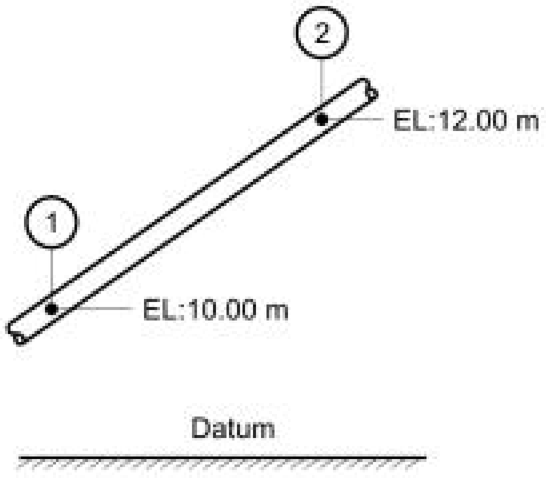


Fig. 4.4

Discharge $Q = A_1V_1 = A_2V_2$

$$Q = \frac{\pi}{4} \times (0.25)^2 \times 1.25 = 0.0614 \text{ m}^3\text{/s}$$

$$= 61.4 \text{ L/s}$$

As $A_1 = A_2$, $V_1 = 1.25$ m/s

$$\gamma = 998 \times 9.81/1000 = 9.79 \text{ kN/m}^3$$

Taking atmospheric pressure as zero,
Total energy at section 1,

$$H_1 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1$$

$$H_1 = \frac{50.0}{9.79} + \frac{(1.25)^2}{2\times 9.81} + 10.00$$

$$= 5.107 + 0.0796 + 10.00 = 15.187 \text{ m}$$

Total energy at section 2,

$$H_2 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$H_2 = \frac{20.0}{9.79} + \frac{(1.25)^2}{2\times 9.81} + 12.00$$

$$= 2.043 + 0.0796 + 12.00 = 14.123 \text{ m}$$

As $H_1 > H_2$, the flow is from section 1 to 2.
Head loss between the sections 1 and 2, i.e.

$$H_L = H_1 - H_2 = 15.187 - 14.123 = \mathbf{1.064\ m}$$

4.3 Figure 4.5 shows a nozzle at the end of a pipe discharging oil from a tank to atmosphere. Estimate the discharge from the nozzle when the head H in the tank is 4.0 m. The loss in the pipe can be taken as 20 $V^2/2g$, where V = velocity in the pipe. The loss of energy in the nozzle can be assumed to be zero. Also determine the pressure at the base of the nozzle.

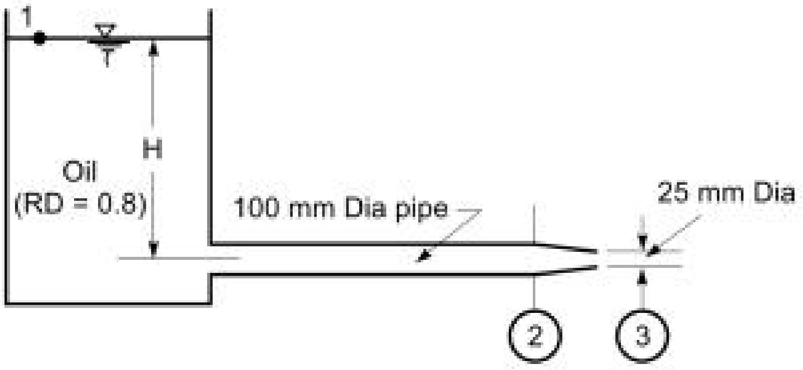


Fig. 4.5

Solution: By continuity criterion,

$$V_2 \frac{\pi}{4} (D)^2 = V_3 \frac{\pi}{4} (d)^2 = Q$$

Velocity in the pipe $V_2 = V_3 \cdot \left(\frac{d}{D}\right)^2 = \left(\frac{25}{100}\right)^2 V_3$

$$= \frac{1}{16} V_3$$

Apply Bernoulli equation to points 1 and 3 with the centre line of the pipe as datum and atmospheric pressure as zero. The velocity at point 1 can be taken as zero.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + Z_3 + H_L$$

$$0 + 0 + H = 0 + \frac{V_3^2}{2g} + 0 + 20\ \frac{V_2^2}{2g}$$

$$H = \frac{V_3^2}{2g} + 20 \times \left(\frac{1}{16}\right)^2 \frac{V_3^2}{2g}$$

$$= 1.07813\ \frac{V_3^2}{2g}$$

As H = 4.0 m, $V_3 = \left(\frac{2 \times 9.81 \times 4.0}{1.07813}\right)^{1/2}$

$$= 8.5319 \text{ m/s}$$

the level of water in the tank (Fig. 4.10). If friction losses are 1.5 m of water, what air pressure must be maintained in the tank to deliver 15 L/s to the roof?

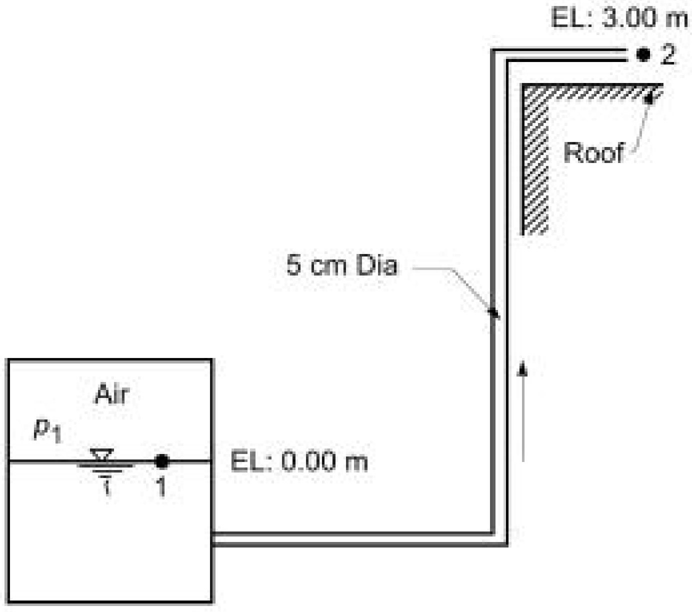


Fig 4.10

Solution:

$$\text{Discharge } Q = 0.015 \text{ m}^3/\text{s} = \frac{\pi}{4} \times (0.05)^2 \times V_2$$

$$V_2 = 7.639 \text{ m/s}$$

By applying Bernoulli equation to points 1 and 2,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + H_L$$

$$\frac{p_1}{\gamma} + 0 + 0 = 0 + \frac{(7.639)^2}{2 \times 9.81} + 3.00 + 1.5$$

$$\frac{p_1}{\gamma} = 7.475 \text{ m and } \gamma = 9.79 \text{ kN/m}^3$$

Hence, $p_1 = 9.79 \times 7.475 = \mathbf{73.18\ kPa}$

4.9 The loss of head in sudden expansion of a pipe is given by $H_L = (V_1 - V_2)^2/2g$ where suffixes 1 and 2 refer to sections before and after the expansion. A 10 cm pipe carrying oil (RD = 0.85) undergoes a sudden expansion to 20 cm diameter in a horizontal stretch of the pipeline. If the pressure difference between sections 2 and 1 is recorded as 660 Pa, estimate the discharge through the pipe and sketch the energy line and piezometric head line at the expansion, by considering the pressure at section 1, $p_1 = 3.0$ kPa.

Solution: Refer to Fig. 4.11.

$$D_1 = 10 \text{ cm}, A_1 = \frac{\pi}{4} \times (0.1)^2 = 7.854 \times 10^{-3} \text{ m}^2$$

$$D_2 = 20 \text{ cm}, A_2 = \frac{\pi}{4} \times (0.2)^2 = 3.142 \times 10^{-2} \text{ m}^2$$

$$Q = V_1 A_1 = V_2 A_2. \text{ Hence } V_2 = \left(\frac{D_1}{D_2}\right)^2 V_1 = \frac{1}{4} V_1$$

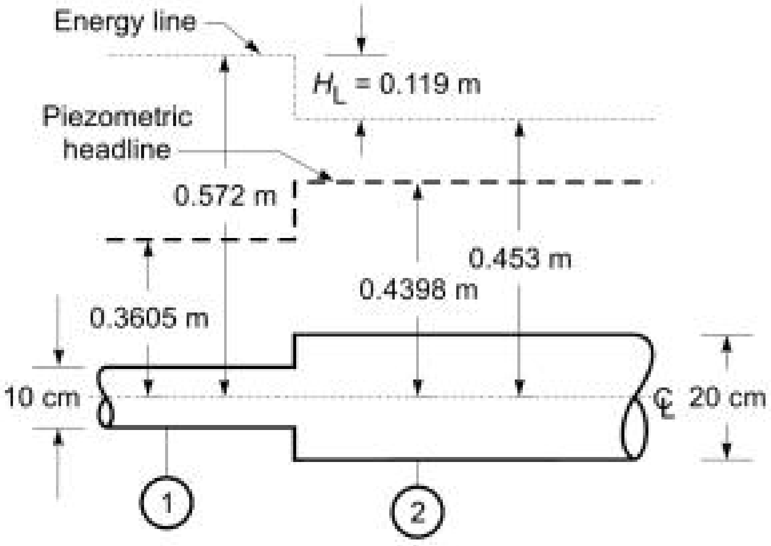


Fig. 4.11

Head loss at the expansion $H_L = (V_1 - V_2)^2/2g$

$$= \frac{(V_1 - V_1/4)^2}{2g} = \frac{9}{16}\frac{V_1^2}{2g}$$

By applying Bernoulli equation to sections 1 and 2

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + H_L$$

$$\left(\frac{p_2}{\gamma} - \frac{p_1}{\gamma}\right) = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + Z_1 - Z_2 - H_L$$

$$\frac{660}{(0.85 \times 9790)} = \frac{V_1^2}{2g}\left(1 - \frac{1}{16} - \frac{9}{16}\right) + 0 = \frac{3}{8}\frac{V_1^2}{2g}$$

$$V_1^2 = \frac{8}{3} \times 2 \times 9.81 \times 0.0793 = 4.1496$$

$$V_1 = 2.037 \text{ m/s}$$

$$\text{Discharge } Q = A_1 V_1 = 7.854 \times 10^{-3} \times 2.037 \text{ m}^3/\text{s}$$

$$= 0.016 \text{ m}^3/\text{s} = \mathbf{16.0\ L/s}$$

$$V_1 = 2.037 \text{ m/s}, \ V_1^2/2g = 0.2115$$

$$V_2 = \frac{2.037}{4} = 0.509 \text{ m/s},$$

$$V_2^2/2g = 0.0132 \text{ m}$$

$$(V_1 - V_2) = 1.528 \text{ m/s},$$

$$H_L = \frac{(V_1 - V_2)^2}{2g} = 0.1190 \text{ m}$$

$$\frac{p_1}{\gamma} = \frac{3000}{0.85 \times 9790} = 0.3605 \text{ m}$$

$$\frac{p_2}{\gamma} = 0.3605 + 660/(0.85 \times 9790)$$

$$= 0.4398 \text{ m}$$

With centre line of pipe as datum, the elevations of energy and piezometric head are as follows:

At Section 1:

$$\text{Energy head} = H_1 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1$$

$$= 0.3605 + 0.2115 + 0 = 0.572 \text{ m}$$

As $Z_2 = Z_1$ and $V = C/r$

$$(p_2 - p_1) = \frac{\rho}{2}(V_1^2 - V_2^2) = \frac{\rho C^2}{2}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$

Thus

$$30 \times 10^3 = \frac{998 \times C^2}{2}\left[\frac{1}{(0.4)^2} - \frac{1}{(0.65)^2}\right]$$

$$= 1937.7\ C^2$$

$$C^2 = 15.48 \text{ and } C = 3.9348$$

Discharge per unit depth

$$q = \int_{r_1}^{r_2} v\,dr$$

$$= \int_{r_1}^{r_2} \frac{C}{r}\,dr = C \ln r_2/r_1$$

$$= 3.9348 \ln \frac{0.65}{0.40}$$

$$= \mathbf{1.910\ m^3/s\ per\ metre\ width}$$

4.16 An irrotational two-dimensional flow leaves a straight duct and enters a bend of the same width, in the horizontal plane as shown in Fig. 4.18. Assuming the velocity distribution in the bend like that in a potential vortex (vr = constant), find the velocities at the inner and outer walls of the bend. Also, estimate the difference in the pressure head between the outer and the inner wall.

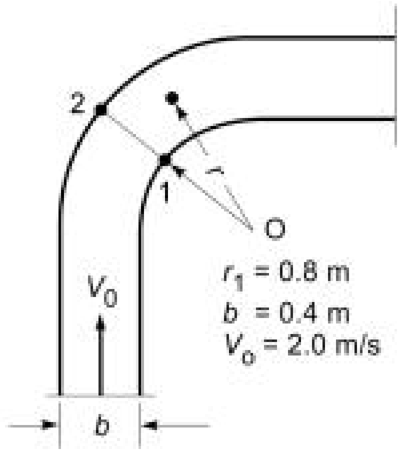


Fig. 4.18

Solution: Consider a unit depth of the duct.

$$v = C/r$$

$$q = bV_0 = \int_{r_1}^{r_2} v\,dr$$

$$= \int_{r_1}^{r_2} C/r\,dr = C \ln \frac{r_2}{r_1}$$

$$C = \frac{bV_0}{\ln(r_2/r_1)} = \frac{0.4 \times 2.0}{\ln(1.2/0.8)} = 1.973$$

V_1 = Velocity at the inner wall = C/r_1

$$= \frac{1.973}{0.80} = \mathbf{2.466\ m/s}$$

V_2 = Velocity at the outer walls = C/r_2

$$= \frac{1.973}{1.20} = \mathbf{1.644\ m/s}$$

Since the flow is irrotational, Bernoulli theorem can be applied across the streamlines also. Applying Bernoulli's theorem to points 1 and 2, located on the inner and outer wall, as in Fig. 4.16,

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1$$

Since $Z_1 = Z_2$ $\left(\frac{p_2 - p_1}{\gamma}\right) = \frac{V_1^2 - V_2^2}{2g}$

$$= \frac{1}{2 \times 9.81}[(2.446)^2 - (1.644)^2]$$

Therefore the difference in pressure head between outer and inner wall is **0.172 m.**

4.17 Figure 4.19 shows pump P pumping 60 L/s of water from a tank. (i) What will be the pressures at points A and B when the pump delivers 10 kW of powers to the flow. Assume losses in the system to be negligible. (ii) What will be the pressure at B when the loss in the inlet up to the pump is negligible and between the pump and the point B, a loss equal to 2 times the velocity head at B takes place.

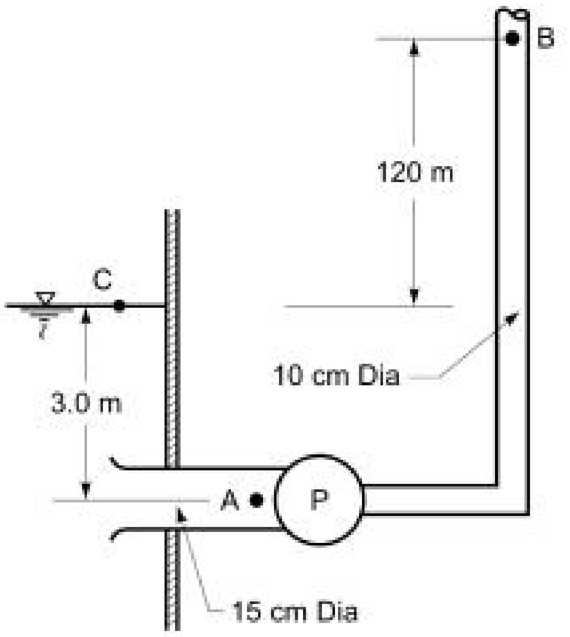


Fig. 4.19

Solution:

Discharge $Q = 0.060\ m^3/s = A_a V_a = A_b V_b$

$$V_a = \frac{0.06}{\frac{\pi}{4} \times (0.15)^2} = 3.395\ \text{m/s},\ \frac{V_a^2}{2g} = 0.588\ \text{m}$$

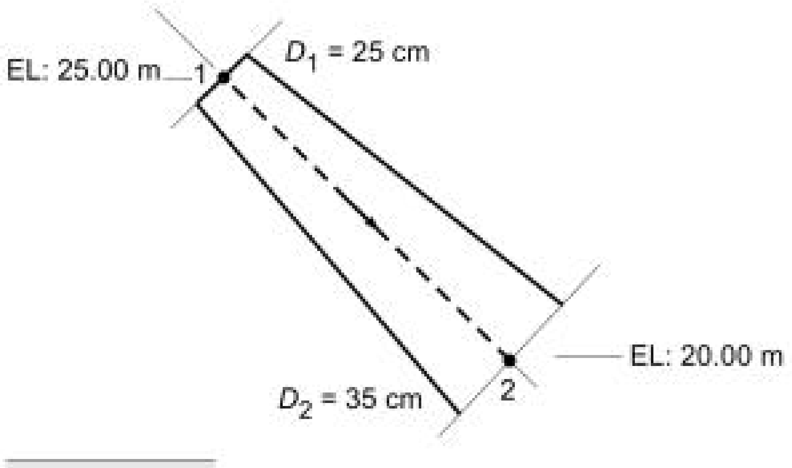


Fig. 4.25

$$V_2 = \frac{0.20}{\frac{\pi}{4} \times (0.35)^2} = 2.079 \text{ m/s}$$

$$(V_1 - V_2) = 1.995 \text{ m/s}$$

$$\alpha_1 = 1.1 \text{ and } \alpha_2 = 1.5$$

By applying Bernoulli equation to points 1 and 2,

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + Z_2 + H_L$$

$$\left(\frac{120}{9.79}\right) + 1.1 \times \frac{(4.074)^2}{2 \times 9.81} + 25.00$$

$$= \frac{p_2}{\gamma} + 1.5 \times \frac{(2.079)^2}{2 \times 9.81} + 20.00 + \frac{1.2 \times (1.995)^2}{2 \times 9.81}$$

$$12.257 + 0.931 + 25.00 = \frac{p_2}{\gamma} + 0.330 + 20.00 + 0.243$$

$$\frac{p_2}{\gamma} = 38.188 - 20.573 = 17.615 \text{ m}$$

$$p_2 = 9.79 \times 17.615 = \mathbf{172.45 \text{ kPa}}$$

4.24 The stream function ψ for the uniform flow past a circular cylinder of radius a in polar coordinates is given by

$$\psi = U_\infty \left[1 - \frac{a^2}{r^2}\right] r \sin \theta$$

where U_∞ is the free stream velocity parallel to the horizontal axis. (i) Determine the velocity on the cylinder. (ii) If p_∞ and p are the pressures at the free stream far away from the cylinder and the pressure on the cylinder (Fig. 4.26), obtain the values of the pressure coefficient $C_p = (p - p_\infty)/\frac{1}{2} \rho U_\infty^2$ on the cylinder at $\theta = 0, \pi/6, \pi/2, 5\pi/6$ and π.

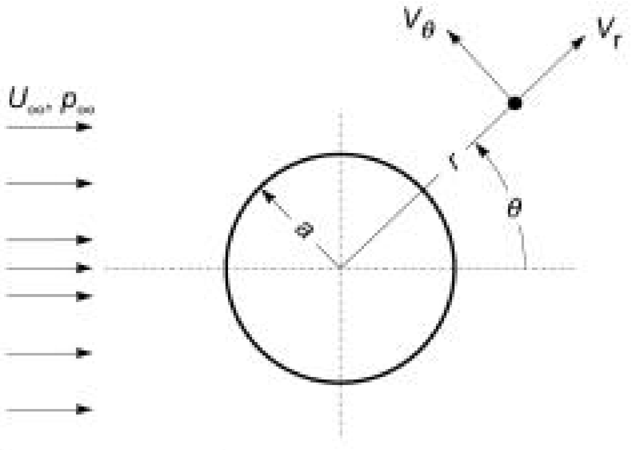


Fig. 4.26

Solution:

$$\psi = U_\infty \left[1 - \frac{a^2}{r^2}\right] r \sin \theta$$

$$v_r = \frac{1}{r} \frac{\partial \psi}{\partial r} \text{ and } v_\theta = -\frac{\partial \psi}{\partial r}$$

On the cylinder $\quad v_r = 0$ and $v_\theta = V = -\left.\frac{\partial \psi}{\partial r}\right|_{r=a}$

Hence $\quad V = -\left. U_\infty \left(1 + \frac{a^2}{r^2}\right) \sin \theta \right|_{r=a}$

$$V = -2U_\infty \sin \theta$$

By applying Bernoulli equation to a point on the free stream and a point on the cylinder at θ, for a 2-D flow

$$\frac{p_\infty}{\gamma} + \frac{V_\infty^2}{2g} = \frac{p}{\gamma} + \frac{V^2}{2g}$$

$$\frac{p - p_\infty}{\gamma} = \frac{1}{2g}(U_\infty^2 - 4U_\infty^2 \sin^2 \theta)$$

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U_\infty^2} = 1 - 4 \sin^2 \theta$$

Values of C_p for various values of θ are as shown in the following:

θ	0	$\pi/6$	$\pi/2$	$5\pi/6$	π
C_p	1	0	–3	0	1

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PROBLEMS

4.1 For the pipeflow system shown in Fig. 4.30 the following data are available:

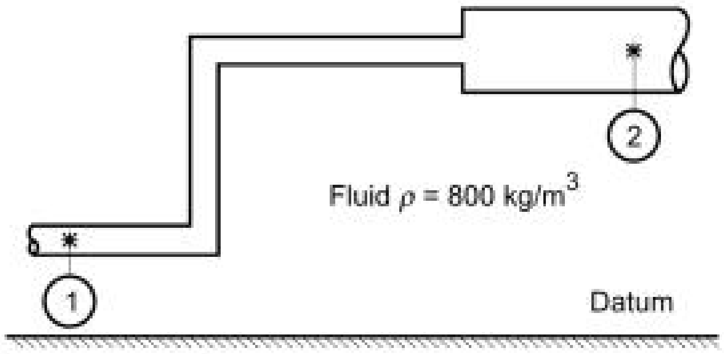


Fig. 4.30

Item	*Point* 1	*Point* 2
Diameter	20 cm	30 cm
Elevation (m)	103.00	106.00
Pressure	55 kPa	75 kPa
Velocity	2.5 m/s	

Determine the direction of flow and the loss of energy between these two points.

(*Ans.* The flow is from 2 to 1. H_L = 5.292 m)

4.2 A 15 cm diameter pipe is reduced to 7.5 cm diameter through gradual contraction. At this contraction the difference between the piezometric heads at the main and the contracted section is 4 cm of mercury. By neglecting losses, calculate the discharge of water.

(*Ans.* Q = 14.9 L/s)

4.3 For the flow system shown in Fig. 4.31 diameter $D_1 = D_3$ = 20 cm, D_2 = 10 cm, D_4 = 7.5 cm. The fluid in the manometer is mercury. The mercury water differential head h_3 = 10 cm. Assuming zero energy loss, find (i) the discharge, (ii) manometer differential head h_1, (iii) velocity of flow at section 4 and (iv) head of water H in the tank.

(*Ans.* Q = 22 L/s; h_1 = 3.03 cm; V_4 = 5.022 m/s; H = 1.285 m)

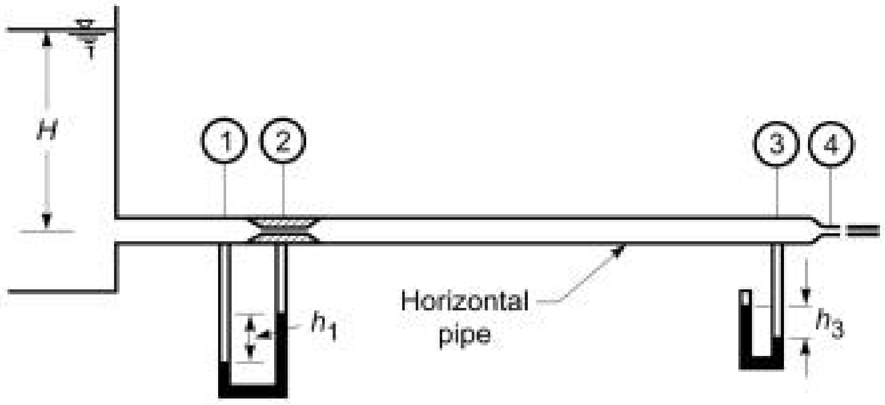


Fig. 4.31

4.4 Two points A and B are located in a long 20 cm diameter pipe. When a downstream valve is completely closed the difference in pressure between B and A, $p_B - p_A$ = 100 kPa. When the valve is open and a discharge of 70 L/s of water is flowing, $p_A - p_B$ = 50 kPa. Calculate the head loss between A and B.

(*Ans.* H_L = 15.32 m)

4.5 A siphon consisting of a 3 cm diameter tube is used to drain water from a tank. The outlet end of the tube is 2.0 m below the water surface in the tank. Neglecting friction, calculate the discharge. If the summit of the siphon is 1.4 m above the water surface in the tank, estimate the pressure at the summit of the siphon.

(*Ans.* Q = 4.428 L/s, p = – 33.29 kPa)

4.6 For the system shown in Fig. 4.32, find the height h to which the jet from the nozzle would rise. If the nozzle and the pipe were to have diameters of 10 cm and 20 cm respectively, calculate the discharge and the velocity in the pipe. Neglect frictional losses.

(*Ans.* h = 2.361 m, Q = 53.5 L/s, V_p = 1.702 m/s)

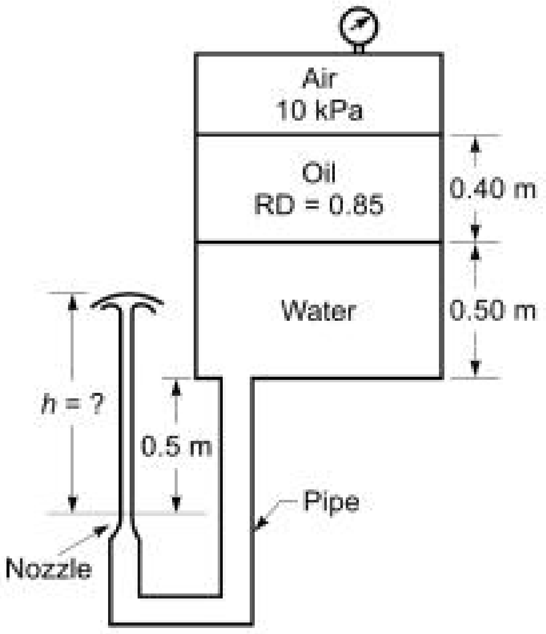


Fig. 4.32

4.7 A 10 cm long nozzle of exit diameter 10 cm is attached to a pipe of 30 cm diameter. The nozzle is vertical. If a water jet issuing out of the nozzle reaches a height of 4.5 m above the nozzle exit, calculate the discharge. Also, by assuming a head loss in the nozzle equal to 10% of the exit velocity

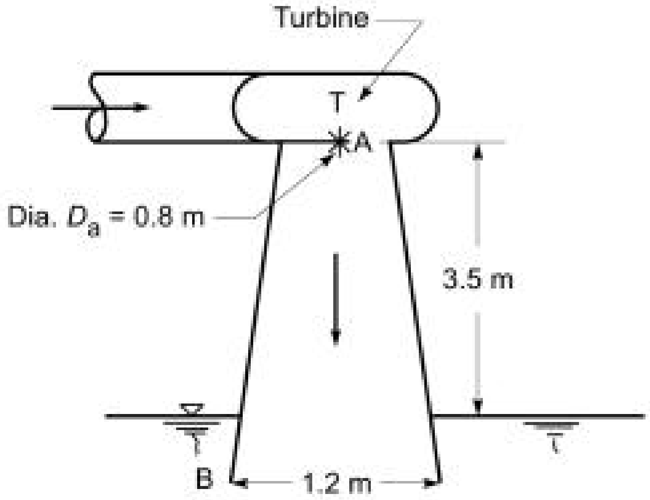


Fig 4.41

of the supply pipe at the turbine. For a discharge of 1.1 m^3/s, the pressure head just upstream of the turbine is 40 m. Estimate the power output of the turbine by assuming 85% efficiency. Also, determine the pressure at the commencement of the draft tube, next to the turbine, which is 3.50 m above the tailwater, (Fig. 4.41)

(*Ans.* **P** = 405 kW; $P_b = -34.79$ kPa)

4.29 A pipeline delivering water from a reservoir is shown in Fig. 4.42. A pump M adds energy to the flow and 45 L/s of water is discharged to atmosphere at the outlet. Calculate the power delivered by the pump. Assume the head loss in the pipe as two times the velocity head at the suction side and 10 times the velocity head in the delivery pipe. Draw a neat sketch showing energy line and hydraulic grade lines.

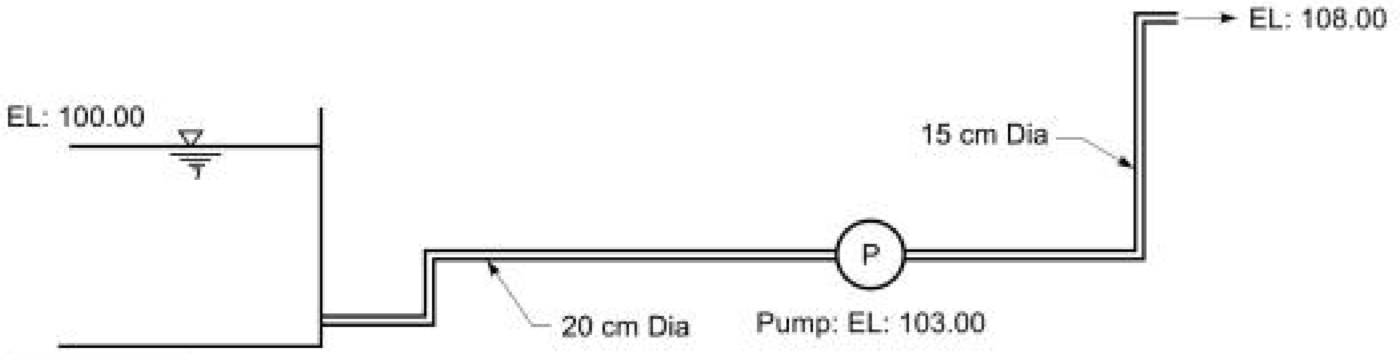


Fig 4.42

(*Ans.* **P** = 5.218 kW)

4.30 Air (ρ = 1.08 kg/m^3) flows past a stationary cylinder. Far away from the cylinder the air velocity V_0 = 40 km/h. Calculate the stagnation pressure on the cylinder. If the velocity v at any point on the cylinder is $v = 2V_0 \sin\theta$ where θ = angle measured from the front stagnation point, calculate the pressure on the surface at angles θ = 30°, 60° and 90°. (Take atmospheric pressure as reference pressure).

[*Ans.* p_s = 66.67 Pa (gauge)
$\theta = 30°$, $p = 0$,
$\theta = 60°$, $p = -133.33$ Pa (gauge)
$\theta = 90°$, $p = -220$ Pa (gauge)]

4.31 For the uniform flow of velocity U_∞ past a circular cylinder of radius a (Fig. 4.43) and circulation Γ the velocity on the cylinder is given by

$$v_\theta = -2U_\infty \sin\theta - \frac{\Gamma}{2\pi a}$$

Obtain an expression for the variation of the pressure coefficient $C_p = (P - p_\infty)/\left(\frac{1}{2}\rho U_\infty^2\right)$ on the cylinder. Also obtain an expression for the location of the stagnation point on the cylinder.

$$\left(Ans.\ C_p = \left(\frac{p-p_\infty}{\frac{1}{2}\rho U_\infty^2}\right) = \left[1-\left(2\sin\theta+\frac{\Gamma}{2\pi U_\infty R}\right)^2\right]\right.$$

$$\left. \text{At stagnation point, } \sin\theta = \frac{\Gamma}{4\pi U_\infty a}\right)$$

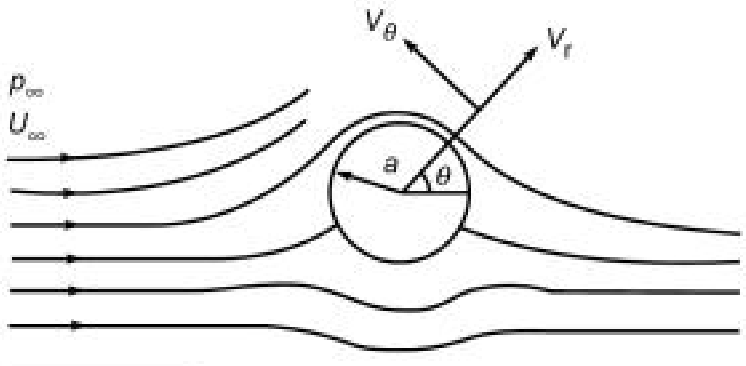


Fig 4.43

$$(48394 \times 0.07069) - R_x = 998\,[0.04418 \times 10 \times \cos 25° + 0.07854 \times 10 \times \cos 35° - 0.1227 \times 1.736]$$

$$3421 - R_x = 998\,(0.4004 + 0.6434 - 0.2130)$$

$$R_x = 3421 - 829 = 2592 \text{ N}$$

By applying momentum equation in the y-direction:

$$0 - R_y = \rho(Q_2V_2 \sin 25° - Q_3V_3 \sin 35°)$$
$$= 998\,(0.04418 \times 10 \times \sin 25° - 0.07854 \times 10 \times \sin 35°)$$
$$= 998\,(0.1867 - 0.4505) = -263$$
$$R_y = 263 \text{ N}$$

The reaction R on the fluid in the control volume is

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(2592)^2 + (263)^2}$$
$$= 2605 \text{ N}$$

Inclination θ of R with the (–x) direction

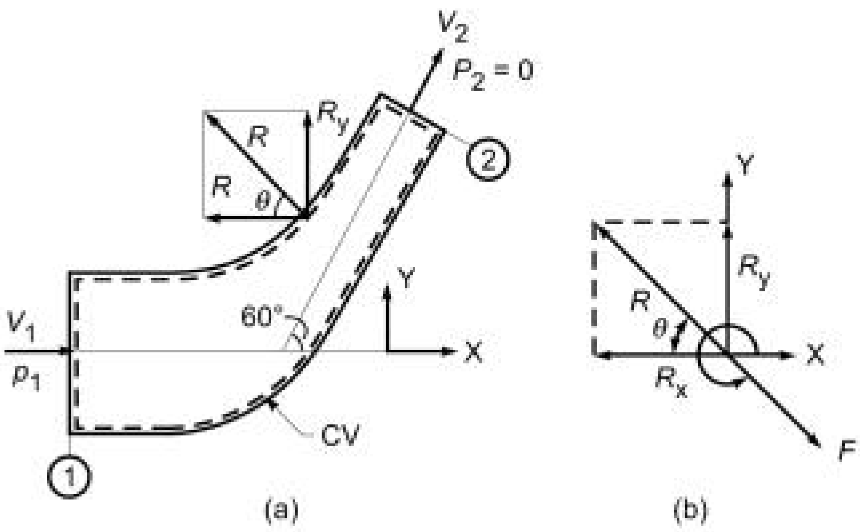
$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{263}{2592} = 5.79°$$

Force F on the body of the junction exerted by the fluid will be equal and opposite to R. Hence **F = 2605 N and will be directed at 5.79° to the positive x-direction**.

5.8 A reducer bend having an outlet diameter of 15 cm discharges freely. The bend, connected to a pipe of 20 cm diameter, has a deflection of 60° and lies in a horizontal plane. Determine the magnitude and direction of force on the anchor block supporting the pipe when a discharge of 0.3 m^3/s passes through the pipe.

Solution: Consider the control volume as shown by the dotted line in Fig. 5.12 (a). At section 2: $p_2 = 0$ = atmospheric pressure.

$$V_2 = \frac{0.3}{\frac{\pi}{4} \times (0.15)^2} = 16.98 \text{ m/s}$$



(a) (b)

Fig. 5.12

$$\text{At section 1: } V_1 = V_2\left(\frac{D_2}{D_1}\right)^2 = 16.98 \times \left(\frac{15}{20}\right)^2$$
$$= 9.55 \text{ m/s}$$

By applying Bernoulli equation to sections 2 and 1,

$$0 + (16.98)^2/(2 \times 9.81) = \frac{p_1}{\gamma} + (9.55)^2/(2 \times 9.81)$$

$$\frac{p_1}{\gamma} = 14.689 - 4.648 = 10.041 \text{ m}$$

$$p_1 = 9790 \times 10.041 = 98301 \text{ Pa}$$

Let R_x and R_y be the reaction of the pipe on the fluid in the control volume in (–x) and y-directions, respectively [Fig. E-5.12 (b)]. By applying momentum equation in x-direction),

$$p_1A_1 - R_x = \rho Q(V_2 \cos 60° - V_1)$$

$$98301 \times \frac{\pi}{4} \times (0.2)^2 - R_x$$
$$= 998 \times 0.3 \times (16.98 \cos 60° - 9.55)$$
$$R_x = 3088.2 + 317.4 = 3405.6 \text{ N}$$

By momentum equation in y-direction,

$$0 + R_y = \rho Q(V_2 \sin 60° - 0)$$
$$R_y = 998 \times 0.3 \times 16.98 \times 0.866 = 4402.7 \text{ N}$$

$$\text{Resultant} \quad R = \sqrt{R_x^2 + R_y^2}$$
$$= \sqrt{(3405.6)^2 + (44027)^2} = 5564.3 \text{ N}$$

Inclined at an angle θ such that $\tan\theta = \frac{R_y}{R_x}$

$$\theta = \tan^{-1}\frac{4402.7}{3405.6} = 52.28°$$

The force F on the pipe is equal and opposite to R and hence **F = 5564 N inclined at (360 – θ) = 307.72° to positive x-axis** (see Fig. 5.12 (b)].

5.9 Water flows upwards through a vertical nozzle of inlet and outlet diameters 0.5 m and 0.25 m respectively as shown in Fig. 5.13. The pressure at section 1 is 15 kPa and the volume rate of flow is 0.5 m^3/s. Find (i) the velocities at section 1 and 2, (ii) pressure at section 2 and (iii) total force acting on the walls of the nozzle [Neglect frictional resistance]

Solution: Refer to Fig. 5.13.

$$A_1 = (\pi/4)\,(0.5)^2 = 0.1964 \text{ m}^2$$
$$A_2 = (\pi/4)\,(0.25)^2 = 0.0491 \text{ m}^2$$
$$V_1 = 0.5/0.1964 = 2.546 \text{ m/s}$$
$$V_2 = 0.5/0.04951 = 10.185 \text{ m/s}$$

By applying Bernoulli's theorem to sections 1 and 2

$$\frac{p_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

5.14 The momentum correction factor is given by $\beta =$

(a) $\dfrac{1}{V^3A}\displaystyle\int_A u^3\,\mathrm{d}A$ (b) $\dfrac{1}{VA}\displaystyle\int_A u\,\mathrm{d}A$

(c) $\dfrac{1}{A^2}\displaystyle\int_A A^2\,\mathrm{d}u$ (d) $\dfrac{1}{V^2A}\displaystyle\int_A u^2\,\mathrm{d}A$

5.15 The momentum correction factor β for triangular velocity distribution for flow between two parallel plates as shown in Fig. 5.23 is

(a) 2.0 (b) 1.50

(c) 1.33 (d) 2.50

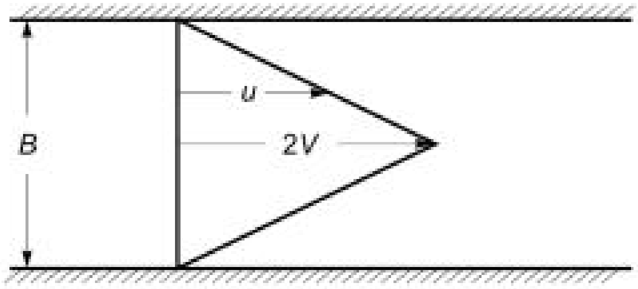


Fig. 5.23

5.16 A nozzle discharging under a head H has an area a and a discharge coefficient $C_d = 1.0$. A vertical plate is acted upon by the fluid force F_j when held across the free jet and by the fluid force F_n when held against the nozzle to stop the flow. The ratio F_j/F_n is

(a) 1/2 (b) 1

(c) $\sqrt{2}$ (d) 2

5.17 Consider the following velocity profiles in a pipeline (Fig. 5.24)

Among these profiles the momentum correction factor would be

(a) least in 4

(b) highest in 1

(c) more in 3 than that for 2

(d) the same in 1, 2, 3 and 4

5.18 The velocity distribution over one half of a cross section is uniform and is zero over the remaining half. The momentum correction factor for this cross section is

(a) 2.0 (b) 4.0

(c) 1.0 (d) 3.0

5.19 A jet strikes a stationary plate normally with a velocity of 8 m/s and the plate suffers a force of 120 N. The power obtained, in kW is

(a) 0.96 (b) 9.4

(c) zero (d) 958

5.20 If all the energy in a jet of velocity 20 m/s issuing as a jet of 15 cm diameter could be extracted, the power available, in kW, is

(a) 70.54 (b) 7.21

(c) 20.39 (d) 705

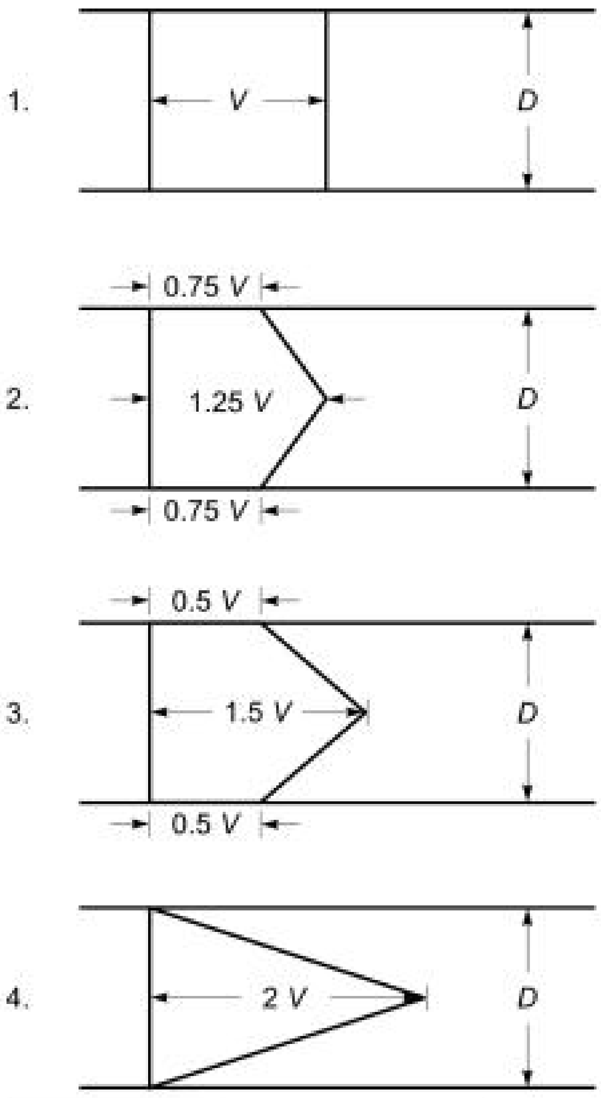


Fig. 5.24 *Question 5.17*

5.21 A horizontal venturimeter has a head loss of 1/10 the upstream pressure head. If the flow is in x-direction, this causes on the venturimeter.

(a) a net force in (– x)-direction

(b) zero force

(c) a net force in x-direction

(d) an oscillating force

Moment of Momentum

5.22 The moment of momentum principle states that in a rotating system

(a) the resultant force exerted by the fluid on the body is equal to the rate of change of angular momentum

(b) the torque exerted by the resultant force is equal to the time rate of change of angular momentum

(c) the torque exerted by the resultant force is equal to the time rate change of linear momentum

(d) the angular moment is conserved

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bottom and 60 cm diameter at the top. Calculate the magnitude and direction of force exerted by the flow on the enlargement.

(*Ans.* F = 21.30 kN acting vertically downwards)

5.22 A sluice gate in a rectangular channel carrying water is so opened to create depths of flow of 1.70 m and 0.25 m on the upstream and downstream of the gate respectively. The discharge intensity is 1.30 m^3/s per metre width and the flow from the sluice gate is in free flow mode. Estimate the force per metre width on the gate.

(*Ans.* F = 8.09 kN)

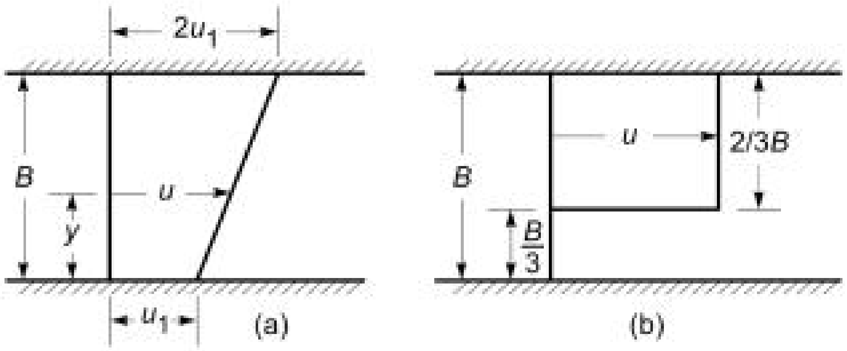
5.23 In a hydraulic jump occurring in a rectangular open channel the discharge is 5.4 m^3/s per metre width. The depth of flow before the jump is 0.3 m. Estimate the depth of flow after the jump by neglecting friction on the channel bottom.

[**Hint:** Use momentum equation and continuity equation].

(*Ans.* y_2 = 4.304 m)

5.24 For the velocity distribution in a two dimensional duct, shown in Fig. 5.31 (a) and (b) calculate the value of the momentum correction factor.

(*Ans.* (a) 1.333, (b) 1.5)



(a) (b)

Fig. 5.31

5.25 Determine the momentum correction factor β for the following velocity distributions in a rectangular open channel of depth of flow y_0.

(a) $u/u_m = y/y_0$ for $y = 0$ to $y = y_0$.

(b) $u/u_m = (y/y_0)^n$ for $y = 0$ to $y = y_0$.

(*Ans.* (a) 1.333, (b) $(n + 1)^2/(2n + 1)$)

5.26 A 20 cm diameter pipe has a nozzle with 10 cm diameter exit attached to it. A jet of water with 15 m/s velocity is discharged, horizontally, from this nozzle into the atmosphere. The momentum correction factor β for the pipe flow is 1.25 and 1.0 for the jet. The kinetic energy correction factor α for the pipe flow is 1.75 and it is 1.0 for the jet. Calculate the force on the nozzle at its junction with the pipe.

(*Ans.* F = 1929 N in x-direction)

5.27 A sluice gate in a rectangular channel is discharging a flow of water in free flow mode. The depths of flow are 2.20 m and 0.40 m on the upstream and downstream of the gate respectively. Assuming zero frictional losses estimate the discharge per unit width of the channel and the force per unit width of the gate.

(*Ans.* q = 2.417 m^3/s/m and F_x = 11.98 kN in x-direction)

Moment of Momentum

5.28 A lawn sprinkler has 1.5 cm diameter nozzles at the two ends of a rotating arm. The nozzles make an angle of 150° with the tangential direction of rotation. The flow enters the sprinkler normally at the centre. The nozzle ends lie on a diameter of 0.8 m and the discharge of water entering the nozzle is measured as 2.0 L/s. Calculate the speed of rotation by

(i) assuming zero frictional resistance

(ii) assuming the torque due to friction as 2.0 N.m

(*Ans.* (i) N = 117 rpm, (ii) N = 57.2 rpm)

5.29 A four-arm lawn sprinkler, shown in Fig. 5.32 has provision for admitting water at the axis of rotation. If the discharge is 1.5 L/s, calculate the steady rotational speed by neglecting friction.

(*Ans.* N = 237.5 rpm)

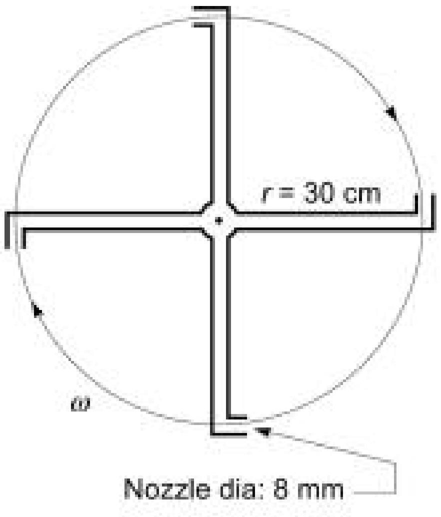


Fig. 5.32

5.30 For the water sprinkler, shown in Fig. 5.33 β = 158°, radius r = 0.50 m, jet diameter = 1.1 cm. If the speed of rotation is 240 rpm calculate the discharge supplied to the sprinkler. Assume zero frictional losses.

(*Ans.* Q = 2.758 L/s)

Elasticity = F_E
Surface tension = F_T
Inertia = F_I

and suffixes m and p stand for model and prototype respectively, strict dynamic similarity means

$$\boxed{\frac{F_{Gm}}{F_{Gp}} = \frac{F_{vm}}{F_{vp}} = \frac{F_{Em}}{F_{Ep}} = \frac{F_{Tm}}{F_{Tp}} = \frac{F_{Im}}{F_{Ip}} = \text{constant}}$$

From the above the following relationships can be derived.

$$\frac{(\text{Inertia force})_m}{(\text{Viscous force})_m} = \frac{(\text{Inertia force})_p}{(\text{Viscous force})_p} = \text{Constant 1}$$

$$\frac{(\text{Inertia force})_m}{(\text{Gravity force})_m} = \frac{(\text{Inertia force})_p}{(\text{Gravity force})_p} = \text{Constant 2}$$

and so on for all the forces.

6.6 IMPORTANT DIMENSIONLESS FLOW PARAMETERS

The following dimensionless parameters representing ratios of forces per unit volume are of great significance in the analysis of fluid flow:

(1) *Reynolds number* = $Re = \dfrac{\text{Inertial force}}{\text{Viscous force}}$

$$= \frac{\rho VL}{\mu} = \frac{VL}{\nu}$$

For dynamic similarity where viscous forces are predominant.

$$\left(\frac{VL}{\nu}\right)_m = Re_m = Re_p = \left(\frac{VL}{\nu}\right)_p$$

(2) *Froude number* = $Fr = \dfrac{(\text{Inertia force})^{1/2}}{(\text{Gravity force})^{1/2}}$

$$= \frac{V}{\sqrt{gL}}$$

For dynamic similarity where gravity forces are predominant.

$$\left(\frac{V}{\sqrt{gL}}\right)_m = (Fr)_m = (Fr)_p = \left(\frac{V}{\sqrt{gL}}\right)_p$$

(3) *Mach number* = $M = \dfrac{(\text{Inertial force})^{1/2}}{(\text{Compressibility force})^{1/2}}$

$$= \frac{V}{\sqrt{E/\rho}} = \frac{V}{C}$$

where C = velocity of sound in the medium.

Where compressibility effects predominate, for dynamic similarity, we have the relations:

$$\left(\frac{V}{C}\right)_m = M_m = M_p = \left(\frac{V}{C}\right)_p$$

(4) *Weber number* = $W = \dfrac{(\text{Inertial force})}{(\text{Surface tension})} = \dfrac{\rho V^2 L}{\sigma}$

(5) *Euler number* = $E = \dfrac{(\text{Inertial force})^{1/2}}{(\text{Pressure force})^{1/2}} = \dfrac{V}{\sqrt{\Delta p/\rho}}$

6.7 MODEL SCALES

Fluid flow models are usually designed to account for one most dominant force, and occasionally for two dominant forces. Thus, if the dominant force is the gravity force, then the Froude number must be the same in the model and prototype. Thus,

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}}$$

As g is the same for both model and prototype,

$$\boxed{\frac{V_m}{V_p} = V_r = \sqrt{L_r}}$$

From this other scales ratios, for such Froude law modelling are developed as shown in Table 6.2.

Table 6.2 *Similitude Scale Ratios*

		Scale Ratios for Laws of	
Parameter	*Dimension*	*Froude*	*Reynolds*
Geometric			
Length	L	L_r	L_r
Area	L^2	L_r^2	L^2
Volume	L^3	L_r^3	L_r^3
Kinematic			
Velocity	LT^{-1}	$(L_r)^{1/2}$	$(\mu_r/L_r\rho_r)$
Time	T	$(L_r)^{1/2}$	$(L_r^2\rho_r/\mu_r)$
Acceleration	LT^{-2}	1	$(\mu_r^2/\rho_r^2 L_r^3)$
Discharge	L^3T	$(L_r)^{5/3}$	$(L_r\mu_r/\rho_r)$
Dynamic			
Mass	M	$(L_r^3\rho_r)$	$(L_r^3\rho_r)$
Force	MLT^{-2}	$(L_r^3\rho_r)$	(μ_r^2/ρ_r)
Pressure	$ML^{-1}T^{-2}$	$(L_r\rho_r)$	$(\mu_r^2/L_r^2\rho_r)$
Momentum	MLT^{-1}	$(L_r^{7/2}\rho_r)$	$(L_r^2\mu_r)$
Energy of work	ML^2T^{-2}	$(L_r^4\rho_r)$	$(L_r\mu_r^2/\rho_r)$
Power	ML^2T^{-3}	$(L_r^{7/2}\rho_r)$	$(\mu_r^3/L_r\rho_r^2)$

Similarly, if viscous forces are predominant the Reynolds number (VL/ν) must be the same in the model and prototype. The scale ratios for Reynolds number modeling are also shown in Table 6.2.

Similarly appropriate scales can be developed for Mach law and Weber number law.

- Froude number similarity is used when there is
  (i) Wave action as in breakwaters and ships.

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By inspection $b = 0$, $c = 0$ and $a = -1$

$$\therefore \quad \pi_2 = P/H$$

π_3 term: $\pi_3 = LH^a g^b \rho^c$

$$[M^0L^0T^0] = [L]\,[L]^a\,[LT^{-2}]^b\,[ML^{-3}]^c$$

By inspection, here also

$$b = 0,\ c = 0 \text{ and } a = -1$$

$$\pi_3 = L/H$$

π_4 term: $\pi_4 = \mu H^a g^b \rho^c$

$$[M^0L^0T^0] = [ML^{-1}\,T^{-1}]\,[L]^a\,[LT^{-2}]^b\,[ML^{-3}]^c$$

By inspection, we have

$$1 + c = 0$$

$$-1 + a + b - 3c = 0$$

$$-1 - 2b = 0$$

$$\therefore \quad c = -1,\ b = -\frac{1}{2} \text{ and } a = 1 - b + 3c = -\frac{3}{2}$$

$$\pi_4 = \frac{\mu}{H^{3/2}\cdot g^{1/2}\rho}$$

π_5 term: $\pi_5 = \sigma H^a g^b \rho^c$

$$[M^0L^0T^0] = [MT^{-2}]\,[L]^a\,[LT^{-2}]^b\,[ML^{-3}]^c$$

$$1 + c = 0$$

$$a + b - 3c = 0$$

$$-2 - 2b = 0$$

$$c = -1,\ b = -1 \text{ and } a = 3c - b = -2$$

Thus $$\pi_5 = \frac{\sigma}{H^2\cdot g\cdot \rho}$$

$$\therefore \quad \frac{\boldsymbol{Q}}{\mathbf{g}\boldsymbol{H}^{5/2}} = \mathbf{fn}\left[\frac{\boldsymbol{P}}{\boldsymbol{H}}, \frac{\boldsymbol{L}}{\boldsymbol{H}}, \frac{\mu}{\boldsymbol{H}^{3/2}\mathbf{g}^{1/2}\rho}, \frac{\sigma}{\rho \mathbf{g}\boldsymbol{H}^2}\right]$$

6.8 A small sphere of density ρ_s and diameter D settles at a terminal velocity V in a liquid of density ρ_f and dynamic viscosity μ. Gravity g is known to be a parameter. Express the functional relationships between these variables in a dimensionless form.

Solution:

$$V = \text{fn}\,[\rho_s, D, \rho_f, \mu, g]$$

List the dimensions of each variable as follows:

V	ρ_s	D	ρ_f	μ	g
$[LT^{-1}]$	$[ML^{-3}]$	$[L]$	$[ML^{-3}]$	$[ML^{-1}\,T^{-1}]$	$[LT^{-2}]$

There are six variables, $n = 6$

and 3 basic dimensions, $m = 3$

Hence there are $(6 - 3) = 3$ dimensionless terms.

Select D, ρ_f and g as the repeating variables

I term: $\pi_1 = VD^a\rho_f^b g^c$

Following the usual procedure

$$[M^0L^0T^0] = [LT^{-1}]\,[L]^a\,[ML^{-3}]^b\,[LT^{-2}]^c$$

$$b = 0$$

$$1 + a - 3b + c = 0$$

$$-1 - 2c = 0$$

Hence $b = 0$, $c = -1/2$, $a = -1/2$

$$\therefore \quad \pi_1 = \frac{V}{\sqrt{gD}}$$

II term: $\pi_2 = \rho_s D^a \rho_f^b g^c$

By inspection, it is easy to see $\pi_2 = \dfrac{\rho_s}{\rho_f}$

III term: $\pi_3 = \mu D^a \rho_f^b g^c$

In this case, we have

$$[M^0L^0T^0] = [ML^{-1}\,T^{-1}]\,[L]^a\,[ML^{-3}]^b\,[LT^{-2}]^c$$

$$1 + b = c$$

$$-1 + a - 3b + c = 0$$

$$-1 - 2c = 0$$

$$b = -1,\ c = -\frac{1}{2} \text{ and } a = 3b - c + 1$$

$$= -\frac{3}{2}$$

$$\therefore \quad \pi_3 = \frac{\mu}{D^{3/2}\rho_f g^{1/2}}$$

Hence $$\frac{\boldsymbol{V}}{\sqrt{\mathbf{g}\boldsymbol{D}}} = \varphi\left[\frac{\rho_s}{\rho_f}, \frac{\mu}{\rho_f \boldsymbol{D}\sqrt{\mathbf{g}\boldsymbol{D}}}\right]$$

6.9 The pressure drop Δp generated by a pump of a given geometry is known to depend upon the impeller diameter D, the rotational speed N, the fluid discharge Q, the fluid density ρ and viscosity μ. Obtain the dimensionless form of the functional relationship.

Solution: $\Delta p = \text{fn}\,(D, N, Q, \rho, \mu)$

List the dimensions of each variable as:

Δp	D	N	Q	ρ	μ
$[ML^{-1}\,T^{-2}]$	$[L]$	$[T^{-1}]$	$[L^3\,T^{-1}]$	$[ML^{-3}]$	$[ML^{-1}T^{-1}]$

There are a total of six variables: $n = 6$

Number of primary dimensions $m = 3$

Hence there are $(6 - 3) = 3$ dimensionless Pi terms.

Select D, N and ρ as repeating variables.

I term: $\pi_1 = \Delta p D^a N^b \rho^c$

The dimensional equation will be

$$[M^0L^0T^0] = [ML^{-1}\,T^{-2}]\,[L]^a\,[T^{-1}]^b\,[ML^{-3}]^c$$

from which we get

$$1 + c = 0$$

$\therefore \qquad \mathbf{F_p = 250\ N}$ (same as in the model)

Power to overcome drag in the prototype:

$$\mathbf{P}_p = F_p \cdot V_p = 250 \times \left(\frac{60 \times 10^3}{3600}\right) = 4167 \text{ W}$$

$$= \mathbf{4.167\ kW}$$

6.19 The pressure drop in a flow meter in which an oil flows at an upstream velocity of 0.9 m/s is to be estimated by model studies. A 1 : 6 scale model using water is used. If the pressure drop in the model is 450 Pa, what will be the prototype pressure drop? If the prototype discharge is 200 L/s what is the model discharge? The following data are relevant:

Item	*Prototype*	*Model*
Density	900 kg/m^3	998 kg/m^3
Viscosity	0.104 Pa.s	1×10^{-3} Pa.s

Solution: The Reynolds model law is applicable.

$$(Re)_m = \frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$$L_m/L_p = L_r = 1/6$$

$$V_r = \frac{\mu_r}{L_r \rho_r}$$

Pressure ratio $\qquad p_r = \rho_r V_r^2 = \dfrac{\mu_r^2}{L_r^2 \rho_r}$

and discharge ratio $\qquad Q_r = V_r \cdot L_r^2 = \dfrac{\mu_r L_r}{\rho_r}$

In the present problem:

$$\mu_r = \frac{1 \times 10^{-3}}{0.104} = 9.615 \times 10^{-3}$$

$$\rho_r = \frac{998}{900} = 1.109$$

$$L_r = 1/6$$

$$\therefore \qquad p_r = \frac{(9.615 \times 10^{-3})^2}{\left(\frac{1}{6}\right)^2 \times 1.109} = 3 \times 10^{-3}$$

$$\therefore \qquad \rho_p = \frac{p_m}{p_r} = \frac{450}{3 \times 10^{-3}} = 150{,}000 \text{ Pa}$$

$$= \mathbf{150\ kPa}$$

$$Q_r = \frac{(9.615 \times 10^{-3}) \times 1/6}{1.109} = 1.445 \times 10^{-3}$$

$$Q_m = Q_p \times 1.445 \times 10^{-3}$$

$$= 200 \times 1.445 \times 10^{-3} = \mathbf{0.289\ L/s}$$

6.20 An underwater device is 1.5 m long, and is to move at 3.5 m/s. A geometrically similar model 30 cm long is tested in a variable pressure wind tunnel at a speed of 35 m/s. Calculate the pressure of air in the model. If the model exhibits a drag force 40 N, calculate the prototype drag force.

[Assume $\rho_{\text{water}} = 998$ kg/m^3, ρ_{air} at standard atmospheric pressure = 1.17 kg/m^3, $\mu_{\text{air}} = 1.90 \times 10^{-5}$ Pa.s, at local atmospheric pressure and $\mu_{\text{water}} = 1.0 \times 10^{-3}$ Pa.s].

Solution: Reynolds model law is applicable. Hence

$$(Re)_m = \frac{\rho_m V_m L_m}{\mu_m} = (Re)_p = \frac{\rho_p V_p L_p}{\mu_p}$$

$$\therefore \text{ If} \qquad \frac{L_m}{L_p} = L_r, \ \frac{\rho_m}{\rho_p} = \rho_r \text{ and } \frac{\mu_m}{\mu_p} = \mu_r$$

$$\frac{V_m}{V_p} = V_r = \frac{\mu_r}{\rho_r L_r}$$

In the present case

$$L_r = \frac{0.30}{1.50} = \frac{1}{5}$$

$$V_r = \frac{35}{3.5} = 10$$

$$\mu_r = \frac{1.90 \times 10^5}{1 \times 10^{-3}} = 1.9 \times 10^{-2}$$

$$\text{Hence} \qquad \rho_r = \frac{\mu_r}{V_r L_r} = \frac{1.9 \times 10^{-2}}{10 \times 1/5} = 0.0095$$

$$\text{Hence} \qquad \rho_{\text{air}} = \rho_m = 998 \times 0.0095 = \mathbf{9.481}$$

This is about 8 times larger than the density at atmospheric pressure. Since at constant temperature, by the equation of state p/ρ = constant.

$$\text{Hence} \quad \frac{(\text{pressure})_{\text{model}}}{(\text{Atmospheric pressure})} = \frac{\rho_{\text{model}}}{\rho_{\text{atmospheric}}}$$

$$p_{\text{model}} = \frac{9.481}{1.17} \times p_a = 8.103\ p_a$$

= 8.103 times local atmospheric pressure

$$\text{Force ratio} = F_r = \rho_r V_r^2 L_r^2$$

$$= \frac{\mu_r^2}{\rho_r} = \frac{(1.9 \times 10^{-2})^2}{0.0095}$$

$$= 0.038$$

$$F_p = \frac{F_m}{0.038} = \frac{40}{0.038} = 1053 \text{ N}$$

$$= \mathbf{1.053\ kN}$$

6.21 A pipe of diameter 1.5 m is required to transport an oil of relative density 0.9 and kinematic viscosity = 3×10^{-2} stoke at a rate of 3.0 m^3/s. If a 15 cm diameter pipe with water at 20°C (ν = 0.01 stoke) is used to model the above flow, find the velocity and discharge in the model.

6.16 In the Froude law of similitude the acceleration ratio a_r =

(a) L_r^2 (b) 1.0

(c) $1/L_r$ (d) $L_r^{-3/2}$

6.17 If Reynolds law of similitude exists between a model and a prototype, then the force ratio F_r =

(a) $L_r^3 \rho_r$ (b) $\mu_r^2 \rho_r^{-1}$

(c) $L_r \mu_r \rho_r^{-1}$ (d) $L_r^{-2} \mu_r \rho_r^{-1}$

6.18 In Reynolds law of similitude the discharge ratio Q_r =

(a) $L_r \mu_r / \rho_r$ (b) $L_r^3 \rho_r$

(c) $L_r^{5/2}$ (d) $L_r \rho_r / \mu_r$

6.19 A hydraulic model of a spillway is constructed with a scale 1 : 16. If the prototype discharge is 2048 m^3/s, then the corresponding discharge in m^3/s for which the model should be tested is

(a) 1 (b) 2

(c) 4 (d) 8

6.20 In the model of a highway bridge constructed to a scale of 1:25, the force of water on the pier was measured as 5 N. The force on the prototype pier will be

(a) 75015 N (b) 76225 N

(c) 78125 N (d) 79165 N

6.21 The fall velocity of a sand grain in water is to be modeled by using particles of the same relative density as sand and a liquid whose kinematic viscosity is 100 times larger than that of water. The diameters of the particles in the model that will have the same fall velocity as the prototype will be

(a) 10 times smaller

(b) 10 times larger

(b) 100 times smaller

(d) 100 times larger

6.22 In a model built on Froude law of similarity a phenomenon lasts for 20 min. If the model scale is 1/25, the duration of the phenomenon in the prototype, in minutes, is

(a) 50 (b) 100

(c) 2500 (d) 4

6.23 If the same fluid is used both in the model and prototype, and if it is desired to have equal Reynolds number and Froude number in the model and prototype, the scale of the model is

(a) V_r (b) $V_r^{1/2}$

(c) 1.0 (d) $V_r^{-1/2}$

where V_r = velocity ratio.

6.24 A harbor model has a horizontal scale of 1/150 and a vertical scale of 1/75. The interval between successive daily high tides in the model will be nearly

(a) 90 min (b) 40 min

(c) 15 min (d) 5 hours

6.25 In the distorted model of a river, the horizontal and vertical scales are L_r and h_r respectively. The discharge ratio will be

(a) $L_r^{1/2} h_r^2$ (b) $L_r h_r^{3/2}$

(c) $L_r^2 h_r^{1/2}$ (d) $L_r^3 h_r^{1/2}$

6.26 A river model is constructed to a horizontal scale of 1 : 1000 and a vertical scale of 1 : 100. If the model discharge were 0.1 m^3/s, then the discharge in the prototype would be

(a) 10^3 m^3/s (b) 10^4 m^3/s

(c) 10^5 m^3/s (d) 10^2 m^3/s

6.27 A model of a weir made to a horizontal scale of 1/40 and vertical scale of 1/9 passes a discharge of 1 Litre/s. The corresponding discharge in the prototype would be

(a) 10.8 Lps (b) 108 Lps

(c) 1080 Lps (d) 10800 Lps

PROBLEMS

Dimensional Analysis

6.1 The Chezy formula for velocity V in an open channel is given by

$$V = C\sqrt{RS_0}$$

where R = hydraulic radius, S_0 = longitudinal slope of the channel and C = Chezy coefficient. If for a channel the value of C in FPS units is 108, what is its value in SI units? (*Ans.* C = 59.6)

6.2 The following is a formula for calculating the maximum flood discharge Q from a catchment of area A:

$$Q = \frac{7000A}{\sqrt{A+4}}$$

viscosity of the fluid and the velocity of the flow. Obtain an expression for the pressure drop in dimensionless form. Estimate the pressure drop in a 20 m long air duct if a model of the duct operating with water produces a pressure drop of 10 kN/m^2 over 10 m length. The scale ratio is 1 : 50.

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$
$$\rho_{\text{air}} = 1.2 \text{ kg/m}^3$$
$$\mu_{\text{water}} = 0.001 \text{ N.s/m}^2$$
$$\mu_{\text{air}} = 0.0002 \text{ N.s/m}^2$$

$$\left(Ans.\ \frac{\Delta p}{\rho V^2} = \text{fn}\left(\frac{\mu}{\rho V D}, \frac{L}{D} \right); \Delta p_{\text{p}} = 133.3 \text{ N / m}^2 \right)$$

6.39 A 1/20 model of a ship having a submerged surface area of 5 m^2 and length of 8 m has a total drag of 20 N when towed through sea water at a velocity' of 1.5 m/s. Calculate the total drag on the prototype when moving at the corresponding speed. The skin friction can be estimated by $F_s = C_f A\rho V^2/2$ where the coefficient $C_f = \dfrac{0.0735}{(Re)^{1/5}}$. [Take $\mu = 1.07 \times 10^{-3}$ Pa.s and $\rho = 1025$ kg/m^3].

(*Ans.* Total drag = 82.17 kN)

6.40 An ocean-going vessel has a length of 80 m and a submerged surface area of 1200 m^2. Its cruising speed is 30 km/h. A 1/20 model of this vessel is tested in a towing tank using fresh water and Froude law of similarity. A total drag force of 18.5 N was measured in the model at a velocity corresponding to the cruising speed. Calculate the total drag on the prototype at the cruising speed. The skin friction is estimated by $F_s = C_f A\rho V^2/2$ where

$$C_f = \frac{0.074}{(Re)^{1/5}}. \text{ Use}$$

for fresh water:

$$\rho = 998 \text{ kg/m}^3$$
$$\mu = 0.001 \text{ Pa.s}$$

and for sea water $\rho = 1025$ kg/m^3

$$\mu = 1.07 \times 10^{-3} \text{ Pa.s}$$

(*Ans.* Total drag = 73.36 kN)

6.41 A model propeller with diameter $d = 0.5$ m is tested in water tunnel at a speed of $n = 360$ rpm when the flow velocity V = 2.5 m/s. The model produces a thrust of 250 N at a torque of 22 Nm. The prototype has a diameter of 4 m and operates at 100 rpm in water. If the significant non-dimensional group is (V/*nd*) only, determine (a) the flow velocity, (b) thrust produced and (c) the applied torque for the prototype.

(*Ans:* (a) $V_p = 0.556$ m/s,
(b) $F_p = 460.8$ kN, (c) $T_p = 324.4$ kN.m)

Distorted Models

6.42 A distorted model of a river has a horizontal scale of 1/750. The slope in the model is 0.0025. If the prototype slope is 0.0001, estimate the prototype discharge corresponding to a model discharge of 0.01 m^3/s.

(*Ans:* $Q_p = 1232$ m^3/s)

6.43 A model of a river has a horizontal scale of 1/500 and vertical scale of 1/50. Fill up the following table pertaining to the above model—prototype pair.

Sl. No.	Parameter	Prototype Value	Model Value
1	Cross-section area	100 m^2	?
2	Longitudinal slope	0.0001	?
3	Manning's roughness coefficient	?	0.05
4	Wave period	1 hour	?
5	Volume of water	?	0.5 m^3
6	Width	?	0. 2 m
7	Velocity	1.5 m/s	?
8	Depth	2.5 m	?

(*Ans:* (1) 0.04 m^2, (2) 0.001, (3) 0.030,
(4) 50.9 s, (5) 6.25 Mm^3, (6) 100 m,
(7) 0.353 m/s, (8) 0.05 m)

6.44 A river carries a discharge of 16,000 m^3/s of water at a depth of 8.0 and slope of 0.0025 when its width is 400 m. Above 15 km reach of this river is to be reproduced in the laboratory where 30 m long space is available. Determine the appropriate horizontal and vertical scales for this model. Also, determine the roughness scale and the model slope.

(*Ans:* $L_r = 1/500$, $h_r = 1/30$,
$n_r = 2.316$ and $S_m = 0.04167$)

Through an analytical procedure Stokes has shown that the net longitudinal force F exerted upon the sphere is

$$\boxed{F = 3\pi D\mu V_0} \tag{7.24}$$

This equation, known as *Stokes Equation*, finds application in the determination of the fall velocity of small particles [For details see Chapter 9].

7.3 VISCOMETERS

A viscometer is a device for determining the viscosity of a liquid. Many of these instruments use laminar flow situations to estimate the viscosity of the liquid. The capillary tube viscometer utilizes the Hagen–Poiseuille equation to estimate the coefficient of viscosity μ of the liquid.

WORKED EXAMPLES

7.1 For a steady laminar flow in a horizontal circular pipe derive expressions for (i) the shear stress at any radial distance from the axis, (ii) the pressure drop along the pipe in terms of the boundary shear stress.

Solution:

(i) Consider a cylindrical element of fluid as shown in Fig. 7.4. Since the flow is steady and uniform, there is no acceleration and, as such, the sum of horizontal forces on the element must be zero. Hence

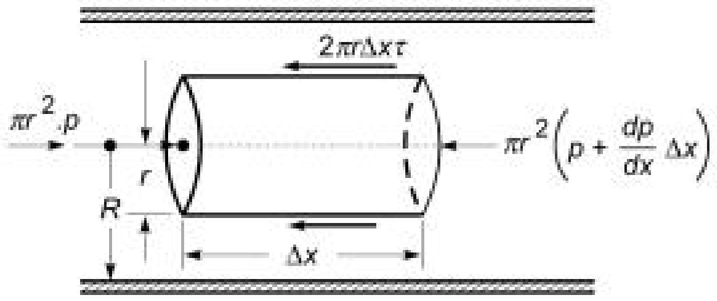

Fig. 7.4

$$\pi r^2 p - \pi r^2\left(p + \frac{\mathrm{d}p}{\mathrm{d}x}\Delta x\right) - 2\pi r\Delta x\tau = 0$$

Note that the shear stress τ at a radial distance r acts over the surface of the cylindrical element, the surface area of which is $(2\pi r \cdot \Delta x)$.

Simplifying $\tau = -\frac{\mathrm{d}p}{\mathrm{d}x}\frac{r}{2}$

At the boundary $r = R$, $\tau = \tau_0 = -\frac{\mathrm{d}p}{\mathrm{d}x}\frac{R}{2}$

At the centre line $r = 0$ $\tau_c = 0$

The variation of the shear stress τ with r is shown in Fig. 7.1.

(ii) $\frac{\mathrm{d}p}{\mathrm{d}x}$ is constant. Hence $\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{\Delta p}{\Delta x} = \frac{\Delta p}{\Delta L}$ over a length L.

The pressure drop $-\Delta P = \frac{2\tau_0 L}{R} = \frac{\boldsymbol{4\tau_0 L}}{\boldsymbol{D}}$

7.2 Derive the Darcy–Weisbach equation for head loss due to a flow in a pipe in terms of the friction factor f. For laminar flow develop relations for the estimation of f and boundary shear stress in terms of flow parameters.

Solution: Considering two sections 1 and 2, distance L apart, as in Fig. 7.5,

$$\frac{p_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + Z_2 + \frac{V_2^2}{2g} + h_f$$

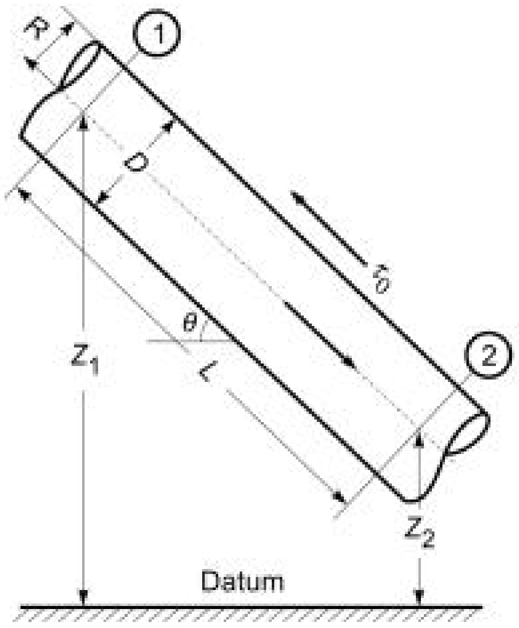

Fig. 7.5

As the flow is uniform $V_1 = V_2 = V$

$$\left(\frac{p_1}{\gamma} + Z_1\right) - \left(\frac{p_2}{\gamma} + Z_2\right) = h_f = -\Delta\left(\frac{p}{\gamma} + Z\right)$$

Considering the force balance between sections 1 and 2

$$p_1\frac{\pi D^2}{4} - p_2\frac{\pi D^2}{4} + \gamma(\pi R^2)L\sin\theta - \tau_0 2\pi RL = 0$$

Since $h_b > h_a$, **the flow is from B towards A, i.e. upwards.**

Head loss $= h_f = h_b - h_a = 56.747 - 42.699$

$= 14.048$ m

Assuming the flow as laminar

$$h_f = \frac{32\mu VL}{\gamma D^2}$$

$$14.048 = \frac{32 \times 0.15 \times V \times 30}{8811 \times (0.03)^2} = 18.159\ V$$

$$V = 0.7736 \text{ m/s}$$

Reynolds number $Re = \dfrac{VD\rho}{\mu} = \dfrac{0.7736 \times 0.03 \times 898.2}{0.15}$

$= 139$

As $Re < 2000$, the flow is laminar and our initial assumption is correct.

Discharge $Q = AV$

$$= \frac{\pi}{4} \times (0.03)^2 \times 0.7736$$

$$= 0.5468 \times 10^{-3} \text{ m}^3\text{/s}$$

$= \mathbf{0.5468\ L/s}$

7.12 Oil of relative density 0.9 and dynamic viscosity 2.5 poises is pumped through a 100 mm diameter pipe 500 m long at a rate of 2 L/s (a) Find the Reynolds number of the flow. (b) Calculate the pressure required at the pump if the outlet end, which is free, is at 20 m above the pump level. (c) What should be the power input if the overall efficiency of the pump set is 65%?

Solution:

$\mu = 2.5$ poise $= 0.25$ Pa.s

$\rho = 0.9 \times 998 = 898.2$ kg/m^3

$D = 100$ mm $= 0.1$ m

$Q = 2$L/s $= 0.002$ m^3/s

$$V = \frac{0.002}{\frac{\pi}{4} \times (0.1)^2} = 0.2546 \text{ m/s}$$

(a) Reynolds number

$$Re = \frac{\rho VD}{\mu}$$

$$= \frac{898.2 \times 0.2546 \times 0.1}{0.25}$$

$= \mathbf{91.49}$

As $Re < 2000$, the flow is laminar.

(b) Head loss due to friction:

$$h_f = \frac{32\mu VL}{\gamma D^2}$$

$$= \frac{32 \times 0.25 \times 0.2546 \times 500}{(898.2 \times 9.81)(0.1)^2}$$

$= 11.55$ m

If A is the pump end and B is the outlet, then

$$\frac{p_A}{\gamma} + 0 + \frac{V^2}{2g} = \frac{p_B}{\gamma} + 20 + \frac{V^2}{2g} + h_f$$

But $p_B = 0$ = atmospheric, as the outlet is free.

Also $h_f = 11.55$ m

$$\frac{p_A}{\gamma} = 20.0 + 11.55 = 31.55 \text{ m}$$

$p_A = 31.55 \times (898.2 \times 9.81)$

$= 278000$ Pa

$= \mathbf{278\ kPa}$

(c) Power required for pumping the fluid

$$\mathbf{P} = \gamma QH$$

where H = overall head

= static head + friction head

$= 31.55$ m

$\mathbf{P} = (898.2 \times 9.81) \times 0.002 \times 31.55$

$= 556$ W

As the overall efficiency of the pump set is 65%

Power input required

$$= \mathbf{P}_i = \frac{556}{0.65} = \mathbf{855.4\ W}$$

7.13 An 8 cm diameter pipe, 300 m long conveys oil of kinematic viscosity 1.5 stokes and mass density 900 kg/m^3. Assuming laminar flow, (a) find the rate of flow if it takes 5.0 kW of power input to a pump set of overall efficiency 60% to drive the flow, (b) verify whether the assumption of laminar flow is correct.

Solution:

(a) Power delivered to the fluid

$= 60\% \times 5000 = 3000$ W

Also, power $\mathbf{P} = \gamma Q h_f$

$$= \frac{\gamma Q 128 \mu QL}{\gamma \pi D^4}$$

$$= \frac{128 \mu Q^2 L}{\pi D^4}$$

Kinematic viscosity

$\nu = 1.5$ stokes $= 1.5 \times 10^{-4}$ m^2/s

Dynamic viscosity

$\mu = \rho\nu = 900 \times 1.5 \times 10^{-4}$

$= 0.135$ Pa.s

$$\mathbf{P} = \frac{128 \times 0.135 \times Q^2 \times 300}{\pi \times (0.08)^4} = 3000$$

$Q^2 = 7.4467 \times 10^{-5}$

$Q = 8.63 \times 10^{-3}$ m^3/s $= \mathbf{8.63\ L/s}$

$$V = \frac{\gamma}{3\mu} d^2 \sin \theta$$

Discharge per unit width

$$= Vd = \frac{\gamma}{3\mu} d^3 \sin \theta \qquad (1)$$

Shear stress on the bed

$$\tau_0 = \gamma d \sin \theta$$

Using result in Eq. 1,

$$\tau_0 = \frac{3\mu V}{d}$$

7.21 Oil of relative density 0.92 and dynamic viscosity 1.05 poise flows between two fixed parallel plates kept 1.2 cm apart. If the mean velocity is 1.40 m/s calculate (a) the maximum velocity (b) the boundary shear stress (c) velocity and shear stress at a distance of 0.2 cm from of the plates and (d) head loss in a distance of 25 m.

Solution:

(a) For two-dimensional laminar flow between parallel plates

$$u_m = \text{maximum velocity} = \frac{3}{2} V$$

$$= \frac{3}{2} \times 1.40 = \mathbf{2.10\ m/s}$$

(b) Since $V = \left(-\frac{dp}{dx}\right) \frac{B^2}{12\mu}$

$$\left(-\frac{dp}{dx}\right) = \frac{12\mu V}{B^2} = \frac{12 \times 0.105 \times 1.40}{(0.012)^2}$$

$$= 12250$$

Boundary shear stress

$$\tau_0 = \left(-\frac{dp}{dx}\right) \frac{B}{2}$$

$$= 12250 \times \frac{0.012}{2}$$

$$= \mathbf{73.5\ Pa}$$

(c) Shear stress τ at any y from the boundary

$$\tau = \left(-\frac{dp}{dx}\right)\left(\frac{B}{2} - y\right)$$

At $y = 0.002$ m:

$$\tau = (12250)\left(\frac{0.012}{2} - 0.002\right) = \mathbf{49\ Pa}$$

Velocity $v = \frac{1}{2\mu}\left(-\frac{dp}{dx}\right)(By - y^2)$

$$= \frac{1}{2 \times 0.105} \times 12250$$

$$[0.012 \times 0.002 - (0.002)^2]$$

$$v = \mathbf{1.167\ m/s}$$

(d) Head loss $h_f = \frac{12\mu VL}{\gamma B^2}$

$$= \frac{12 \times 0.105 \times 1.4 \times 25}{(0.92 \times 998 \times 9.81)(0.012)^2}$$

$$= \mathbf{34.0\ m}$$

OBJECTIVE QUESTIONS

7.1 The equations of motion for laminar flow of a real fluid are known as
- (a) Euler's equations
- (b) Bernoulli equation
- (c) Navier–Stokes equation
- (d) Hagen–Poiseuille equation

7.2 Identify the *incorrect* statement
- (a) Laminar flow is rotational
- (b) Loss of head in laminar flow in a pipe is proportional to square of the velocity
- (c) In laminar flow the friction factor *f* is directly proportional to the Reynolds number.
- (d) In laminar flow in a conduit the shear stress varies linearly across the pipe.

7.3 In a two-dimensional, steady, horizontal, uniform laminar flow the shear gradient in the normal direction is equal to
- (a) the velocity gradient in the normal direction.
- (b) the velocity gradient in the longitudinal direction
- (c) the pressure gradient in the normal direction.
- (d) the pressure gradient in the direction of flow.

7.4 An oil of kinematic viscosity 0.25 stokes flows through a pipe of diameter 10 cm.
The flow is critical at a velocity of
- (a) 7.2 m/s
- (b) 5.0 m/s
- (c) 0.5 m/s
- (d) 0.72 m/s

difference of 6 kPa. Find the time taken to collect 50 cm^3 of liquid at the capillary outlet.

(*Ans.* T = 8 min 50.5 s)

7.13 A liquid of dynamic viscosity 0.07 Pa.s and relative density 0.86 flows through an inclined pipe of 2 cm diameter. A discharge of 13 L/min is to be sent through the pipe in such a manner that the pressure along the length is constant. Find the required inclination of the pipe.

$\left(\textbf{Hint: } \text{For constant pressure the slope} = \sin\theta = \frac{h_f}{L} \right)$

(*Ans.* $\theta = 27° \, 18' \, 17''$)

7.14 Determine a suitable diameter for a horizontal pipeline which is to convey 50 L/s of an oil through a distance of 250 m, if the pressure loss is not to exceed 150 kPa. At the operating temperature the relative density of the oil is 0.9 and the dynamic viscosity 1.3 Pa.s.

(*Ans.* D = 25.7 cm; in practice the next larger standard size is used.)

7.15 Calculate the least diameter of a pipe to carry 10 L/s of an oil of density 900 kg/m^3 and dynamic viscosity 1.5 poise with a permissible energy gradient in laminar flow regime of 0.03.

(*Ans.* D = 12.3 cm)

7.16 An oil of relative density 0.92 and dynamic viscosity 0.9 poise flows though a 10 cm diameter pipe 30 m long. Determine the largest flow that can be passed through this pipe while maintaining laminar regime. What is the head loss between the two ends of the pipe under this flow?

(*Ans.* Q = 15.4 L/s and h_L = 1.88 m)

7.17 A liquid of relative density 0.85 flows in a pipe 8 cm is diameter with a velocity of 0.8 m/s in laminar regime. If two pressure gauges located at the ends of a 12 m long horizontal stretch of pipe record a pressure difference of 60 kPa, estimate the (i) viscosity of the fluid and (ii) Reynolds number of the flow.

(*Ans.* μ = 1.25 Pa.s; Re = 43.4)

7.18 For a steady fully developed laminar flow of an oil of density ρ_f through two pipes in series as shown in Fig. 7.10 find the ratio h_1/h_2 of the manometer fluid deflections Consider only friction losses in the pipes.

$$\left(\textit{Ans.} \ \frac{h_1}{h_2} = \left(\frac{D_2}{D_1}\right)^4 \left(\frac{L_1}{L_2}\right) \right)$$

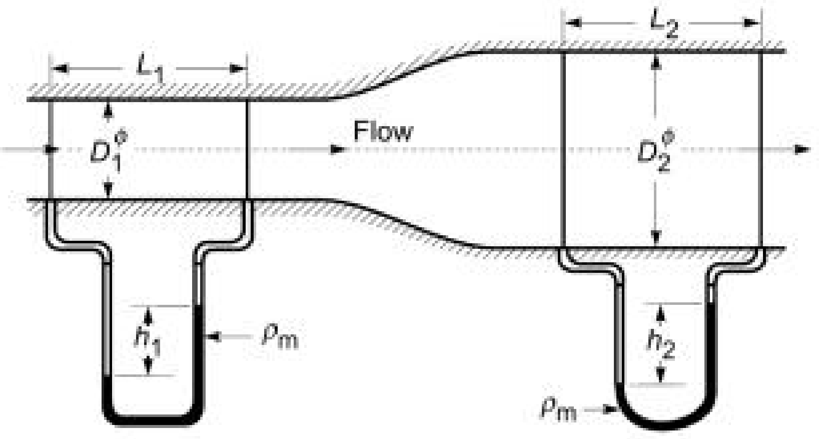

Fig. 7.10

7.19 Calculate the force F required on the piston to discharge 500 mm^3/s of water through a syringe (Fig. 7.11) taking into account the frictional loss in the syringe needle only. Assume fully developed laminar flow in the needle. (Take $\mu_{\text{water}} = 10^{-3}$ Ns/m^2)

(*Ans.* F = 1.024 N)

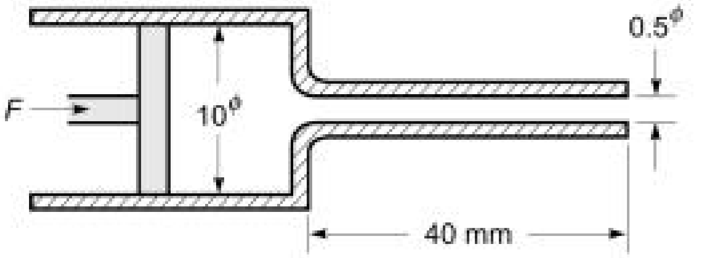

Fig. 7.11

7.20 A piston of area A is forced with a force F into a cylinder full of oil of density ρ and viscosity μ. If there are n tubular openings of diameter d in the piston, show that the rate of descent of the piston is

$$v = \frac{n\pi F d^4}{128 \mu A^2 L}$$

where L = length of the piston. Assume that no oil leaks between the piston and the cylinder. Further, laminar flow can be assumed through the tubular openings.

7.21 A pump delivers a lubricating oil of relative density 0.90 and dynamic viscosity 0.85 poise through 25 m of 30 mm pipe to a tank whose oil surface is 10 m higher than the oil surface of the supply tank. The pump efficiency is 70%. (a) Estimate the power input required to pump oil at a rate of 250 L/min (b) What power input is required to pump the liquid at a Reynolds number of 2000 in this system?

(*Ans.* (a) **P** = 3.18 kW; (b) **P** = 3.6 kW)

Integrating with the boundary condition ($x = 0$, $\delta = 0$)

$$\delta = \sqrt{2(\beta/\alpha)}\,\frac{x}{\sqrt{Re_x}}$$

or

$$\frac{\delta}{x} = \frac{\sqrt{2(\beta/\alpha)}}{\sqrt{Re_x}} \qquad (8.8)$$

Substituting the value of δ in Eq. (8.7) for τ_0, and simplifying,

$$C_f = \frac{\tau_0}{\rho U^2/2} = \sqrt{2\alpha\beta}\,\frac{1}{\sqrt{Re_x}} \qquad (8.9)$$

Knowing that the drag force on side of the plate, for a plate of unit width

$$F_D = \int_0^L \tau_0\,\mathrm{d}x$$

$$F_D/(0.5\ \rho U^2 L) = C_{Df} = \sqrt{2\alpha\beta}\,/\sqrt{Re_L} \qquad (8.10)$$

Example 8.5 illustrates in detail the use of this method for a specific $f(\eta)$.

In Table 8.1, some of the commonly adopted forms of $u/U = f(\eta)$ and the corresponding boundary layer parameters δ, C_f, C_{Df}, obtained by using the Karman momentum integral equations, are given. After studying the Example 8.5, the reader is advised to derive all the elements listed in Table 8.1 as a good exercise.

Table 8.1 *Laminar Boundary Layer Solutions for Various Velocity Profiles*

($y/\delta = \eta$)

$u/U = f(\eta)$	$(\delta/x)\sqrt{Re_x}$	$(\delta^*/x)\sqrt{Re_x}$	$C_f\sqrt{Re_x}$	$C_{Df}\sqrt{Re_L}$
Exact (Blasius)	5.00	1.729	0.664	1.328
$2\eta - 2\eta^3 + \eta^4$	5.84	1.752	0.686	1.372
$\frac{3}{2}\eta - \frac{1}{2}\eta^3$	4.64	1.740	0.626	1.292
$2\eta - \eta^2$	5.48	1.826	0.730	1.460
$\sin\left(\frac{\pi}{2}\eta\right)$	4.80	1.741	0.654	1.308
η	3.46	1.730	0.577	1.154

8.5 BOUNDARY CONDITIONS FOR A PROPER f(η)

A proper function $u/U = f(\eta)$ must satisfy the following essential and desirable boundary conditions:

		Essential	Desirable
At the wall,	$y = 0$;	$u = 0$	$\partial^2 u/\partial y^2 = 0$
At	$y = \delta$;	$u = U$	$\partial u/\partial y = \partial^2 u/\partial y^2 = 0$

8.6 TRANSITION FROM LAMINAR BOUNDARY LAYER

As the flow passes down the plate, i.e. as Re_x increases the boundary layer thickness increases and soon it becomes unstable. Turbulence persists and grows in the boundary layer at higher values of Re_x. It is generally believed that the transition from laminar to turbulent boundary layer takes place between $Re_x = 1.3 \times 10^5$ and 4×10^6, with the mean value of $Re_x = (Re_x)_{crit} = 5 \times 10^5$ taken as the commonly accepted *critical Reynolds number.*

In a flow past a long plate, the initial part in the boundary layer up to x_{crit} will be laminar and then onwards the flow of the boundary layer will be turbulent.

8.7 TURBULENT BOUNDARY LAYER

The turbulent boundary layer will have much more steeper velocity gradients at the boundary than the laminar boundary layer. The velocity distribution is logarithmic and could be conveniently expressed in the form of a power law, $u/U = (y/\delta)^{1/n}$ over a range of Reynolds number. The power n can be 5 to 10, depending on the Reynolds number range.

Next to the boundary, in a turbulent boundary layer over a smooth bed, there exists a thin layer called as *laminar sublayer*.

For Re_x between 5×10^6 and 2×10^7 the velocity distribution can be expressed by the 1/7 power law, $u/U = (y/\delta)^{1/7}$. The turbulent boundary layer characteristics found by experiments and analytical calculations, to be valid for $5 \times 10^5 < Re_x < 2 \times 10^7$, are

$$\boxed{\delta/x = 0.377/Re_x^{1/5}} \qquad (8.11)$$

$$\boxed{C_f = 0.059/Re_x^{1/5}} \qquad (8.12)$$

$$\boxed{C_{Df} = 0.074/Re_L^{1/5}} \qquad (8.13)$$

The above formulae assume the boundary layer to be turbulent from $x = 0$. To account for initial laminar boundary layer, C_{Df} can be calculated by

$$\boxed{C_{Df} = (0.074/Re_L^{1/5}) - (1700/Re_L)} \qquad (8.14)$$

For higher Reynolds numbers ($10^7 < Re_x < 10^9$), the logarithmic form of velocity distribution in the turbulent boundary layer is more appropriate. The boundary layer shear coefficients are expressed by the following formulae given by *Schlichting*:

$$\boxed{C_f = 0.370/(\log Re_x)^{2.58}} \qquad (8.15)$$

$$\boxed{C_{Df} = 0.455/(\log Re_L)^{2.58}} \qquad (8.16)$$

At $y = \delta$, $u = U \sin(\pi/2) = U$

$$\frac{\partial u}{\partial y} = \left(\frac{U\pi}{2\delta}\right) \cos(\pi/2) = 0$$

$$\frac{\partial^2 u}{\partial y^2} = -\frac{U\pi^2}{4\delta^2} \sin(\pi/2) \neq 0$$

Hence all essential and one desirable boundary condition are satisfied.

(b) $u = U[1 + (y/\delta) - 2(y/\delta)^2]$

$$\frac{\partial u}{\partial y} = U\left[\frac{1}{\delta} - \frac{4y}{\delta^2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = U[-4/\delta^2]$$

At $y = 0$, $u = U \neq 0$

$$\frac{\partial^2 u}{\partial y^2} = -4U/\delta^2 \neq 0$$

At $y = \delta$, $u = 0 \neq U$

$$\frac{\partial u}{\partial y} = -3U/\delta \neq 0$$

$$\frac{\partial^2 u}{\partial y^2} = -4U/\delta^2 \neq 0$$

None of the essential and desirable boundary conditions is satisfied. **Hence this is not a proper velocity distribution in a laminar boundary layer.**

8.4 If the velocity distribution in a laminar boundary layer on a flat plate is

$$u/U = a + b\eta + c\eta^2 + d\eta^3$$

where $\eta = y/\delta$, determine the coefficients a, b, c and d.

Solution:

Let $u/U = f(\eta)$

The requisite boundary conditions are:

At $y = 0$, i.e. $\eta = 0$ $u = 0$ i.e. $f(\eta) = 0$

$\dfrac{\partial^2 u}{\partial y^2} = 0$ i.e. $f''(\eta) = 0$

At $y = \delta$, i.e. $\eta = 1$, $u = U$ i.e. $f(\eta) = 1$

$\dfrac{\partial u}{\partial y} = 0$ i.e. $f'(\eta) = 0$

$\dfrac{\partial^2 u}{\partial y^2} = 0$ i.e. $f''(\eta) = 0$

Here

$$f(\eta) = a + b\eta + c\eta^2 + d\eta^3$$
$$f'(\eta) = b + 2c\eta + 3d\eta^2$$
$$f''(\eta) = 2c + 6d\eta$$

Therefore

at $\eta = 0$, $f(\eta) = 0$ i.e. $\boldsymbol{a = 0}$

$f''(\eta) = 0$ i.e. $\boldsymbol{c = 0}$

at $\eta = 1$, $f(\eta) = 1$ i.e. $a + b + c + d = 1$

i.e. $b + d = 1$

$f'(\eta) = 0$ i.e. $b + 2c + 3d = 0$

i.e. $b + 3d = 0$

Solving $\boldsymbol{b = \dfrac{3}{2}}$ and $\boldsymbol{d = -\dfrac{1}{2}}$

Thus $\boldsymbol{\dfrac{u}{U} = \dfrac{3}{2}\eta - \dfrac{1}{2}\eta^3}$ where $\eta = y/\delta$

8.5 Given the velocity distribution in a laminar boundary layer on a flat plate as

$$\frac{u}{U} = 2(y/\delta) - 2(y/\delta)^3 + (y/\delta)^4$$

obtain expressions for the boundary layer thickness, shear intensity and force on one side of the plate.

Solution: By Karman momentum integral equation

$$\tau_0 = \rho U^2 \frac{\partial}{\partial x}\left(\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy\right)$$

put $y/\delta = \eta$, $dy = \delta\, d\eta$ and the limits of η are 0 and 1.

$$\frac{u}{U} = f(\eta) = 2\eta - 2\eta^3 + \eta^4$$

Substituting,

$$\tau_0 = \rho U^2 \frac{d\delta}{dx} \int_0^1 f(\eta)(1 - f(\eta))d\eta$$

But,

$$\int_0^1 f(\eta)(1 - f(\eta))d\eta$$

$$= \int_0^1 (2\eta - 2\eta^3 + \eta^4)(1 - 2\eta - 2\eta^3 + \eta^4)d\eta$$

$$= \int_0^1 (2\eta - 4\eta^2 - 2\eta^3 + 9\eta^4 - 4\eta^5 + 4\eta^6 - 4\eta^7 - \eta^8)d\eta$$

$$= 37/315$$

Therefore, $\tau_0 = \dfrac{37}{315}\rho U^2 \dfrac{d\delta}{dx}$

From the boundary conditions for a laminar boundary layer

$$\tau_0 = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{uU}{\delta}\left[\frac{df(\eta)}{d\eta}\right]_{\eta=0}$$

$$f(\eta) = 2\eta - 2\eta^2 + \eta^4$$

$$\left[\frac{df(\eta)}{d\eta}\right]_{\eta=0} = [2 - 4\eta + 4\eta^3]_{\eta=0} = 2$$

$\therefore$ $\tau_0 = 2\mu U/\delta$

Laminar boundary layer exists in the first 25 cm of the plate.

(ii) At x_{crit}, $\dfrac{\delta}{x_{\text{crit}}} = \dfrac{5.0}{\sqrt{Re_{(\text{crit})}}}$

The boundary layer thickness at the edge of the laminar boundary layer

$$\delta_c = \frac{5.0 \times 0.25}{\sqrt{5 \times 10^5}} = 1.768 \times 10^{-3}\text{m}$$

$$= \mathbf{1.768\ mm}$$

At the trailing edge, as the Reynolds number is $> 10^7$, the thickness of the turbulent boundary layer is obtained by putting $x = L = 20$ m in the equation

$$\frac{\delta_t}{x} = \frac{0.22}{(Re_x)^{1/6}}$$

$$\delta_t = \frac{0.22 \times 20}{(4 \times 10^7)^{1/6}} = 0.238 \text{ m}$$

$$= \mathbf{23.8\ cm}$$

(iii) The shear stress coefficient in the turbulent boundary with $Re > 10^7$ is

$$C_f = \frac{0.370}{(\log Re_x)^{2.58}}$$

Here $x = L = 20$ m and $Re_x = Re_L = 4 \times 10^7$

$$C_f = \frac{0.370}{(\log 4 \times 10^7)^{2.50}} = 1.9742 \times 10^{-3}$$

$$\tau_o = C_f \frac{\rho U^2}{2}$$

$$= 1.9742 \times 10^{-3} \times \frac{998 \times 2^2}{2} = \mathbf{3.94\ Pa}$$

8.12 A 1.80 m wide and 5.0 m long plate moves through a stationary air of density 1.22 kg/m^3 and viscosity 1.8×10^{-4} poise at a velocity of 1.75 m/s parallel to its length. Determine the drag force on one side of the plate by assuming (i) laminar flow condition and (ii) turbulent flow condition.

Solution:

$$\mu = 1.8 \times 10 \text{ poise} = 1.8 \times 10 \text{ Pa.s}$$

$$\nu = \mu/\rho = 1.8 \times 10^{-5}/1.22$$

$$= 1.475 \times 10^{-5} \text{ m}^2\text{/s}$$

Reynolds number,

$$Re_L = \frac{UL}{\nu} = \frac{1.75 \times 5}{1.475 \times 10^{-5}} = 5.932 \times 10^5$$

(i) *For laminar boundary layer*

Drag coefficient

$$C_{\text{Df}} = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{5.932 \times 10^5}}$$

$$= 1.724 \times 10^{-3}$$

$$F_D = C_{\text{Df}}(\text{area}) \frac{\rho U^2}{2}$$

$$= 1.724 \times 10^{-3} \times (1.8 \times 5.0) \times \frac{1.22 \times (1.75)^2}{2}$$

$$= \mathbf{0.029\ N}$$

(ii) *For a completely turbulent boundary layer:*

Since Re_L is between 5×10^5 and 10^7, the 1/7th power law-is applicable. Thus the drag coefficient

$$C_{\text{Df}} = \frac{0.074}{Re_L^{1/5}} = \frac{0.074}{(5.932 \times 10^5)^{1/5}} = 5.183 \times 10^{-3}$$

Drag force on one side of the plate

$$F_D = C_{\text{Df}} \times \text{area} \times \frac{\rho U^2}{2}$$

$$= 5.183 \times 10^{-3} \times (1.8 \times 5.0) \times \frac{1.22 \times (1.75)^2}{2}$$

$$= \mathbf{0.0871\ N}$$

8.13 A flat barge 35 m long and 10 m wide moves at 1.5 m/s in sea water ($\rho = 1025$ kg/m^3 and $\mu = 0.001$ Pa.s). Estimate the frictional drag on the bottom of the barge for (a) a smooth boundary and (b) a rough boundary with roughness height of 2.0 mm.

Solution: The relative flow past the bottom of the barge is similar to the flow past a flat surface.

$$Re_L = \frac{UL}{\nu} = \frac{1.5 \times 35.0}{(0.001/1025)} = 5.38 \times 10^7$$

The flow is turbulent in the boundary layer.

(a) Smooth surface

The drag coefficient, appropriate to this Reynolds number, is

$$C_{\text{Df}} = \frac{0.455}{(\log Re_L)^{2.58}} - \frac{1700}{Re_L}$$

$$= \frac{0.455}{(\log 5.38 \times 10^7)^{2.58}} - \frac{1700}{5.38 \times 10^7}$$

$$= 2.3248 \times 10^{-3} - (3.16 \times 10^{-5}) = 0.00229$$

F_D = drag force on the bottom of the barge

$$= C_{\text{Df}} \times \text{area} \times \frac{\rho U^2}{2}$$

$$= 0.00229 \times (35 \times 10) \times \frac{1025 \times (1.5)^2}{2}$$

$$= \mathbf{924\ N}$$

8.10 If the velocity distribution in a turbulent boundary layer is $u/U = (y/\delta)^{1/10}$ the displacement thickness δ^* is given by $\delta^*/\delta =$

(a) 0.3 (b) 0.125

(c) 0.091 (d) 0.0758

8.11 In a boundary layer at a certain location the boundary layer thickness $\delta = 0.5$ cm, displacement thickness $\delta^* = 0.15$ cm and the momentum thickness $\theta = 0.0585$ cm. The shape factor H at this location is

(a) 2.564 (b) 0.39

(c) 3.333 (d) 8.547

8.12 In a laminar boundary layer the shear stress τ_0 at a location x is given by $\tau_0 =$

(a) $\mu \left.\dfrac{\partial u}{\partial x}\right|_{y=0}$ (b) $\mu \left.\dfrac{\partial u}{\partial y}\right|_{y=0}$

(c) $\mu \left.\dfrac{\partial u}{\partial y}\right|_{y=\delta}$ (d) $\mu \left.\dfrac{\partial u}{\partial x}\right|_{y=x}$

8.13 In a laminar boundary layer the nominal thickness varies with the longitudinal distance x as

(a) $x^{-1/2}$ (b) $x^{-1/5}$

(c) x (d) $x^{1/2}$

8.14 In a laminar boundary layer over a flat plate the ratio of shear stresses τ_1 and τ_2 at two sections 1 and 2 at distances from the leading edge such that $x_2 = 5\,x_1$, is given by $\tau_1/\tau_2 =$

(a) $\sqrt{5}$ (b) $\dfrac{1}{\sqrt{5}}$

(c) 1.0 (d) 5.0

8.15 If δ_1 and δ_2 denote boundary layer thicknesses at a point distance x from the leading edge when the Reynolds numbers are 100 and 256 respectively, then the ratio of δ_1/δ_2 will be

(a) 0.625 (b) 1.6

(c) 2.56 (d) 4.90

8.16 The mean drag coefficient C_{Df} for a laminar boundary layer over a flat plate was found to be 0.015. If all other flow factors remain the same and the length of the plate is decreased to 1/4 of its original value, the drag coefficient C_{Df} would be equal to

(a) 0.015 (b) 0.060

(c) 0.030 (d) 0.0075

8.17 In the case of flow over a flat plate the growth of the boundary layer δ/x

(a) decreases with an increase in the kinematic viscosity

(b) increases with an increase in the free stream velocity

(c) decreases with an increase in the free stream velocity only if the boundary layer is laminar

(d) increases with an increase in the kinematic viscosity in both laminar and turbulent boundary layers.

8.18 If the velocity u in a turbulent boundary layer varies as $y^{1/7}$, the growth of the boundary layer thickness δ/x varies as

(a) $Re_x^{-1/5}$ (b) $Re_x^{-1/2}$

(c) $Re_x^{-4/5}$ (d) Re_x^{-1}

8.19 The rate of growth of the boundary layer thickness with the longitudinal distance on a flat plate

(a) is faster when the boundary layer is laminar than when it is turbulent

(b) is the same whether the boundary layer is laminar or turbulent

(c) is faster in a turbulent boundary layer when compared to that in a laminar boundary layer

(d) depends only on the aspect ratio of the plate

8.20 In a turbulent boundary layer on a flat plate the velocity distribution is given by the 1/7th power law. If the boundary layer over the whole plate is turbulent, the ratio of the shear stresses τ_1 and τ_2 at two sections 1 and 2 at distances from the leading edge $x_2 = 5\,x_1$ is given by $\tau_1/\tau_2 =$

(a) 1.379 (b) 1.258

(c) 2.236 (d) 1.308

8.21 The ratio of the coefficient of friction drag in laminar boundary layer compared to that in turbulent boundary layer is proportional to

(a) $R_L^{1/2}$ (b) $R_L^{1/5}$

(c) $R_L^{3/10}$ (d) $R_L^{3/10}$

8.22 In a turbulent boundary layer over a flat plate with Reynolds number $10^7 < Re_L < 10^9$ the growth of the boundary layer thickness is given by d/x is proportional to

(a) $Re_x^{1/5}$ (b) $Re_x^{-1/5}$

(c) $Re_x^{-1/6}$ (d) $Re_x^{-3/10}$

8.23 The laminar sublayer exists

(a) only in laminar boundary layers

(b) in all turbulent boundary layers

(c) only in smooth turbulent boundary layers

(d) only in rough fully developed turbulent boundary layers

8.24 On a flat plate, point A is at the mid-section and point B is at the trailing edge. The shear stresses τ_A at A and τ_B at B are such that

(a) $\tau_A > \tau_B$

(b) $\tau_B > \tau_A$

8.23 A 1/25 model of an ocean-going ship was towed in a towing tank containing fresh water. The model had the same Froude number as the prototype. The model had a length of 3 m, wetted surface area of 2 m^2 and was towed at a speed to reproduce the prototype speed of 9 m/s. What is the prototype drag corresponding to a measured total drag of 16 N in the model?

[For sea water: $\rho_w = 1025$ kg/m^3 and $\mu_s = 1.07 \times 10^{-3}$ Pa.s

For fresh water $\rho_w = 998$ kg/m^3 and $\mu_w = 1.00 \times 10^{-3}$ Pa.s].

(*Ans.* $F_{Dp} = 186.1$ kN)

The boundary layer thickness δ is estimated by

$$\boxed{\delta / x = 0.22 / Re_x^{1/6}} \tag{8.17}$$

If correction for initial laminar boundary layer is applied then,

$$\boxed{C_{\mathrm{Df}} = 0.455/(\log Re_L)^{2.58} - 1700/Re_L} \tag{8.18}$$

The term $1700/Re_L$ is so small that omitting it does not cause any appreciable error.

In Fig. 8.3, the values of C_{Df} are plotted against the Reynolds number Re_L as obtained from various equations, e.g. 8.4, 8.13, 8.14 and 8.16 for various regimes of flow.

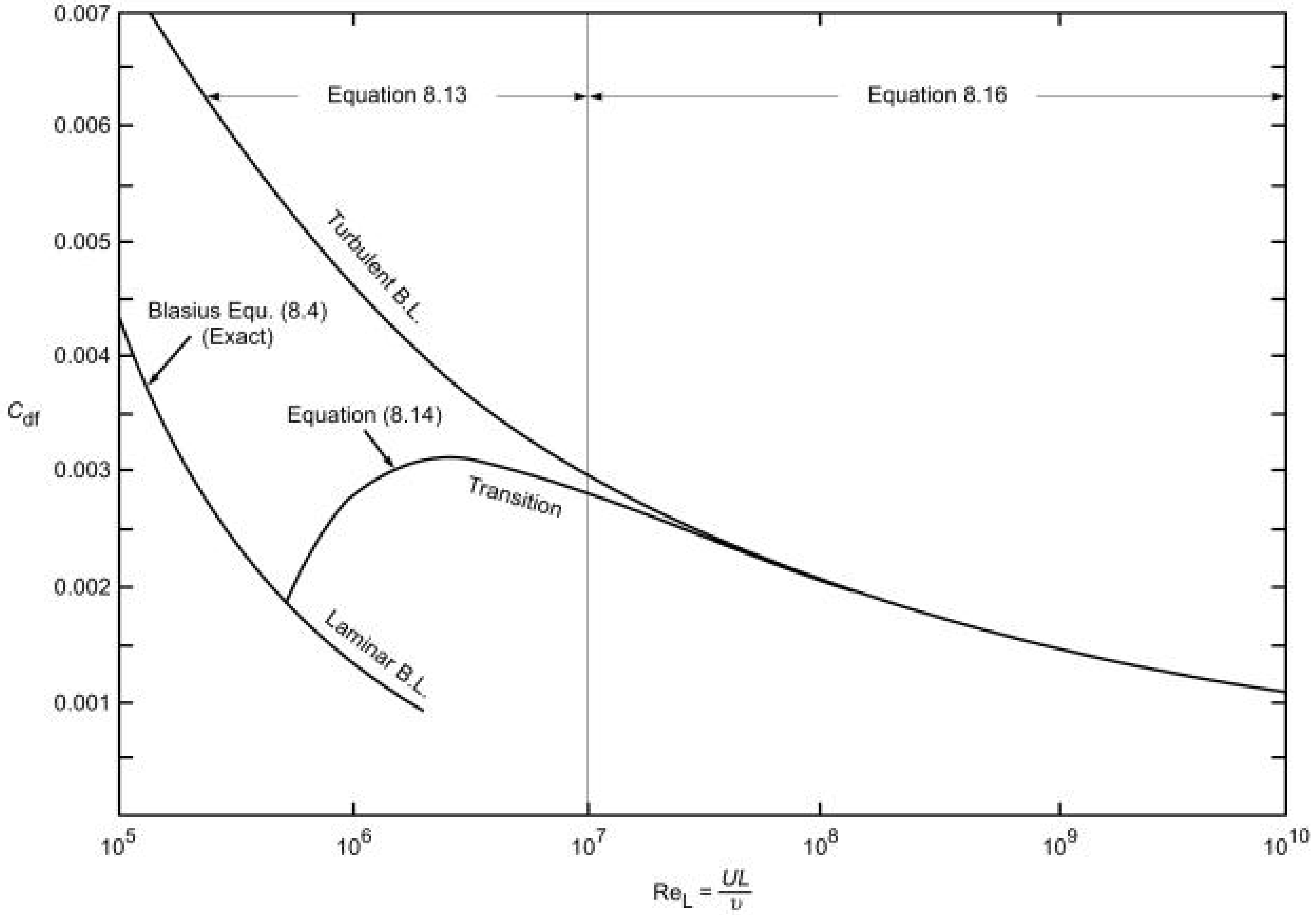

Fig. 8.3 *Drag Coefficients for Plates Kept Parallel to Flow*

8.8 LAMINAR SUBLAYER

The laminar sublayer is usually very thin and its thickness δ' is found by experiments to be

$$\boxed{\delta' = 11.6\, \nu / u_*} \tag{8.19}$$

where $u_* = \sqrt{\tau_0/\rho}$ = shear velocity.

If the roughness magnitude of a surface ε is very small compared to δ', i.e. $\varepsilon << \delta'$, then such a surface is said to be *hydrodynamically smooth*. Roughness does not have any influence in such flows while the viscous effects predominate. Usually $\varepsilon/\delta' < 0.25$ is taken as the criterion for hydrodynamically smooth surface (Fig. 8.4). If the laminar sublayer thickness δ' is very small compared to roughness height ε, (i.e. $\varepsilon >> \delta'$), in such flows viscous effects are not important and the boundary is said to be *hydrodynamically rough*. Usually $\varepsilon/\delta' > 6$ is taken as the criterion for hydrodynamically rough boundaries.

In the region $0.25 < \varepsilon/\delta' < 6$, the boundary is in the transition regime and both viscosity and roughness control the flow.

Rough flat plate: For flow on a completely rough flat plate the local friction coefficient C_{f} and total drag coefficient C_{Df} are given by

$$\boxed{C_{\mathrm{f}} = 1/(2.87 + 1.58 \log (x/\varepsilon))^{2.5}} \tag{8.20}$$

$$\boxed{C_{\mathrm{Df}} = 1/(1.89 + 1.62 \log (L/\varepsilon))^{2.5}} \tag{8.21}$$

8.9 ESTABLISHMENT OF FLOW IN A PIPE

When a flow enters a pipe from a reservoir a boundary layer forms in the pipe at the entrance. The thickness of

Laminar boundary layer exists in the first 25 cm of the plate.

(ii) At x_{crit}, $\dfrac{\delta}{x_{\text{crit}}} = \dfrac{5.0}{\sqrt{Re_{(\text{crit})}}}$

The boundary layer thickness at the edge of the laminar boundary layer

$$\delta_c = \frac{5.0 \times 0.25}{\sqrt{5 \times 10^5}} = 1.768 \times 10^{-3}\text{m}$$

$$= \mathbf{1.768\ mm}$$

At the trailing edge, as the Reynolds number is $> 10^7$, the thickness of the turbulent boundary layer is obtained by putting $x = L = 20$ m in the equation

$$\frac{\delta_t}{x} = \frac{0.22}{(Re_x)^{1/6}}$$

$$\delta_t = \frac{0.22 \times 20}{(4 \times 10^7)^{1/6}} = 0.238 \text{ m}$$

$$= \mathbf{23.8\ cm}$$

(iii) The shear stress coefficient in the turbulent boundary with $Re > 10^7$ is

$$C_f = \frac{0.370}{(\log Re_x)^{2.58}}$$

Here $x = L = 20$ m and $Re_x = Re_L = 4 \times 10^7$

$$C_f = \frac{0.370}{(\log 4 \times 10^7)^{2.50}} = 1.9742 \times 10^{-3}$$

$$\tau_o = C_f \frac{\rho U^2}{2}$$

$$= 1.9742 \times 10^{-3} \times \frac{998 \times 2^2}{2} = \mathbf{3.94\ Pa}$$

8.12 A 1.80 m wide and 5.0 m long plate moves through a stationary air of density 1.22 kg/m^3 and viscosity 1.8×10^{-4} poise at a velocity of 1.75 m/s parallel to its length. Determine the drag force on one side of the plate by assuming (i) laminar flow condition and (ii) turbulent flow condition.

Solution:

$$\mu = 1.8 \times 10^{-4} \text{ poise} = 1.8 \times 10^{-5} \text{ Pa.s}$$

$$\nu = \mu/\rho = 1.8 \times 10^{-5}/1.22$$

$$= 1.475 \times 10^{-5} \text{ m}^2/\text{s}$$

Reynolds number,

$$Re_L = \frac{UL}{\nu} = \frac{1.75 \times 5}{1.475 \times 10^{-5}} = 5.932 \times 10^5$$

(i) *For laminar boundary layer*

Drag coefficient

$$C_{Df} = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{5.932 \times 10^5}}$$

$$= 1.724 \times 10^{-3}$$

$$F_D = C_{Df}(\text{area})\ \frac{\rho U^2}{2}$$

$$= 1.724 \times 10^{-3} \times (1.8 \times 5.0) \times \frac{1.22 \times (1.75)^2}{2}$$

$$= \mathbf{0.029\ N}$$

(ii) *For a completely turbulent boundary layer:*

Since Re_L is between 5×10^5 and 10^7, the 1/7th power law-is applicable. Thus the drag coefficient

$$C_{Df} = \frac{0.074}{Re_L^{1/5}} = \frac{0.074}{(5.932 \times 10^5)^{1/5}} = 5.183 \times 10^{-3}$$

Drag force on one side of the plate

$$F_D = C_{Df} \times \text{area} \times \frac{\rho U^2}{2}$$

$$= 5.183 \times 10^{-3} \times (1.8 \times 5.0) \times \frac{1.22 \times (1.75)^2}{2}$$

$$= \mathbf{0.0871\ N}$$

8.13 A flat barge 35 m long and 10 m wide moves at 1.5 m/s in sea water ($\rho = 1025$ kg/m^3 and $\mu = 0.001$ Pa.s). Estimate the frictional drag on the bottom of the barge for (a) a smooth boundary and (b) a rough boundary with roughness height of 2.0 mm.

Solution: The relative flow past the bottom of the barge is similar to the flow past a flat surface.

$$Re_L = \frac{UL}{\nu} = \frac{1.5 \times 35.0}{(0.001/1025)} = 5.38 \times 10^7$$

The flow is turbulent in the boundary layer.

(a) Smooth surface

The drag coefficient, appropriate to this Reynolds number, is

$$C_{Df} = \frac{0.455}{(\log Re_L)^{2.58}} - \frac{1700}{Re_L}$$

$$= \frac{0.455}{(\log 5.38 \times 10^7)^{2.58}} - \frac{1700}{5.38 \times 10^7}$$

$$= 2.3248 \times 10^{-3} - (3.16 \times 10^{-5}) = 0.00229$$

F_D = drag force on the bottom of the barge

$$= C_{Df} \times \text{area} \times \frac{\rho U^2}{2}$$

$$= 0.00229 \times (35 \times 10) \times \frac{1025 \times (1.5)^2}{2}$$

$$= \mathbf{924\ N}$$

8.10 If the velocity distribution in a turbulent boundary layer is $u/U = (y/\delta)^{1/10}$ the displacement thickness δ^* is given by $\delta^*/\delta =$

(a) 0.3 (b) 0.125

(c) 0.091 (d) 0.0758

8.11 In a boundary layer at a certain location the boundary layer thickness $\delta = 0.5$ cm, displacement thickness $\delta^* = 0.15$ cm and the momentum thickness $\theta = 0.0585$ cm. The shape factor H at this location is

(a) 2.564 (b) 0.39

(c) 3.333 (d) 8.547

8.12 In a laminar boundary layer the shear stress τ_0 at a location x is given by $\tau_0 =$

(a) $\mu \left.\frac{\partial u}{\partial x}\right|_{y=0}$ (b) $\mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$

(c) $\mu \left.\frac{\partial u}{\partial y}\right|_{y=\delta}$ (d) $\mu \left.\frac{\partial u}{\partial x}\right|_{y=x}$

8.13 In a laminar boundary layer the nominal thickness varies with the longitudinal distance x as

(a) $x^{-1/2}$ (b) $x^{-1/5}$

(c) x (d) $x^{1/2}$

8.14 In a laminar boundary layer over a flat plate the ratio of shear stresses τ_1 and τ_2 at two sections 1 and 2 at distances from the leading edge such that $x_2 = 5\,x_1$, is given by $\tau_1/\tau_2 =$

(a) $\sqrt{5}$ (b) $\frac{1}{\sqrt{5}}$

(c) 1.0 (d) 5.0

8.15 If δ_1 and δ_2 denote boundary layer thicknesses at a point distance x from the leading edge when the Reynolds numbers are 100 and 256 respectively, then the ratio of δ_1/δ_2 will be

(a) 0.625 (b) 1.6

(c) 2.56 (d) 4.90

8.16 The mean drag coefficient C_{Df} for a laminar boundary layer over a flat plate was found to be 0.015. If all other flow factors remain the same and the length of the plate is decreased to 1/4 of its original value, the drag coefficient C_{Df} would be equal to

(a) 0.015 (b) 0.060

(c) 0.030 (d) 0.0075

8.17 In the case of flow over a flat plate the growth of the boundary layer δ/x

(a) decreases with an increase in the kinematic viscosity

(b) increases with an increase in the free stream velocity

(c) decreases with an increase in the free stream velocity only if the boundary layer is laminar

(d) increases with an increase in the kinematic viscosity in both laminar and turbulent boundary layers.

8.18 If the velocity u in a turbulent boundary layer varies as $y^{1/7}$, the growth of the boundary layer thickness δ/x varies as

(a) $Re_x^{-1/5}$ (b) $Re_x^{-1/2}$

(c) $Re_x^{-4/5}$ (d) Re_x^{-1}

8.19 The rate of growth of the boundary layer thickness with the longitudinal distance on a flat plate

(a) is faster when the boundary layer is laminar than when it is turbulent

(b) is the same whether the boundary layer is laminar or turbulent

(c) is faster in a turbulent boundary layer when compared to that in a laminar boundary layer

(d) depends only on the aspect ratio of the plate

8.20 In a turbulent boundary layer on a flat plate the velocity distribution is given by the 1/7th power law. If the boundary layer over the whole plate is turbulent, the ratio of the shear stresses τ_1 and τ_2 at two sections 1 and 2 at distances from the leading edge $x_2 = 5\,x_1$ is given by $\tau_1/\tau_2 =$

(a) 1.379 (b) 1.258

(c) 2.236 (d) 1.308

8.21 The ratio of the coefficient of friction drag in laminar boundary layer compared to that in turbulent boundary layer is proportional to

(a) $R_L^{1/2}$ (b) $R_L^{1/5}$

(c) $R_L^{3/10}$ (d) $R_L^{3/10}$

8.22 In a turbulent boundary layer over a flat plate with Reynolds number $10^7 < Re_L < 10^9$ the growth of the boundary layer thickness is given by d/x is proportional to

(a) $Re_x^{1/5}$ (b) $Re_x^{-1/5}$

(c) $Re_x^{-1/6}$ (d) $Re_x^{-3/10}$

8.23 The laminar sublayer exists

(a) only in laminar boundary layers

(b) in all turbulent boundary layers

(c) only in smooth turbulent boundary layers

(d) only in rough fully developed turbulent boundary layers

8.24 On a flat plate, point A is at the mid-section and point B is at the trailing edge. The shear stresses τ_A at A and τ_B at B are such that

(a) $\tau_A > \tau_B$

(b) $\tau_B > \tau_A$

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frictional forces are predominant as for example: flat plate flow, airplane wings and hydrofoils.

(3) A = wetted area. Customarily this is used for water crafts such as boats, barges and ships.

The variation of the drag coefficient for certain interesting and commonly used objects is discussed in the following sub-sections.

9.2.1 Sphere

The variation of the drag coefficient C_D with Reynolds number $Re = \dfrac{V_0 D}{\nu}$ where D = diameter of the sphere, is illustrated in Fig. 9.2. The variation is studied in three regimes:

(i) *Very small Reynolds numbers*

For $Re < 1.0$, the fluid motion is known as *creeping motion*. The total drag is given by Stokes' law, as

$$\boxed{F_D = 3\pi D \mu V_0} \tag{9.2}$$

Writing
$$F_D = C_D \left(\frac{\pi D^2}{4}\right)\frac{\rho V_0^2}{2} \tag{9.3}$$

$$\boxed{C_D = \frac{24\rho}{V_0 D/\mu} = \frac{24}{Re}}$$

Stokes' law is valid for $Re \le 1.0$ and its application to determine fall velocities of small particles is given in Examples 9.1 to 9.4.

(ii) *Reynolds number:* $1 \le Re \lesssim 2 \times 10^5$

In this regime, increasing Reynolds number reflects decreasing role of viscosity and C_D drops from a value of 24 at $Re = 1$ to about 0.4 at $Re \approx 10^4$. Then onwards, C_D is essentially constant, and at around $Re = 10^5$, $C_D = 0.5$. The region $10^4 \lesssim Re \lesssim 2 \times 10^5$ marks laminar boundary layer flow. In the range $1 < Re < 100$, C_D can be approximated by

$$\boxed{C_D = \frac{24}{Re}\left(1 + \frac{3}{16} Re\right)^{1/2}} \tag{9.4}$$

(iii) *Transition Region:* $(Re \approx 2 \times 10^5)$

Around $Re = 2 \times 10^5$ the laminar boundary layer undergoes transition to turbulent boundary layer. Consequently, the boundary layer separation line which occurred at around 85° angle shifts downstream to an angular position of about 120° causing a reduction in wake area. This is reflected in a rapid reduction in the value of C_D from 0.5 to 0.2. The critical Reynolds number depends upon boundary roughness and free stream turbulence, and an average value of 2×10^5 is generally adopted.

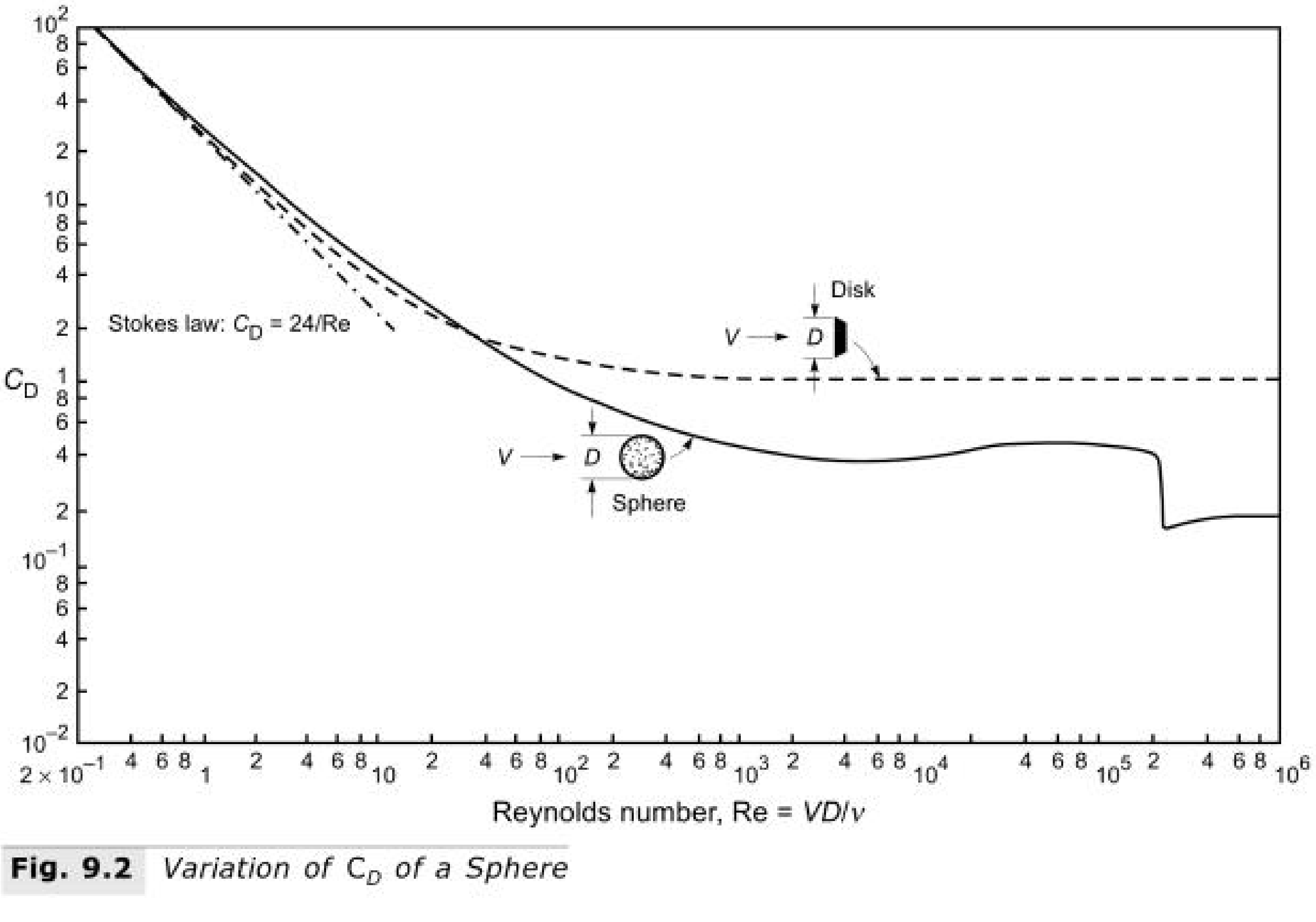

Fig. 9.2 *Variation of C_D of a Sphere*

9.31 The overall drag of an aircraft of weight W and wing area S is given by

$$C_D = a + bC_L^2$$

where a and b are constants. The maximum drag in horizontal flight will be

(a) $6W\sqrt{ab}$ (b) $4W\sqrt{ab}$
(c) $2W\sqrt{ab}$ (d) $W\sqrt{ab}$

9.32 The optimum efficiency of a lifting vane is limited by the

(a) onset of stall
(b) separation from the leading edge
(c) separation from the leading edge
(d) more rapid increase in C_D than in C_L

9.33 A streamlined body is one for which

(a) the skin friction is zero
(b) the skin friction is minimum
(c) the thickness of the body is minimum
(d) the separation point occurs on the far downstream part of the body

PROBLEMS

9.1 Find the terminal settling velocity of a 0.8 mm diameter sediment particle (RD = 2.65) in water ($\nu = 1.03 \times 10^{-6}$ m^2/s). The drag coefficient for $1.0 < Re < 200$ can be taken as

$$C_D = 24\left(1+\frac{3}{16}Re\right)^{1/2}/Re$$

(*Ans.* V_{0t} = 12.7 cm/s)

9.2 If a raindrop of 1 mm diameter has a terminal velocity of 4.0 m/s, (1) what is the Reynolds number of the flow? (2) What is the magnitude of the drag coefficient C_D? [Air: ρ_a = 1.20 kg/m^3, $\mu_a = 1.80 \times 10^{-5}$ Ns/m^2
Water: ρ_w = 998 kg/m^3, $\mu_w = 1.0 \times 10^{-3}$ Ns/m^2]

(*Ans*: Re_D = 267, C_D = 0.679)

9.3 Determine the largest diameter and the corresponding terminal velocity of particle that will obey Stokes' law in the following cases:

(a) Polysterene (RD = 1.05) spheres settling in air (ρ_a = 1.2 kg/m^3, $\nu_a = 1.5 \times 10^{-5}$ m^2/s)
(b) Polysterene (RD = 1.05) spheres settling in water ($\nu_w = 1 \times 10^{-6}$ m^2/s)
(c) Aluminum (RD = 2.8) spheres settling in glycerene (ρ_g = 1260 kg/m^3, μ_g = 1.49 Pa.s)

(*Ans.* (a) D_m = 0.078 mm; V_{0t} = 0.1925 m/s,
(b) D_m = 0.332 mm; V_{0t} = 3 mm/s
(c) D_M = 1.282 cm; V_{0t} = 9.22 cm/s)

9.4 A 2 mm sphere made of stainless steel (RD = 7.8) is observed to have a fall velocity of 9.5 mm/s in a liquid of density 1260 kg/m^3. Estimate the kinematic viscosity of the liquid.

(*Ans.* $\nu = 1.188 \times 10^{-3}$ m^2/s)

9.5 Determine the terminal velocity at which an air bubble of 3 mm diameter will rise in SAE 30 oil. What is the corresponding Reynolds number? (ρ_f = 917 kg/m^3 and μ_f = 0.29 Pa.s]

(*Ans.* V_{0t} = 1.55 cm/s;
Re = 0.147]

9.6 A helium filled spherical balloon rises in air at a terminal velocity of 3 m/s. Estimate its diameter if the total weight of the balloon is 210 N. [ρ_{air} = 1.2 kg/m^3, ρ_{helium} = 0.20 kg/m^3, $\nu_a = 1.5 \times 10^{-5}$ m^2/s. For $10^4 < Re < 3 \times 10^5$, C_D = 0.5 and for $Re > 3 \times 10^5$, C_D = 0.2]

(*Ans.* D = 3.5 m)

9.7 A hydrogen balloon is 2.0 m in diameter and contains hydrogen of density ρ_h = 0.066 kg/m^3. The balloon is found to rise at a terminal speed in air of density 0.95 kg/m^3 when the combined weight of empty balloon and payload is 24.5 N. Estimate the terminal speed. [$\nu_a = 2.3 \times 10^{-5}$ m^2/s. C_D = 0.5 for $10^4 \le Re \le 3 \times 10^5$ and C_D = 0.2 for $Re > 3 \times 10^5$]

(*Ans.* V_{0t} = 6.3 m/s)

9.8 Calculate the weight of ball of diameter 15 cm which is just supported by a vertical air stream of velocity 10 m/s, ρ_a = 1.25 kg/m^3 and ν_a = 1.5 stoke. The variation of C_D with Reynolds number Re is as follows:

Re	10^4	10^5	$> 3 \times 10^5$
C_D	0.4	0.5	0.2

(*Ans.* W = 0.4418 N)

9.9 What is the maximum load that can be carried by a 7.0 m diameter hemispherical parachute without exceeding the terminal velocity of 5 m/s? [ρ_a = 1.2 kg/m^3. For a hemisphere C_D = 0.34 and 1.33 depending upon the aspect of the hemisphere].

(*Ans.* F_D = 768 N)

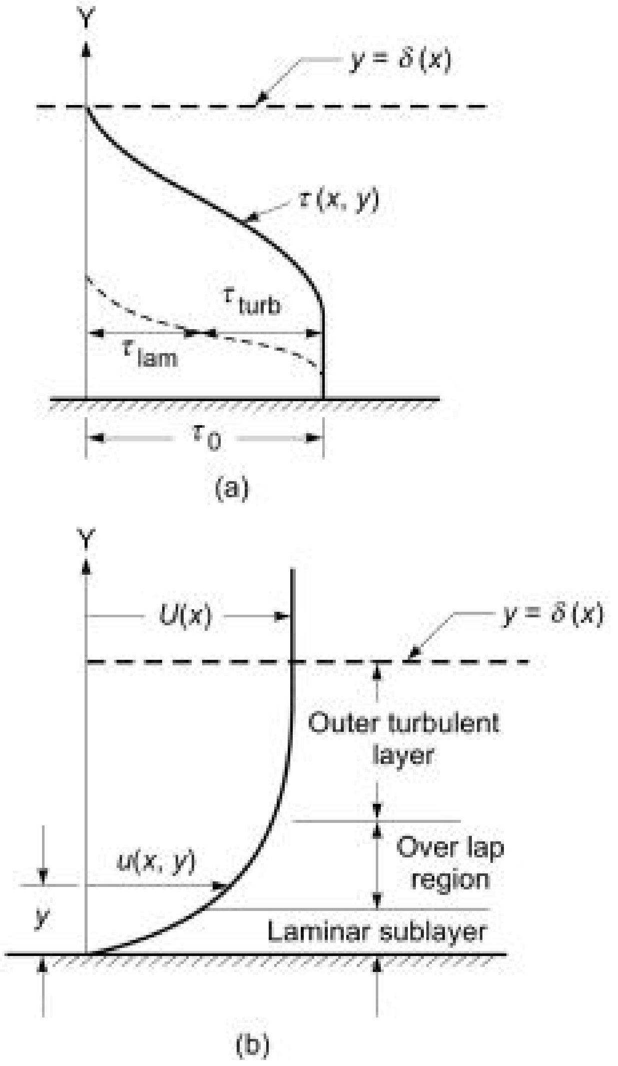

Fig. 10.1 *Turbulent Flow Near a Wall*

$\delta = r_0 = D/2$ = constant. The shear stress varies linearly with the distance from the boundary to become zero at the centre

$$\boxed{\tau = \tau_0\,(1 - y_0/r_0)} \qquad (10.8)$$

where r_0 = radius of the pipe.

In a horizontal pipe of diameter D carrying a steady flow there will be pressure drop $(p_2 - p_1)$ in a length L of the pipe. Equating the frictional resistance to the difference in pressure forces.

$$(p_1 - p_2)\,\frac{\pi D^2}{4} = \tau_0\,\pi D L \qquad (10.9)$$

or $$\tau_0 = (p_1 - p_2)\,\frac{D}{4L} = \left(-\frac{\Delta p \cdot D}{4L}\right) \qquad (10.10)$$

Designating $$\tau_0 = \frac{f}{4}\,\frac{\rho V^2}{2} \qquad (10.11)$$

where f = Darcy–Weisbach friction factor,

$$\sqrt{\tau_0/\rho} = u_* = V\sqrt{f/8} \qquad (10.12)$$

and $$(p_1 - p_2) = (-\,\Delta p) = f\,\frac{L}{D}\,\frac{\rho V^2}{2} \qquad (10.13)$$

For sloping pipes, $(-\Delta p)$ will be replaced by change in piezometric head and Eq. (10.13) can be written as

$$\boxed{\left(\frac{p_1}{\gamma} + Z_1\right) - \left(\frac{p_2}{\gamma} + Z_2\right) = h_f = \frac{f\,LV^2}{2gD}} \qquad (10.14)$$

Here h_f = drop in piezometric head in a distance L of the pipe due to friction. Equation (10.14) is known as *Darcy–Weisbach formula.*

10.2.1 Hydrodynamically Smooth and Rough Pipes

On the basis of relative magnitudes of surface protrusions ε and thickness of laminar sublayer δ', a pipe surface is classified as follows:

(a) Hydrodynamically smooth:

$$\varepsilon/\delta' \le 0.25 \qquad (10.15)$$

i.e $$\boxed{\frac{u_*\varepsilon}{\nu} \le 3.0} \qquad (10.15a)$$

(b) Surface in transition:

$$0.25 < \frac{\varepsilon}{\delta'} < 6.0 \qquad (10.16)$$

i.e. $$\boxed{3 < \frac{u_*\varepsilon}{\nu} < 70} \qquad (10.16a)$$

(c) Hydrodynamically rough:

$$\varepsilon/\delta' > 6.0 \qquad (10.17)$$

i.e. $$\boxed{\frac{u_*\varepsilon}{\nu} \ge 70} \qquad (10.17a)$$

It may be noted that the laminar sublayer thickness δ' could be expressed in terms of pipe radius, as

$$\frac{\delta'}{r_0} = \frac{11.6\nu}{r_0 u_*} = \frac{11.6\nu}{\frac{D}{2}V\sqrt{f/8}} = \frac{65.6}{Re\sqrt{f}} \qquad (10.18)$$

Hence $$\boxed{\frac{\varepsilon}{\delta'} = \frac{\varepsilon}{r_0}\,\frac{r_0}{\delta'} = \frac{Re\sqrt{f}}{(r_0/\varepsilon)\,(65.6)}} \qquad (10.19)$$

Thus $\dfrac{\varepsilon}{\delta'} < 0.25$ corresponds to $\dfrac{Re\sqrt{f}}{(r_0/\varepsilon)} < 17$

$\dfrac{\varepsilon}{\delta'} \ge 6.0$ corresponds to $\dfrac{Re\sqrt{f}}{(r_0/\varepsilon)} \ge 400$

10.2.2 Velocity Distribution

(a) *Local velocity*: For turbulent flow in a pipe of diameter D (radius $r_0 = D/2$), the velocity distribution at any distance y from the boundary, the local velocity u is given by the following expressions:

$$\left(\frac{4.50-4.20}{u_*}\right) = 5.75 \log \frac{0.30}{(0.30-0.10)}$$

$$= 1.01252$$

$$u_* = 0.2963 \text{ m/s}$$

The mean velocity V is related to u_m as

$$\frac{u_m - V}{u_*} = 3.75$$

$$\frac{4.50 - V}{0.2963} = 3.75$$

$$V = 3.389 \text{ m/s}$$

The discharge $$Q = \frac{\pi D^2}{4} V$$

$$= \frac{\pi \times (0.6)^2}{4} \times 3.389$$

$$= \mathbf{0.958\ m^3/s}$$

10.8 In a fully rough turbulent flow in a 15 cm diameter pipe the centre line velocity is 2.50 m/s and the local velocity at mid-radius is 2.28 m/s. Find the discharge and the height of the roughness projections.

Solution: For rough turbulent flow

$$\frac{u_m - u}{u_*} = 5.75 \log \frac{r_0}{y}$$

$$\frac{2.50 - 2.28}{u_*} = 5.75 \log \left(\frac{r_0}{r_0/2}\right) = 1.731$$

$$u_* = 0.1271 \text{ m/s}$$

The mean velocity V is related to u_m as

$$\frac{u_m - V}{u_*} = 3.75$$

$$2.50 - V = 3.75 \times (0.1271) = 0.4766$$

$$V = 2.0234 \text{ m/s}$$

The discharge $$Q = \left(\frac{\pi}{4} D^2\right) \times V$$

$$Q = \frac{\pi}{4} \times (0.15)^2 \times 2.0234$$

$$= 0.0358 \text{ m}^3\text{/s}$$

$$= \mathbf{35.8\ L/s}$$

For rough turbulent flow

$$\frac{u_m}{u_*} = 5.75 \log \frac{r_0}{\varepsilon} + 8.5$$

$$\frac{2.50}{0.1271} = 5.75 \log \frac{r_0}{\varepsilon} + 8.5$$

$$\log \frac{r_0}{\varepsilon} = 1.9425$$

$$\frac{r_0}{\varepsilon} = 87.61,$$

$$\varepsilon = \frac{(0.15/2)}{87.61} = \mathbf{8.56 \times 10^{-4}\ m}$$

The height of roughness projections $\varepsilon = 0.856$ mm

10.9 A pipe of 25 cm diameter carries air ($\rho = 1.22$ kg/m^3, $\nu = 1.5 \times 10^{-5}$ m^2/s) at an average velocity of 8 m/s. The equivalent sand grain roughness of the pipe is 0.50 mm. Calculate the friction factor f by assuming the flow to be fully rough-turbulent. What is the shear stress at the boundary?

Solution: Reynolds number

$$Re = \frac{VD}{\nu} = \frac{8.0 \times 0.25}{1.5 \times 10^{-5}} = 1.33 \times 10^5$$

The friction factor f in fully rough-turbulent flow is

$$\frac{1}{\sqrt{f}} = 2.0 \log \frac{r_0}{\varepsilon} + 1.74$$

$$= 2.0 \log \left(\frac{0.125}{0.50 \times 10^{-3}}\right) + 1.74$$

$$= 6.536$$

$$f = 0.0234$$

Shear stress at the boundary

$$u_* = \sqrt{\tau_0/\rho} = V_0\sqrt{f/8}$$

$$\tau_0 = \frac{\rho V_0^2 f}{8} = \frac{1.22 \times (8.0)^2 \times 0.0234}{8}$$

$$= \mathbf{0.2285\ Pa}$$

10.10 A 30 cm diameter pipeline carries water at a velocity of 3.5 m/s. If the friction factor $f = 0.028$, (a) determine the roughness height ε by assuming fully-rough turbulent flow to exist, (b) Find the shear stress at the wall, and (c) Find at a radial distance of 5 cm from the axis, the shear stress and the value of the velocity gradient. [$\nu = 0.8 \times 10^{-6}$ m^2/s].

Solution:

(a) For rough-turbulent flow:

$$\frac{1}{\sqrt{f}} = 2 \log \frac{r_0}{\varepsilon} + 1.74$$

$$\frac{1}{\sqrt{0.028}} = 2 \log \frac{r_0}{\varepsilon} + 1.74$$

$$2 \log \frac{r_0}{\varepsilon} = 4.236,$$

$$\frac{r_0}{\varepsilon} = 131.24$$

10.21 In turbulent flow through pipes, the maximum velocity u_m and the mean velocity V are related as $(u_m - V)/u_* =$

(a) 3.75 for smooth boundaries only
(b) 3.75 for both smooth and rough boundaries
(c) 5.75 for both smooth and rough boundaries
(d) 3.75 for rough boundaries only

10.22 In a turbulent flow through a pipe of radius r_0, the radial distance at which the local velocity is equal to the mean velocity is

(a) $r = 0.1\ r_0$
(b) $0.223\ r_0$
(c) $0.777\ r_0$
(d) $0.5\ r_0$

10.23 In a pipe flow the shear velocity u_* is related to friction factor f and mean velocity V as $u_*/V =$

(a) $\sqrt{f/8}$
(b) $\sqrt{8/f}$
(c) $\sqrt{8g/f}$
(d) $\dfrac{1}{\sqrt{f}}$

10.24 The Darcy–Weisbach friction factor f is related to boundary shear τ_0 as $f =$

(a) $\dfrac{8\rho V^2}{\tau_0}$
(b) $\dfrac{\tau_0}{8\rho V^2}$
(c) $\dfrac{8\tau_0}{\rho V^2}$
(d) $\dfrac{\rho V^2}{8\tau_0}$

10.25 The head loss in 100 m length of a 0.1 m diameter pipe ($f = 0.02$) carrying water is 10 m. The boundary shear stress, in kPa, is

(a) 9.79
(b) 0.298
(c) 0.1958
(d) 0.0245

10.26 In laminar flow through a pipe the Darcy–Weisbach friction factor f is given by $f =$

(a) $\dfrac{64}{Re}$
(b) $\dfrac{24}{Re}$
(c) $\dfrac{16}{Re}$
(d) $\dfrac{3}{16} Re$

10.27 In a turbulent pipe flow the boundary is hydrodynamically smooth if

(a) $\varepsilon_s/\delta' \le 0.25$
(b) $\varepsilon_s/\delta' < 6.0$
(c) $\delta'/\varepsilon_s \le 0.25$
(d) $\varepsilon_s/\delta' \ge 6.0$

10.28 The Blasius equation for friction factor f in turbulent flow through pipes relates f to the Reynolds number Re as $f =$

(a) $64/Re$
(b) $0.316/Re^{1/4}$
(c) $1.328/Re^{1/2}$
(d) $0.316/Re^{1/5}$

10.29 In a fully rough-turbulent pipe flow, the friction factor f is

(a) a function of Re and ε_s/D
(b) a function of Re only
(c) a function of ε_s/D only
(d) independent of Re and ε_s/D

where Re = Reynolds number and ε_s/D = relative roughness.

10.30 For hydrodynamically smooth pipes the friction factor f

(a) is a constant
(b) is a function of ε_s/r_0 only
(c) is a function of Re and ε_s/r_0 only
(d) is a function of Re only

where Re = Reynolds number and ε_s/r_0 = relative roughness.

10.31 In a fully developed rough-turbulent regime in pipe flow,

(a) rough and smooth pipes have the same friction factor
(b) the laminar sublayer is thicker than the roughness projections
(c) the friction factor is independent of the Reynolds number
(d) the friction factor is independent of the relative roughness ε_s/r_0

10.32 A turbulent pipe flow is said to be in the transitional regime, if the friction factor f is

(a) independent of Reynolds number
(b) dependent on both Reynolds number and relative roughness
(c) independent of both Reynolds number and relative roughness
(d) independent of the relative roughness

10.33 In a Moody diagram the third parameter is ε_s/D. Here ε_s refers to

(a) the equivalent uniform sand grain roughness
(b) an arbitrarily chosen roughness
(c) median size of a non-uniform sand grain roughness
(d) mean height of actual roughness of commercial pipes

10.34 Two pipes are identical in diameter and carry the same fluid. One of the pipes is rough and the other has a very smooth inside surface. If both the pipes have the same friction factor, when carrying the same discharge, then

(a) the flow is rough-turbulent regime
(b) the roughness magnitudes are much smaller than the laminar sublayer
(c) the flow must be laminar
(d) the flow is in transitional regime from smooth to rough flow

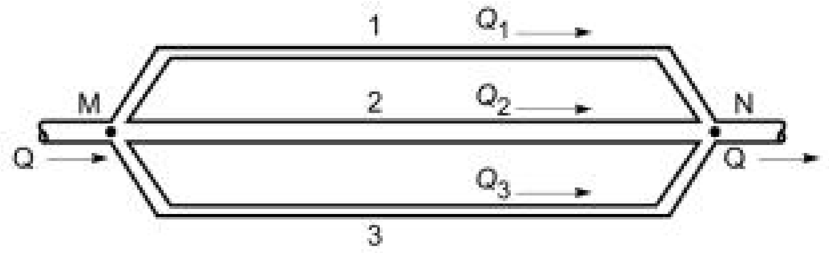

Fig. 11.4 *Pipes in Parallel*

Head loss

$$\boxed{\begin{aligned} H_{L1} = H_{L2} = H_{L3} &= \left(\frac{p_M}{\gamma} + Z_M\right) - \left(\frac{p_N}{\gamma} + Z_N\right) \\ &= h_M - h_N \end{aligned}} \tag{11.13}$$

It is usual to consider minor losses as equivalent lengths in parallel pipe flow problems and as such

$$\boxed{H_L = h_f = \frac{fLV^2}{2gD}}$$

Two types of problems can be recognised:

(1) Given piezometric heads at M and N (h_M and h_N) to find Q_1, Q_2, ... etc. This is a straight forward problem, especially if f of each pipe is known.
(2) Given total discharge Q, to determine the discharge division (i.e. to find Q_1, Q_2, ... etc.).

Three solution procedures are available to solve this problem

1. Exact method
2. Equivalent Pipe method
3. Trial and Error method

(1) Exact Method

Let there be N pipes in parallel.

Put $h_f = r_1 Q_1^2 = r_2 Q_2^2 = \ldots = r_i Q_i^2 = \ldots = r_n Q_n^2$

In this

$$r_i = \frac{8 f_i L_i}{\pi^2 g D_i^5} \quad \left(\text{obtained from } h_f = \frac{f_i L_i V_i^2}{2gD_i}\right) \tag{11.14}$$

Thus
$$Q_2 = \sqrt{\frac{r_1}{r_2}}\, Q_1$$

$$Q_3 = \sqrt{\frac{r_1}{r_3}}\, Q_1$$

....

$$Q_i = \sqrt{\frac{r_1}{r_i}}\, Q_1 \tag{11.15}$$

By continuity, total discharge

$$Q = Q_1 + Q_2 + Q_3 + \ldots + Q_i + \ldots + Q_n = \sum_1^n Q_i$$

Substituting Q_i from Eq. (11.15)

$$Q = Q_1\left[1 + \sqrt{\frac{r_1}{r_2}} + \sqrt{\frac{r_1}{r_3}} + \ldots + \sqrt{\frac{r_1}{r_i}} + \ldots + \sqrt{\frac{r_1}{r_N}}\right]$$

$$= Q_1 \sqrt{r_1}\left[\sum_1^N \frac{1}{\sqrt{r_i}}\right]$$

Thus
$$Q_1 = \frac{Q}{\sqrt{r_1}}\left[\frac{1}{\sum_1^N \frac{1}{\sqrt{r_i}}}\right] \tag{11.16}$$

Knowing Q_1, other discharges Q_i can be found from Eq. 11.15. In addition, from Eq. 11.14 h_f can be found.

(2) Equivalent Pipe Method

This is an alternative exact method. In this an equivalent pipe is used as a replacement of the set of parallel pipes. Examples 11.12 and 11.16 illustrate the use of this method.

(3) Trial and Error Method

(i) Assume a trial discharge Q_1' in pipe 1, (Fig. 11.4).
(ii) Using Q_1' find $h_f' = \dfrac{f L_1 V_1'^2}{2gD_1}$
(iii) Using h_f' as common head find Q_2' and Q_3'.
(iv) Find $Q' = (Q_1' + Q_2' + Q_3')$
(v) Assume the correct discharge is split among the pipes in the same ratio as $Q_1' : Q_2' : Q_3'$, thus $Q_1 = K\, Q_1'$, $Q_2 = K\, Q_2'$ and $Q_3 = K\, Q_3'$ where $K = \left(\dfrac{Q}{Q'}\right)$
(vi) Check the accuracy of this assumption by calculating h_{f1}, h_{f2} and h_{f3}. If the difference is greater than 5% repeat step (iv) onwards. The Example 11.12 illustrates this method.

11.3.4 Branching Pipes

In this at a junction three or more pipes lead off to different terminals. A typical branched pipe problem is the three-reservoir problem (Fig. 11.5). Here at the junction J the continuity equation must be satisfied, i.e. the flow into the junction = flow out of the junction.

i.e. $D_2 = \sqrt{2}\, D_1$

(b) Head loss $h_L = \dfrac{(V_1 - V_2)^2}{2g} = \dfrac{V_1^2}{2g}\,(1 - x^2)^2$

$$= \left(1 - \frac{1}{2}\right)^2 \frac{V_1^2}{2g} = \frac{1}{4}\,\frac{V_1^2}{2g}$$

Differential pressure head

$$= \frac{\Delta p}{\gamma} = \frac{V_1^2}{2g}\,(1 - x^4) - h_L$$

$$= \frac{V_1^2}{2g}\left(1 - \frac{1}{4} - \frac{1}{4}\right) = \frac{1}{2}\cdot\frac{V_1^2}{2g}$$

11.3 Three pipes with diameter, length and friction factor values of (D_1, L_1, f_1); (D_2, L_2, f_2) and (D_3, L_3, f_3) are connected in parallel between two points A and B in a pipeline. If an equivalent pipe (D_e, L_e, f_e) is to replace the set of parallel pipes, obtain an expression for estimating the equivalent pipe parameters.

Solution: Since there is a common head drop for the parallel pipes

$$h_f = \frac{f_1 L_1 V_1^2}{2gD_1} = \frac{f_2 L_2 V_2^2}{2gD_2} = \frac{f_3 L_3 V_3^2}{2gD_3} \qquad (1)$$

The sum of the flows = total discharge

or $\quad Q = Q_1 + Q_2 + Q_3 \qquad (2)$

From Eq. (1) $V_1 = \left(\dfrac{D_1}{f_1 L_1}\right)^{1/2} \sqrt{2g \cdot h_f}$

$$Q_1 = \frac{\pi}{4}\left(\frac{D_1^5}{f_1 L_1}\right)\sqrt{2g \cdot h_f}$$

Similarly $\quad Q_2 = \dfrac{\pi}{4}\left(\dfrac{D_2^5}{f_2 L_2}\right)\sqrt{2g \cdot h_f}$

$$Q_3 = \frac{\pi}{4}\left(\frac{D_3^5}{f_3 L_3}\right)\sqrt{2g \cdot h_f}$$

Also for the equivalent pipe

$$Q = \frac{\pi}{4} D_e^2 V_e$$

and the head loss

$$h_f = \frac{f_e L_e}{D_e}\,\frac{V_e^2}{2g}$$

or $\quad Q = \dfrac{\pi}{4}\left(\dfrac{D_e^5}{f_e L_e}\right)^{1/2} \sqrt{2g \cdot h_f}$

Thus from Eq. (2)

$$\left(\frac{D_e^5}{f_e L_e}\right)^{1/2} = \left(\frac{D_1^5}{f_1 L_1}\right)^{1/2} + \left(\frac{D_2^5}{f_2 L_2}\right)^{1/2} + \left(\frac{D_3^5}{f_3 L_3}\right)^{1/2} \qquad (3)$$

[***Note:*** For the equivalent pipe, out of the 3 variables D_e, L_e and f_e, given any two, the third one can be solved by Eq. (3).]

11.4 A compound piping system consists of 1800 m of 50 cm, 1200 m of 40 cm and 600 m of 30 cm diameter pipes of the same material connected in series.

(i) What is the equivalent length of a 40 cm pipe of the same material?

(ii) What is the equivalent size of a pipe 3600 m long?

(iii) If the three pipes are in parallel, what is the equivalent length of a 50 cm pipe?

Solution: It is assumed that f is same for all the pipes.

(i) $\quad \dfrac{f_e L_e}{D_e^5} = \dfrac{f_1 L_1}{D_1^5} + \dfrac{f_2 L_2}{D_2^5} + \dfrac{f_3 L_3}{D_3^5}$

$$f_e = f_1 = f_2 = f_3$$

$$\frac{L_e}{D_e^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

$$\frac{L_e}{(0.4)^5} = \frac{1800}{(0.5)^5} + \frac{1200}{(0.4)^5} + \frac{600}{(0.3)^5}$$

$\boldsymbol{L_e = 4318.2}$ **m**

(ii) $\quad \dfrac{3600}{D_e^5} = \dfrac{1800}{(0.5)^5} + \dfrac{1200}{(0.4)^5} + \dfrac{600}{(0.3)^5}$

$\boldsymbol{D_e = 38.57}$ **cm**

(iii) Three pipes in parallel:

$$Q_0 = Q_1 + Q_2 + Q_3$$

and $\quad h_{f1} = h_{f2} = h_{f3} = h_{fe}$

$$\therefore \quad \left(\frac{f_1 L_1}{D_1}\right)\frac{V_1^2}{2g} = \left(\frac{f_2 L_2}{D_2}\right)\frac{V_2^2}{2g}$$

$$= \left(\frac{f_3 L_3}{D_3}\right)\frac{V_3^2}{2g} = \left(\frac{f_e L_e}{D_e}\right)\frac{V_e^2}{2g}$$

But $\quad f_1 = f_2 = f_3 = f_e$

$$\therefore \quad \frac{V_2}{V_1} = \sqrt{\frac{L_1}{L_2}\cdot\frac{D_2}{D_1}} = \sqrt{\frac{1800 \times 0.40}{1200 \times 0.50}} = 1.0954$$

Similarly

$$\frac{V_3}{V_1} = \sqrt{\frac{1800 \times 0.3}{600 \times 0.5}} = 1.3416$$

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$$\therefore \quad (H_a - H_b) = r_1 Q_1^2 - r_2 Q_2^2 = 100.00 - 98.00 = 2.0$$

$$(H_b = H_c) = r_2 Q_2^2 + r_3 Q_3^2 = 98.00 - 90.00 = 8$$

Substituting the value of r_1, r_2 and r_3

$$136 Q_1^2 - 68 Q_2^2 = 2$$

$$68 Q_2^2 + 136 Q_3 = 8$$

Put $Q_3 = m Q_2$ resulting in $Q_1 = (m - 1)\, Q_2$. Substituting in the above equations

$$Q_2^2\,[136\,(m-1)^2 - 68] = 2 \qquad (1)$$

$$Q_2^2\,[68 + 136\, m^2] = 8 \qquad (2)$$

Dividing (1) by (2)

$$\frac{136(m-1)^2 - 68}{68 + 136 m^2} = 0.25$$

$$\frac{2(m-1)^2 - 1}{1 + 2m^2} = 0.25$$

On simplifying $m^2 - 2.667\, m + 0.5 = 0$

$$\therefore \quad m = \frac{2.667 \pm \sqrt{(2.667)^2 - 4 \times 0.5}}{2} = 2.46$$

[***Note:*** *only positive root is relevant as m < 1 gives negative values of* Q_1.]

Substituting in Eq. (2)

$$Q_2^2 = [68 - 138\,(2.46)^2] = 8$$

Hence $Q_2 = \mathbf{0.095\ m^3/s}$

$$Q_3 = m Q_2 = 2.46\,(0.095) = \mathbf{0.234\ m^3/s}$$

$$Q_1 = (m - 1)\, Q_2 = 1.46\,(0.095) = 0.139\ \text{m}^3/\text{s}$$

$$H_f = \text{piezometric head at the junction} = H_c + r_3 Q_3^2$$

$$= 90.00 + 136\,(0.234)^2 = \mathbf{97.447\ m}$$

11.20 For supplying water to a colony, it is intended to store water in a large tank whose water surface elevation is presently at 85.37 mRL. This tank is being supplied by a pipeline from a junction J, the friction loss in meters of water in this pipe being 0.6 $Q^{1.95}$, where Q is the discharge in m^3/s. Junction J receives water under pressure and without leakage from (i) a large reservoir I whose water surface elevation is constant at 100.00 mRL through a pipe characterised by $h_f = 0.35\ Q^2$; and (ii) a second large reservoir II with its constant water level at 96 mRL through a pipe characterised by $h_f = 0.46\ Q^{1.85}$. Neglecting losses other than due to pipe friction, determine the incoming flow into the tank in the colony.

Solution: Let the tank in the colony be designated as III and the pipelines from reservoirs to junctions be designated as 1, 2 and 3 as in Fig. 11.16.

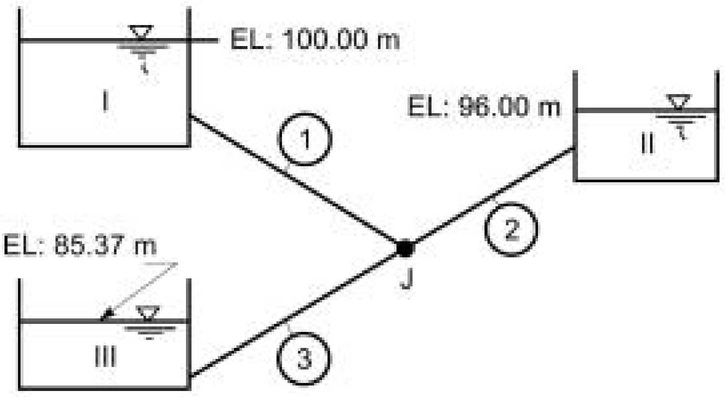


Fig. 11.16

Let H_j = Elevation of hydraulic gradeline at J.
For all pipes:

$$h_f = rQ^n \text{ with different values of } r \text{ and } n.$$

$$\text{Average value } \bar{n} = \frac{2.00 + 1.85 + 1.95}{3} = 1.933$$

A trial and error method is adopted.

[***Note:*** In this problem $h_f = rQ^n$. Hence $Q = (h_f/r)^{1/n}$. The correction ΔH_j is calculated as

$$\Delta H_j = (\bar{n} \times \Delta Q)/(\Sigma|Q/h_f|).]$$

First trial Assume H_j = 95.0 mRL

Pipe	*r*	*n*	*Estimated h_f (m)*	*$Q = (h_f/r)^{1/n}$ (m^3/s)*	*$\lvert Q/h_f \rvert$*
1	0.35	2.0	100.00 – 95.00 = 5.00	+ 3.780	0.760
2	0.46	1.85	96.00 – 95.00 = 1.00	+ 1.522	1.522
3	0.60	1.95	95.00 – 85.37 = 9.63	– 4.151	0.431
				Σ = +1.151	Σ = 2.713

$$\Delta Q = \text{Inflow} - \text{outflow} = +\ 1.151\ \text{m}^3/\text{s}$$

$$\Delta H_j = \text{Correction to assumed } H_j = (\bar{n} \times \Delta Q)/(\Sigma|Q/h_f|)$$

$$= +\ \frac{1.933 \times 1.151}{2.713} = 0.82\ \text{m}$$

New H_j (for 2nd trial) = 95.00 + 0.82 = 95.82 mRL

Second trial Assume H_j = 95.82 mRL

Pipe	*r*	*n*	*Estimated h_f (m)*	*$Q = (h_f/r)^{1/n}$ (m^3/s)*	*$\lvert Q/h_f \rvert$*
1	0.35	2.0	100.00 – 95.82 = 4.18	+3.456	0.827
2	0.46	1.85	96.00 – 95.82 = 0.18	+0.602	3.346
3	0.60	1.95	95.82 – 85.37 = 10.45	–4.329	0.414
				Σ = –0.271	4.587

$$\Delta Q = -\ 0.271\ \text{m}^3/\text{s}$$

$$\Delta H_j = \frac{1.933 \times 0.271}{4.587} = -\ 0.11\ \text{m}$$

New H_j (for 3rd trial) = 95.82 – 0.11 = 95.71 mRL

Third trial Assume H_j = 95.71 mRL.

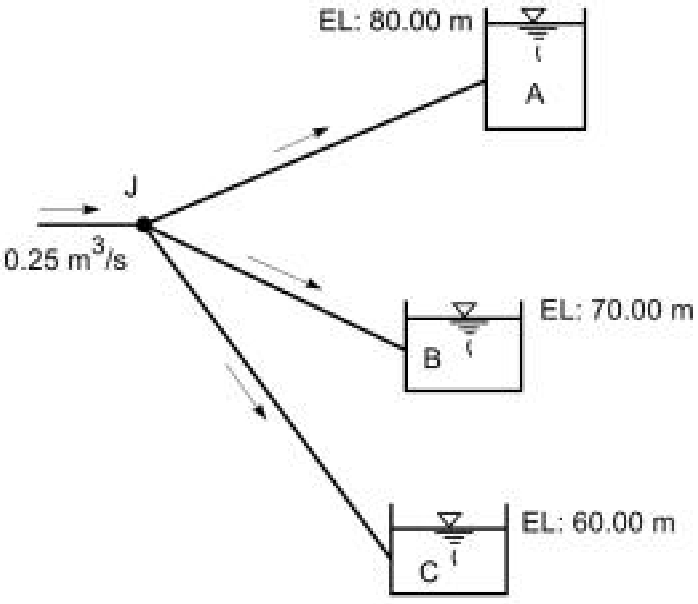


Fig. 11.19 *Example 11.23*

First trial

Let H_j = Elevation of hydraulic grade line at J
= 150.0 m

Pipe	*r* $\left(=\frac{8fL}{\pi^2 g D^5}\right)$	h_f *(m)*	$Q=\sqrt{h_f/r}$ *(m³/s)*	*Q* *(L/s)*	$\lvert Q/h_f\rvert$ *(Q in L/s)*
JA	10328	150 – 80 = 70	– 0.0823	–82.3	1.18
JB	12910	150 – 70 = 80	– 0.0787	–78.7	0.98
JC	15493	150 – 60 = 90	– 0.0762	–76.2	0.85
				Σ = –237.2	Σ = 3.01

$$\Delta Q = \text{Inflow} + \Sigma\ \text{Outflow} = 250 - 237.2 = 12.8\ \text{L/s}$$

$$\Delta H_j = \text{Correction to assumed } H_j = \frac{2\Delta Q}{\Sigma|Q/h_f|}$$

$$= \frac{2\times 12.8}{3.01} = 8.50\ \text{m}$$

New H_j = 158.5 m

Second trial

$$H_j = 158.5\ \text{m}$$

Pipe	*r*	h_f *(m)*	$Q=\sqrt{h_f/r}$ *(m³/s)*	*Q* *(L/s)*	$\lvert Q/h_f\rvert$ *(Q in L/s)*
JA	10328	158.5 – 80 = 78.5	–0.0872	–87.2	1.11
JB	12910	158.5 – 70 = 88.5	–0.0828	–82.8	0.94
JC	15493	158.5 – 60 = 98.5	–0.0797	–79.7	0.81
				Σ = –249.7	Σ = 2.86

$$\Delta Q = 250 - 249.7$$
$$= +\,0.3\ \text{L/s}$$

$$\Delta H_j = +\,\frac{0.3\times 2}{2.86} = +\,0.2\ \text{m}$$

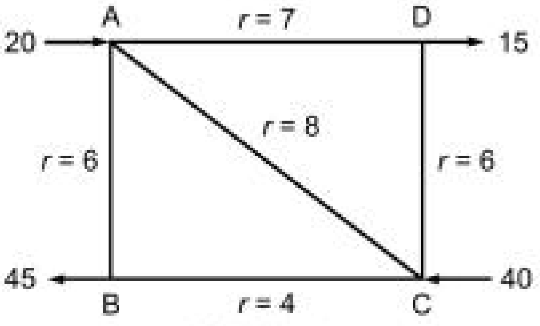
As ΔQ is very small no further iterations are necessary and ΔQ is distributed equally to the 3 pipes.

Hence

$$Q_{JA} = \mathbf{87.3\ L/s}$$
$$Q_{JB} = \mathbf{82.9\ L/s}$$
$$Q_{JC} = \mathbf{79.8\ L/s}$$

The elevation of the hydraulic gradient line at the junction is 158.5 + ΔH_j = **158.7 m.**

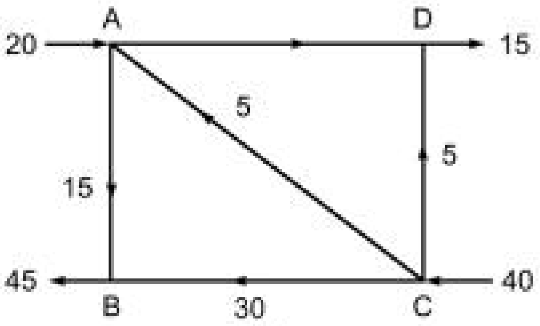
11.24 For the network shown in Fig. 11.20(G) the head loss is given by $h_f = rQ^2$. The values of r for each pipe, and the discharge into or out of various nodes are shown in the sketch. The discharges are in an arbitrary unit. Obtain the distribution of discharge in the network.



(G) (Given)

Fig. 11.20 *Example 11.24*

Solution: Flow direction is assumed positive clockwise for all loops. A first trial set of discharges is selected to satisfy continuity at each node [Fig. 11.20(a)), (i.e. flow into a node = flow out of the node). The Hardy–Cross method is used to find the corrections ΔQ.



(a)

Fig. 11.20(a)

First Trial

Line	*Loop ABC* rQ^2	$\lvert 2rQ\rvert$
AB	$-6\times 15^2 = -1350$	$2\times 6\times 15 = 180$
CB	$4\times 30^2 = +3600$	$2\times 4\times 30 = 240$
CA	$-8\times 5^2 = -200$	$2\times 8\times 5 = 80$
	Σ = 2050	Σ = 500

Correction ΔQ for loop ABC

$$\Delta Q = -\frac{2050}{500}$$

$$\Delta Q = -\,4$$

$$\frac{v^2}{2g} = \frac{H}{\left(1+\frac{fL}{D}\left(\frac{d}{D}\right)^4\right)} \qquad (2)$$

Power of jet $\quad \mathbf{P} = \gamma Q \dfrac{v^2}{2g}$

From Eq. (1) $\quad \dfrac{v^2}{2g} = H - \dfrac{fL}{D}\dfrac{V^2}{2g}$

Thus $\quad \mathbf{P} = \gamma \dfrac{\pi}{4} d^2 v \left[H - \dfrac{fL}{D}\cdot\left(\dfrac{d}{D}\right)^4\cdot\dfrac{v^2}{2g}\right]$

For maximum power $\dfrac{d\mathbf{P}}{dv} = 0$

i.e.
$$H - 3\frac{fL}{D}\left(\frac{d}{D}\right)^4\cdot\frac{v^2}{2g} = 0 \qquad (3)$$

$$\frac{v^2}{2g} = \frac{H}{3\frac{fL}{D}\left(\frac{d}{D}\right)^4}$$

But by Eq. (2), $\quad \dfrac{v^2}{2g} = \dfrac{H}{1+\frac{fL}{D}\cdot\left(\frac{d}{D}\right)^4}$

Hence $\quad \dfrac{H}{3\frac{fL}{D}\left(\frac{d}{D}\right)^4} = H\Big/\left[1+\dfrac{fL}{D}\left(\dfrac{d}{D}\right)^4\right]$

or $\quad 1 + \dfrac{fL}{D}\left(\dfrac{d}{D}\right)^4 = 3\dfrac{fL}{D}\left(\dfrac{d}{D}\right)^4$

$$d^4 = \frac{D^5}{2fL}$$

$$d = \left(\frac{D^5}{2fL}\right)^{1/4}$$

[***Note:*** From Eq. (3) at the condition of maximum power transmission, $h_f = 1/3\ H$.]

11.28 A 50 cm diameter pipe conveys water for purposes of power generation. The pipe is 1000 m long and has a head of 100 m at the inlet. If a nozzle, discharging into atmosphere is fitted at the outlet, estimate the diameter of the nozzle for maximum transmission of power. What is the magnitude of the maximum power? [Assume $f = 0.019$].

Solution: For the condition of maximum power, the diameter of the nozzle

$$d = \left(\frac{D^5}{2fL}\right)^{1/4} = \left(\frac{(0.5)^5}{2\times 0.019\times 1000}\right)^{1/4}$$
$$= 16.93 \text{ cm}$$

Head loss $\quad h_f = \dfrac{H}{3} = \dfrac{100}{3} = 33.33$ m

$$h_f = \frac{fLV^2}{2gD} = \frac{0.019\times 1000}{2\times 9.81\times 0.5}\times V^2 = 33.33$$

V = Velocity in the pipe = 4.148 m/s

Discharge $\quad Q = \dfrac{\pi}{4}\times(0.5)^2\times 4.148$

$= 0.8146\ \text{m}^3/\text{s}$

Maximum Power

$$\mathbf{P}_m = \gamma Q(H - h_f)$$
$$= 9.79\times 0.8146\times\frac{2}{3}\times 100 \text{ kW}$$
$$= \mathbf{531.6\ kW}$$

11.29 A pipeline 60 cm in diameter takes off from a reservoir whose water surface elevation is 150 m above datum. The pipe is 5000 m long and is laid completely at the datum level. In the last 1200 m of the pipe, water is withdrawn by a series of pipes at a uniform rate of 0.088 m³/s per 300 m. Find the pressure at the end of the pipeline. Assume $f = 0.02$ and the pipe to have a dead end.

Solution: First an expression for head loss in a pipe having a uniform withdrawal of q^* m³/s per metre length is derived.

Consider a section at a distance x from the start of the uniform withdrawal at q^* per metre length (see Fig. 11.23).

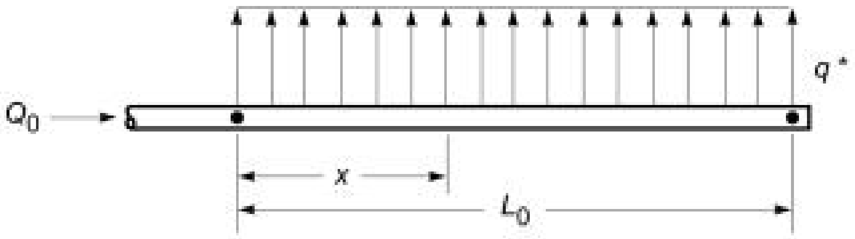


Fig. 11.23

Discharge $\quad Q_x = Q_0 - q^*x$

In a small distance dx,

$$dh_f = \frac{fLV^2}{2gD} = \frac{f(Q_0 - q^*x)^2}{2g\times\left(\frac{\pi}{4}\right)^2\times D^5}$$

$$dx = \frac{8f}{\pi^2 gD^5}(Q_0 - q^*x)^2\,dx$$

$$h_f = \int_0^{L_0} dh_f = \frac{-8f}{3\pi^2 gD^5}\ \frac{1}{q^*}\ [(Q_0 - q^*x)^3]_0^{L0}$$

$$= \frac{8f}{3\pi^2 gD^5}\ \frac{1}{q^*}\ [Q_0^3 - (Q_0 - q^*L_0)^3]$$

OBJECTIVE QUESTIONS

11.1 Hydraulic grade line for flow in a pipe of constant diameter is
(a) always above the centreline of the pipe
(b) always above the energy grade line
(c) always sloping downwards in the direction of the flow
(d) coincides with the pipe centreline

11.2 The head loss in a sudden expansion from 6 cm diameter pipe to 12 cm diameter pipe, in terms of velocity V_1 in the 6 cm pipe is
(a) $\frac{15}{16}\frac{V_1^2}{2g}$ (b) $\frac{3}{4}\frac{V_1^2}{2g}$
(c) $\frac{1}{4}\frac{V_1^2}{2g}$ (d) $\frac{9}{16}\frac{V_1^2}{2g}$

11.3 The head loss caused due to sudden expansion from area A_1 to area A_2 causing velocity to change from V_1 to V_2 is
(a) $\left(1-\frac{A_1}{A_2}\right)^2\frac{V_1^2}{2g}$ (b) $\left(1-\frac{A_1}{A_2}\right)^2\frac{V_2^2}{2g}$
(c) $\left(1-\frac{A_2}{A_1}\right)^2\frac{V_2^2}{2g}$ (d) $\left(1-\frac{A_1}{A_2}\right)\frac{V_1^2}{2g}$

11.4 A pipeline 5 cm diameter is reduced abruptly to 2.5 cm diameter at a section to enable measurement of flow. The loss head at the contraction is 0.5 m. The mean velocity at the reduced section will be about.
(a) 6.3 m/s (b) 3.0 m/s
(c) 4.0 m/s (d) 4.80 m/s

11.5 In a sudden contraction, the velocity head changes from 0.5 m to 1.25 m. The coefficient of contraction is 0.66. The head loss is this contraction is
(a) 0.133 m (b) 0.332 m
(c) 0.644 m (d) 0.750 m

11.6 The minor loss due to sudden contraction is due to
(a) flow contraction
(b) expansion of flow after sudden contraction
(c) boundary friction
(d) cavitation

11.7 A pipe has a well rounded entrance from a reservoir. If the head loss at the entrance is expressed as $K\frac{V^2}{2g}$, the value of K would be about
(a) 0.02 (b) 0.2
(c) 0.5 (d) 1.0

11.8 The loss at the exit of a submerged pipe in a reservoir is
(a) $0.5\frac{V^2}{2g}$ (b) $\frac{V^2}{2g}$
(c) $0.1\frac{V^2}{2g}$ (d) negligibly small

11.9 Minor losses in a pipe flow are those losses
(a) which are insignificantly small
(b) which can be neglected always
(c) caused by local disturbance due to pipe fittings
(d) caused by frictional resistance

11.10 Minor losses are usually neglected when they comprise 5% or less of the head loss due to friction. In a 10 cm diameter pipe ($f = 0.02$), the minimum length of the pipeline at which one can ignore minor losses due to sharp edged square entrance from a reservoir and a submerged square outlet to another reservoir, is
(a) 100 m (b) 15 m
(c) 150 m (d) 12000 m

11.11 In a 15 cm pipe line, the minor losses add up to $15\frac{V^2}{2g}$. The length of a pipe of 15 cm diameter ($f = 0.02$) equivalent to this loss is
(a) 75 m (b) 200 m
(c) 7.5 m (d) 150 m

11.12 Two identical pipes of length L, diameter D and friction factor f, are connected in *series* between two reservoirs. The size of a pipe of length L and of the same friction factor f, equivalent to the above pipeline, is
(a) 0.5 D (b) 0.87 D
(c) 1.15 D (d) 1.40 D

11.13 Two pipe systems in series are said to be equivalent when
(a) the average diameter in both systems is the same
(b) the average friction factor in both systems is the same
(c) total length of the pipe is the same in both the systems.
(d) the discharge under the same head is the same in both systems.

11.14 Two identical pipes of length L, diameter D and friction factor f, are connected in *parallel* between

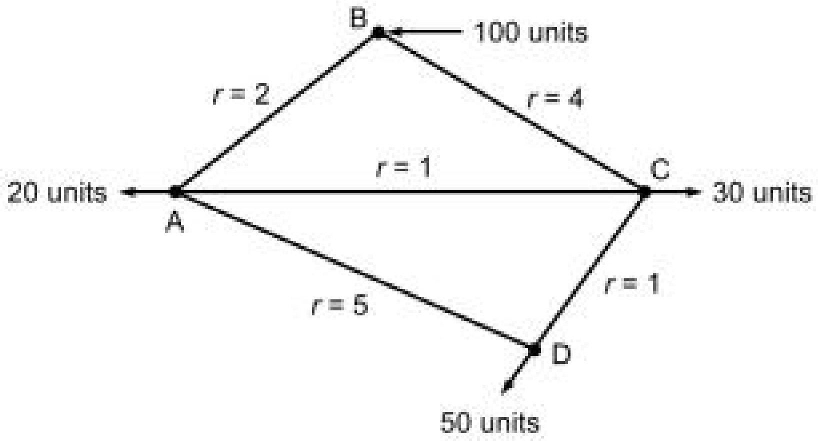


Fig. 11.32 *Related to Problem 11.29*

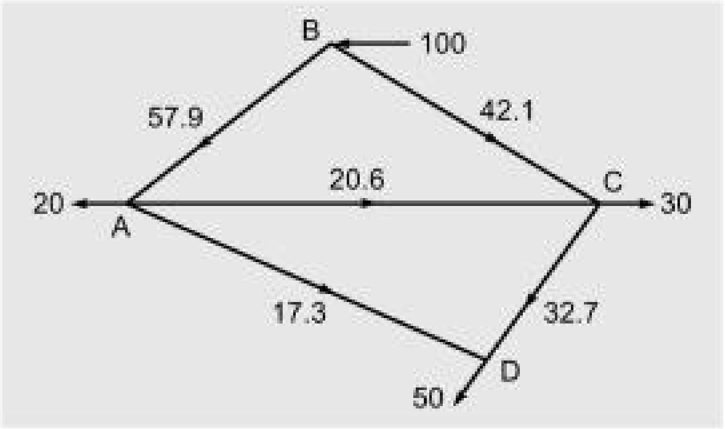


Fig. 11.32(a) *Answers to Problem 11.29*

11.30 If a conservative contaminant enters the steady state network of Fig. 11.33 at node D with a concentration C_d, and at the node E with a concentration 0.5 C_d, find the concentration of contaminant in the flows leaving the nodes E and F. Water entering the nodes A and B contains no contaminant. Assume perfect mixing at the nodes. The discharges entering the nodes and in the pipes are shown in the figure in arbitrary units.

(*Ans*: $C_f = 0.0893\ C_d$; $C_e = 0.2608\ C_d$)

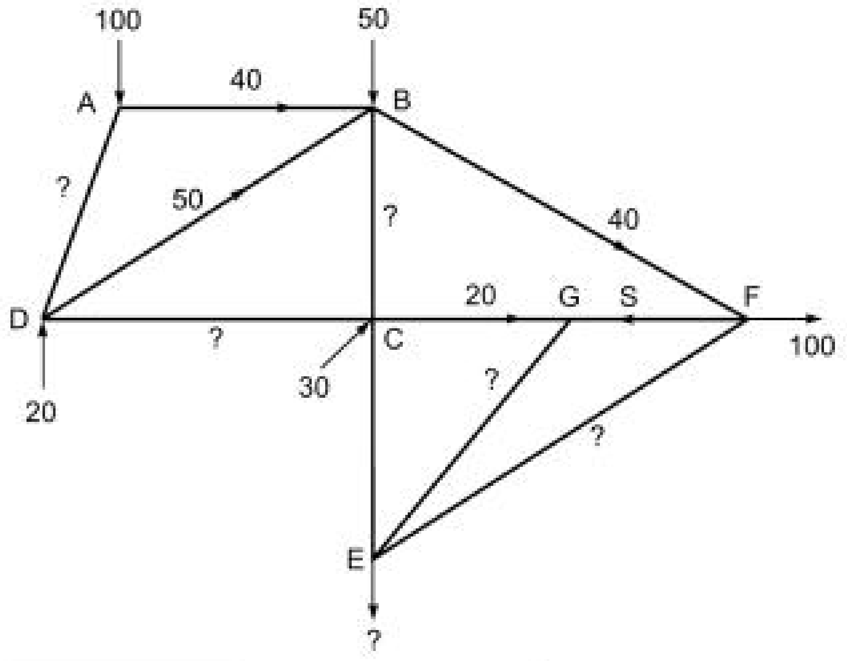


Fig. 11.33 *Problem 11.30*

[*Note:* The discharges in the pipes and at the nodes are indicated by numerals in the Figure. The arrows indicate the direction of flow.]

11.31 A pipe 10 cm in diameter and 400 m long has a nozzle fixed at the discharge end. The nozzle discharges at atmospheric pressure. If the pipe is horizontal and the head at the pipe inlet is 45 m, determine the maximum power that could be transmitted. What would be the corresponding size of the nozzle? (Assume $f = 0.02$).

(*Ans*. $\mathbf{P}_{max} = 4.42$ kW; $d = 28.1$ mm)

11.32 A 1000 m long pipe supplies water to a turbine which develops a brake power of 100 kW. The efficiency of the turbine is 90%. The head at the turbine inlet is 150 m and at the upstream and of the pipe it is 200 m. If $f = 0.02$, estimate the discharge and the velocity in the pipe. What is the diameter of the pipe?

(*Ans*. $Q = 75.66$ L/s, $V = 2.973$ m/s; $D = 18.0$ cm)

11.33 The supply of water from a reservoir to a nozzle situated 108 m below the reservoir free surface is through a 300 mm diameter pipe of 600 m length. The friction factor of the pipe is $f = 0.02$. Find the greatest possible power of the issuing jet.

(*Ans*. $\mathbf{P}_m = 209.4$ kW)

11.34 Kerosene ($\rho = 804$ kg/m^3) is pumped from one tank to another through a 100 m long 5 cm diameter pipe. The total minor loss coefficient in the system can be taken as $\Sigma k = 10.0$. If the total static lift is 5 m, calculate the power required to pump 0.3 m^3/min. Assume $f = 0.02$.

(*Ans*. $\mathbf{P} = 0.85$ kW)

11.35 Calculate the power required to pump oil (RD = 0.85) at a rate of 20 L/s from a storage sump to an elevated tank. The static lift is 20 m and the pipe is 10 cm in diameter and 250 m long. Assume $f = 0.02$. (Include entrance and exit losses).

(*Ans*. $\mathbf{P} = 6.16$ kW)

11.36 In a closed loop air drying system the conduit is 0.50 m × 0.30 m rectangular section and has a total length of 85 m. The minor losses can be accounted for by an equivalent length of 15 m. What power is required to circulate air through this loop at 25 m/s? Assume $f = 0.02$ and $\rho = 1.20$ kg/m^3.

(*Ans*. $\mathbf{P} = 8.91$ kW)

11.37 A fire brigade pump delivers water through a hose of 200 m length and 8 cm diameter to a nozzle

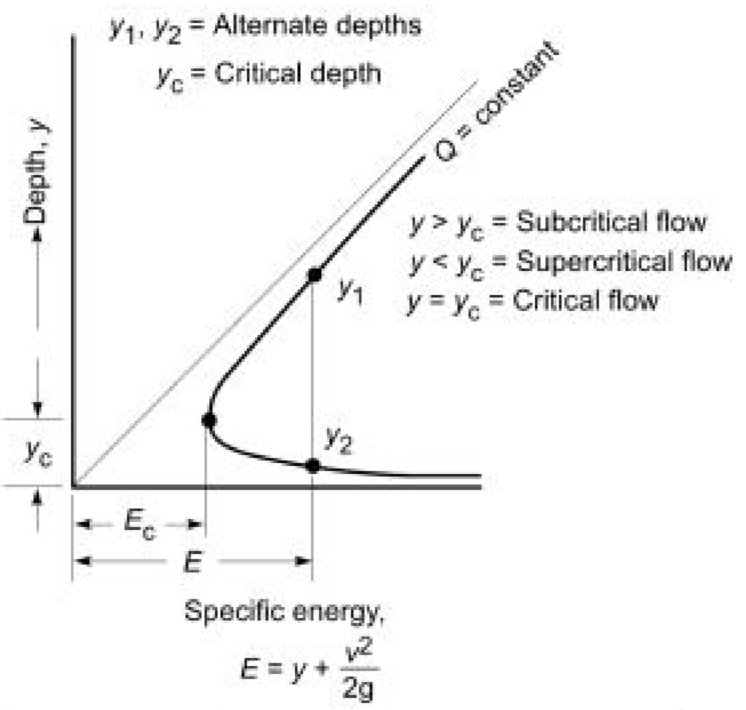

y_1, y_2 = Alternate depths
y_c = Critical depth
Q = constant
Depth, y
$y > y_c$ = Subcritical flow
$y < y_c$ = Supercritical flow
$y = y_c$ = Critical flow
y_1
y_2
y_c
E_c
E
Specific energy,
$E = y + \frac{v^2}{2g}$

Fig. 12.2 *Specific Energy Diagram*

known as the *critical depth*. It can also be shown that the discharge for a given specific energy is maximum at the critical depth. Using subscript 'c' to denote the critical flow condition, at the critical depth y_c

$$\boxed{\frac{Q^2}{g} = \frac{A_c^3}{T_c}} \tag{12.18}$$

where T_c = top width at critical depth.

This relationship is used to define the Froude number F for open channels. At critical depth $F_c = 1$ and hence

$$\frac{(Q/A_c)^2}{g\,(A_c/T_c)} = F_c^2 = 1.$$

At any other depth y, the Froude number is given by

$$\boxed{F = \frac{V}{\sqrt{g\,(A/T)}}} \tag{12.19}$$

[*Note:* In this chapter the notation for the Froude number is F instead of Fr as used in Chapter 6. This is a commonly used notation in open cannel flow.]

12.3.9 Computation of Critical Depth

The critical depth y_c is calculated by the relation $\frac{Q^2}{g} = \frac{A_c^3}{T_c}$. For rectangular and triangular sections simple explicit expressions for y_c are obtained.

Rectangular channel:
$$\boxed{y_c = \left(\frac{q^2}{g}\right)^{1/3}} \tag{12.20}$$

where $q = Q/B$ = discharge intensity.

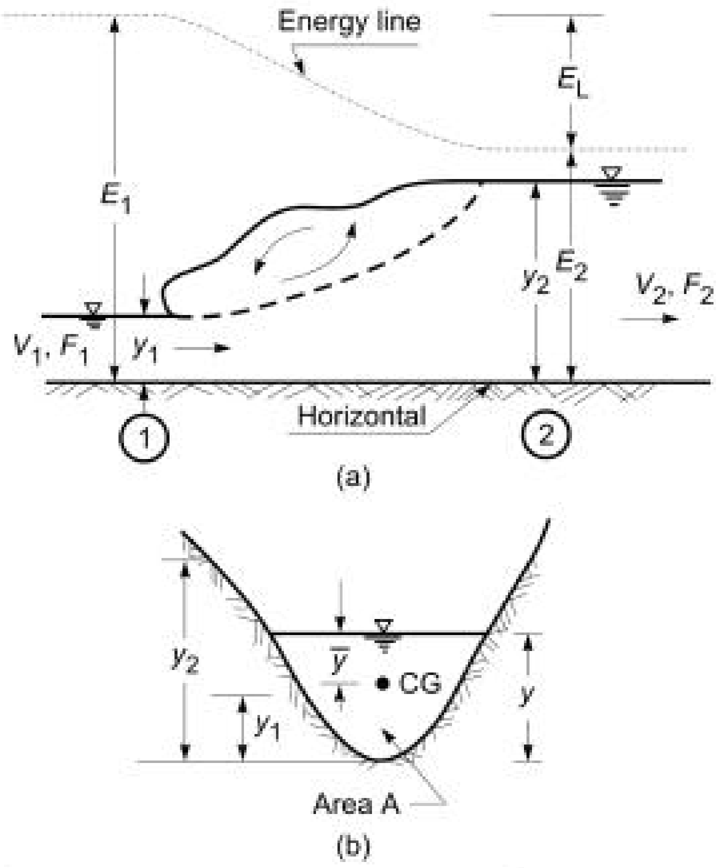
Triangular section:
$$\boxed{y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5}} \tag{12.21}$$

For other sections, a trial and error procedure is required for the evaluation of y_c. For details regarding the critical flow computation and use of specific energy relationship for solving problems relating to transition, the reader is advised to refer to Ref. 12.1.

12.4 RAPIDLY VARIED FLOW

12.4.1 Hydraulic Jump

Hydraulic jump is a case of rapidly varied flow. This phenomenon occurs when a super-critical flow stream tries to reach its alternate depth in sub-critical mode. In the process it loses substantial energy and falls short of the alternate depth. The depths on either side of the jump are known as *sequent depths* (Fig. 12.3).



(a)

(b)

Fig. 12.3 *Hydraulic Jump*

Consider a horizontal, frictionless channel of any arbitrary shape (Fig. 12.3) By applying momentum equation in the flow direction to a control volume encompassing sections 1 and 2

$$P_1 - P_2 = M_2 - M_1 \tag{12.22}$$

where P = pressure force on a section = $\gamma A\,\bar{y}$

M = momentum flux passing a section = $\rho\,Q\,V = \rho\,Q^2/A$

In the above A = cross sectional area and $\bar{y}$ = depth of center of gravity of the area from the water surface.

or $\quad (3.5 + 2y_0)^{2/3} = 1.804\, y_0^{5/3}$

By *trial and error,* $\boldsymbol{y_0}$ **= 1.48 m**

[***Note:*** The determination of normal depth of flow involves trial and error solution in most of the cases).

12.8 A circular drainage pipe 0.80 m in diameter conveys a discharge at a depth of 0.30 m. If the pipe is laid on a slope of 1 in 900, estimate the discharge. Manning's $n = 0.015$.

Solution: Referring to Fig. 12.7

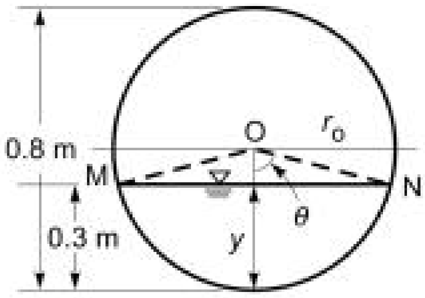


Fig. 12.7

$$D = 0.80 \text{ m}$$
$$y = 0.30 \text{ m}$$

Area of flow section

A = Area of sector OMN—Area of triangle OMN

$$= \frac{1}{2} r_0^2 \cdot 2\theta - \frac{1}{2} (2r_0 \sin \theta)\, r_0 \cos \theta$$

$$= \frac{D^2}{8} (2\theta - \sin 2\theta)$$

Also $\quad \cos \theta = \left(\frac{D}{2} - y\right)\Big/(D/2) = 1 - \frac{2y}{D}$

Hence $\quad 2\theta = 2 \cos^{-1}\left(1 - \frac{2y}{D}\right)$

$$= 2 \cos^{-1}\left(1 - \frac{2 \times 0.3}{0.8}\right) = 2.636 \text{ rad}$$

$$\sin 2\theta = 0.4841$$

Area $\quad A = \frac{(0.8)^2}{8} (2.636 - 0.4841)$

$$= 0.1722 \text{ m}^2$$

Wetted perimeter $P = \frac{1}{2} D\, (2\theta)$

$$= \frac{0.8}{2} \times 2.636 = 1.055 \text{ m}$$

Hydraulic radius $R = A/P = \frac{0.1722}{1.055} = 0.1633$ m

By Manning's formula

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2}$$

$$= \frac{1}{0.015} \times (0.1722)\,(0.1633)^{2/3}\,(1/900)^{1/2}$$

$= 0.1143$ m^3/s = **114.3 L/s**

12.9 A triangular channel has a side slope of 1.5 horizontal: 1 vertical and is laid on a longitudinal slope of 1 in 1650. Assuming Manning's $n = 0.013$, estimate the normal depth required to pass a discharge of 0.30 m^3/s.

Solution: Referring to Fig. 12.8

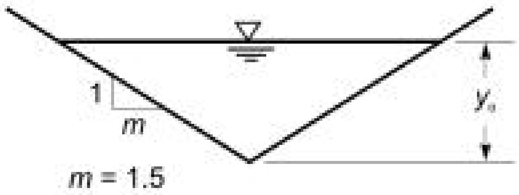


Fig. 12.8 *Triangular channel*

Let y_0 be the normal depth.

Here $\quad m = 1.5$

$$A = my_0^2 = 1.5\, y_0^2$$

$$P = 2y_0\sqrt{m^2 + 1} = 2y_0\sqrt{(1.5)^2 + 1}$$

$$= 3.6056\, y_0$$

$$R = A/P = \frac{1.5\, y_0^2}{3.6056\, y_0} = 0.416\, y_0$$

By Manning's formula,

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2}$$

$$0.30 = \frac{1}{0.013} \times (1.5\, y_0^2)\,(0.416\, y_0)^{2/3}\,(1/1650)^{1/2}$$

$$0.30 = 1.583\, y_0^{8/3}$$

$\boldsymbol{y_0}$ **= 0.54 m**

12.10 A trapezoidal channel is to be designed to convey 50 m^3/s of water at a velocity of 2.0 m/s. The bed width to depth ratio is to be 8.0 and the side slopes are to be 1 horizontal : 1 vertical. It will be lined with a material whose $n = 0.02$. Calculate the bed width, depth of flow and slope of the channel.

Solution:

$$m = 1.0,\ Q = 50.0 \text{ m}^3\text{/s}$$
$$V = 2.0 \text{ m/s}$$

Area $\quad A = \frac{Q}{V} = \frac{50.0}{2.0} = 25.0$ m^2

But $\quad A = (B + my)y$

$$= (8y + y)y = 9y^2 = 25.0$$

$y = \left(\frac{25}{9}\right)^{1/2}$ **= 1.667 m**

$B = 8y$ = **13.33 m**

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12.47 The sequent depths in a hydraulic jump formed in a horizontal rectangular channel are 0.2 m and 2.0 m. The length of the jump is about

(a) 50 m (b) 12 m
(c) 8 m (d) 2 m

12.48 In a hydraulic jump occurring in a horizontal rectangular channel the sequent depths are 0.25 m and 1.25 m.

The energy loss in this jump is

(a) 0.8 m (b) 1.0 m
(c) 1.25 m (d) 1.50 m

12.49 The discharge per metre width at the foot of a spillway is 10 m^3/s at a velocity of 20 m/s. A perfect free jump will occur at the foot of the spillway when the tail water depth is nearly

(a) 4.5 m (b) 5.00 m
(c) 6.50 m (d) 8.50 m

12.50 A hydraulic jump occurs near the junction of

(a) mild slope and milder slope
(b) steep slope and mild slope
(c) mild slope and steep slope
(d) steep slope and steeper slope

[The sequence of slopes is in the order mentioned]

Gradually Varied Flow

12.51 The differential equation of the gradually varied flow can be written by using Manning's formula for the case of a wide rectangular channel as $\dfrac{dy}{dx}$ =

(a) $S_0 \dfrac{1-(y_0/y)^{3.33}}{1-(y_c/y)^3}$ (b) $S_0 \dfrac{1-(y_c/y)^{3.33}}{1-(y_0/y)^3}$

(c) $S_0 \dfrac{1-(y_0/y)^3}{1-(y_c/y)^3}$ (d) $S_0 \dfrac{1-(y_0/y_c)^3}{1-(y_c/y)^{3.33}}$

12.52 If E = specific energy at a section in a gradually varied flow, then dE/dx =

(a) $S_0 + S_f$ (b) $S_0 - S_f$
(c) $S_f - S_0$ (d) $S_f/S_0 - 1$

where S_f = energy slope and S_0 = bed slope.

12.53 A long prismatic channel ends in an abrupt drop. For an initial subcritical flow the water surface profile

(a) starts from the critical depth line and joins the normal depth line asymptotically
(b) starts from the critical depth at the drop and decreases in the upstream direction.
(c) lies wholly above the normal depth line
(d) lies wholly below the critical depth line

12.54 If in a gradually varied flow dy/dx is positive, then dE/dx

(a) is always negative
(b) is always positive
(c) is positive if $y/y_c > 1$
(d) is negative if $y > y_c$

12.55 A 3 m wide rectangular channel flowing at its normal depth of 0.8 m carries a discharge of 9.5 m^3/s. The channel slope is

(a) steep (b) critical
(c) mild (d) none of the above

12.56 In an M$_1$ type of gradually varied flow profile

(a) $y_0 > y > y_c$ (b) $y_0 > y_c > y$
(c) $y > y_0 > y_c$ (d) $y_c > y_0 > y$

12.57 In an M$_2$ type of gradually varied flow profile

(a) $y_0 > y > y_c$ (b) $y > y_0 > y_c$
(c) $y_0 > y_c > y$ (d) $y_c > y > y_0$

12.58 In an S$_1$ type of gradually varied flow

(a) $y_c > y > y_0$ (b) $y > y_c > y_0$
(c) $y_c > y_0 > y$ (d) $y > y_0 > y_c$

12.59 The flow will be in supercritical state in the following profiles

(a) M_3, S_3 and M_1 (b) M_2, S_1 and M_3
(c) S_2, S_3 and M_3 (d) S_1, S_2 and S_3

12.60 The following types of gradually varied flow profiles do not exist.

(a) C_2, H_1 and A_1 (b) C_3, H_2 and A_2
(c) C_2, S_1 and A_3 (d) C_2, H_2 and A_1

12.61 Fig 12.27 shows a gradually varied flow situation in an open channel with a break in bed slope. Types of water surface profiles occurring from left to right are

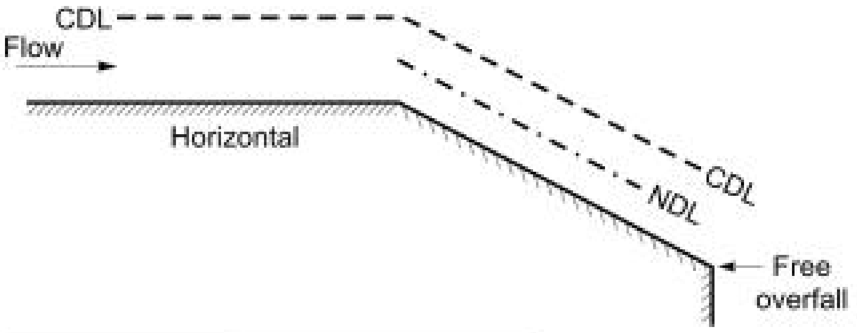

Fig. 12.27 *Question 12.61*

(a) H$_3$, S$_3$ (b) H$_2$, S$_2$
(c) H$_2$, M$_3$ (d) H$_3$, M$_2$

12.62 A sluice gate is operated in a long open channel with a steep slope as shown in Fig. 12.28 in such a manner that its opening is smaller than the normal depth.

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Table 12.6 *Dimensions of Efficient Channels–Problem 12.42*

Prob.	Q (m^3/s)	B_e (m)	y_e (m)	m	V (m/s)	P_e (m)	A_e (m^2)	n	S_0
(i)	15			2.0				0.015	0.0004
(ii)				1.5	1.5			0.020	0.0009
(iii)				1.5		12.0		0.020	0.0006
(iv)				1.0			5.0	0.015	0.0004
(v)			1.2	1.5				0.018	0.001
(vi)		1.5		1.0				0.014	0.0004
(vii)	6.0			0				0.015	0.0008
(viii)	3.0	0						0.015	0.0008

Answers to Problem 12.42

Prob.	Q (m^3/s)	B_e (m)	y_e (m)	m	V (m/s)	P_e (m)	A_e (m^2)	n	S_0
(i)	15.0	0.991	2.10	2.0	1.376	10.383	10.90	0.015	0.0004
(ii)	1.256	0.84	0.20	1.5	1.50	0.842	0.837	0.020	0.0009
(iii)	30.94	1.726	2.85	1.5	1.551	12.0	19.952	0.020	0.0006
(iv)	5.87	1.370	1.654	1.0	1.175	6.047	5.0	0.015	0.0004
(v)	3.79	0.727	1.20	1.5	1.250	5.053	3.032	0.018	0.001
(vi)	8.01	1.50	1.811	1.0	1.337	6.621	5.994	0.014	0.0004
(vii)	6.0	1.190	2.380	0	2.118	9.522	2.833	0.015	0.0008
(viii)	3.0	0	1.544	1.0	1.059	4.366	2.832	0.015	0.0008

E. Specific Energy and Critical Depth

12.43 A circular channel 1.20 m in diameter is laid on a slope of 0.008. The Manning's n for the channel can be taken as 0.018. Calculate the specific energy when the depth of flow is 0.60 m.

(*Ans.* $E = 0.853$ m)

12.44 A rectangular channel 1.5 m wide carries a discharge of 9.0 m^3/s at a depth of 2.0 m. Calculate the (a) specific energy and (b) depth alternate to the given depth of 2.0 m.

(*Ans.* $E = 2.4587$ m; $y_2 = 1.214$ m)

12.45 A rectangular channel 2.0 m wide carries a discharge of 6.0 m^3/s. If the specific energy is 1.8 m calculate the alternate depths.

(*Ans.* $y_1 = 0.625$ m; $y_2 = 1.626$ m)

12.46 In a rectangular channel 2.0 m wide the two alternate depths to a given specific energy are 0.30 m and 1.2 m respectively. Calculate the discharge.

(*Ans.* $Q = 2.446$ m^3/s)

12.47 For a constant specific energy of 3.0 m, what maximum flow may occur in a rectangular channel of 4.5 m bed width?

(*Ans.* $Q_m = 39.87$ m^3/s)

12.48 What is the lowest possible specific energy for a water discharge of 12 m^3/s to flow through a 3.0 m wide rectangular channel?

(*Ans.* $E_{min} = 1.766$ m)

12.49 A rectangular channel 12.0 m wide carries 200 m^3/s. Find the critical depth and critical velocity. What slope will produce this critical velocity in the channel if $n = 0.02$?

(*Ans.* $S_0 = 4.682 \times 10^{-3}$; $y_c = 3.048$ m; $V_c = 5.468$ m/s)

12.50 What is the maximum discharge that may be carried by a 3.2 m wide rectangular channel at a specific energy of 1.8 m?

(*Ans.* $Q_m = 13.18$ m^3/s)

12.51 Fill in the missing data in Table 12.7, connected with critical depth computation in a rectangular channel.

Table 12.7 *Data on Calculation of y_c in a Rectangular Channel–Problem 12.51*

Prob.	Q (m^3/s)	B (m)	y_c (m)	E_c (m)
(i)			2.6	0.40
(ii)	10.48		0.70	
(iii)	6.0	2.0		
(iv)		5.0		1.2

Answers to Problem 12.51

Prob.	Q (m^3/s)	B (m)	y_c (m)	E_c (m)
(i)	5.15	2.6	0.40	0.60
(ii)	10.48	4.0	0.70	1.05
(iii)	6.0	2.0	0.917	1.376
(iv)	14.0	5.0	0.80	1.2

$$R = \text{hydraulic radius} = \frac{\text{Area of flow}}{\text{wetted perimeter}} = A/P$$

S_0 = bottom slope of the channel

12.3.2 Uniform Flow Resistance Formulas:

Manning's formula The most widely used resistance equation for uniform flow, known as *Manning*'s *formula*, is

$$\boxed{V = \frac{1}{n} R^{2/3} S_0^{1/2}} \qquad (12.1)$$

where V = average velocity of flow and n = a roughness coefficient known as Manning's roughness coefficient. This coefficient is essentially a function of the boundary surface and has the dimensions of $[L^{-1/3}\,T]$. It varies from 0.013 for a smooth float finished concrete surface to 0.025 for irregular channels in excavated rock. Natural streams and rivers may have a high value of n ranging from 0.020 to as high as 0.13.

Chezy formula Another popular resistance equation, known as *Chezy formula*, is

$$\boxed{V = C\sqrt{RS_0}} \qquad (12.2)$$

where C = Chezy coefficient.

Darcy–Weisbach resistance equation The Darcy–Weisbach resistance equation, developed for pipe flow, could be used for representing open channel resistance as

$$\boxed{V = \sqrt{8g/f}\,\sqrt{RS_0}} \qquad (12.3)$$

where f = Darcy–Weisbach friction factor the variation of which is given by the Moody Diagram as

$$\boxed{f = \text{fn}\left[\left(Re = \frac{4RV}{\nu}\right), \left(\frac{4R}{\varepsilon_s}\right)\right]} \qquad (12.4)$$

where ε_S = equivalent sand grain roughness of the surface.

12.3.3 Relationship between *n*, *C* and *f*

The Manning's n, Chezy's C and Darcy–Weisbach's f are related as

$$C = \sqrt{8g/f} = \frac{1}{n} R^{1/6} \qquad (12.5)$$

Thus

$$f = \left(\frac{n^2}{R^{1/3}}\right)(8g) \qquad (12.6)$$

and

$$\boxed{n = \frac{R^{1/6}}{C}} \text{ and } \boxed{n = \frac{R^{1/6}\sqrt{f}}{\sqrt{8g}}} \qquad (12.7)$$

It should be noted that Manning's formula is applicable to fully developed rough turbulent flow regimes only.

12.3.4 Uniform Flow Computation

The discharge $Q = AV$ and thus

$$\boxed{Q = \frac{1}{n} AR^{2/3} S_0^{1/2}} \qquad (12.8)$$

Usually the uniform flow parameters are designated with a subscript 0. For channel cross sections whose top width is constant (e.g. rectangular) or increases with depth. (e.g. triangular, trapezoidal, parabolic) there is only one depth at which a given discharge will flow as uniform flow in a channel of known slope S_0. Such a depth is known as *normal depth*, y_0. This is a very important parameter in all open channel computations.

Uniform flow computations are relatively simple. The available relations are:

(a) Manning's formula (In this chapter Manning's formula is used in all cases unless specifically mentioned otherwise).
(b) Continuity equation and
(c) Geometry of the channel.

The following five types of basic problems can arise:

Problem type	*Given*	*Required*	*Method of solution*
1	y_0, n, S_0, Geometric elements (GE)	Q and V	Direct (explicit)
2	Q, y_0, n, GE	S_0	Direct
3	Q, y_0, S_0, GE	n	Direct
4	Q, n, S_0, GE	y_0	Trial and error
5	Q, y_0, n, S_0	GE	Trial and error

Typical geometric elements of common open channel cross sections are shown in Table 12.1.

12.3.6 Maximum Velocity and Discharge

Channels with closing top, i.e. those in which the top width decreases with depth in a certain range, do have an interesting property in which the velocity and discharge can reach a maximum value at finite values of depth.

For maximum discharge $\frac{dQ}{dy} = 0$ (12.9)

i.e.

$$\boxed{\frac{d}{dy}(AR^{2/3}) = 0}$$

Similarly for maximum velocity $\frac{dV}{dy} = 0$ (12.10)

i.e.

$$\boxed{\frac{d}{dy}(R) = 0}$$

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considerable use is the *broad-crested weir*. These weirs are sturdy structures with finite crest width in the direction of the flow (Fig. 12.5).

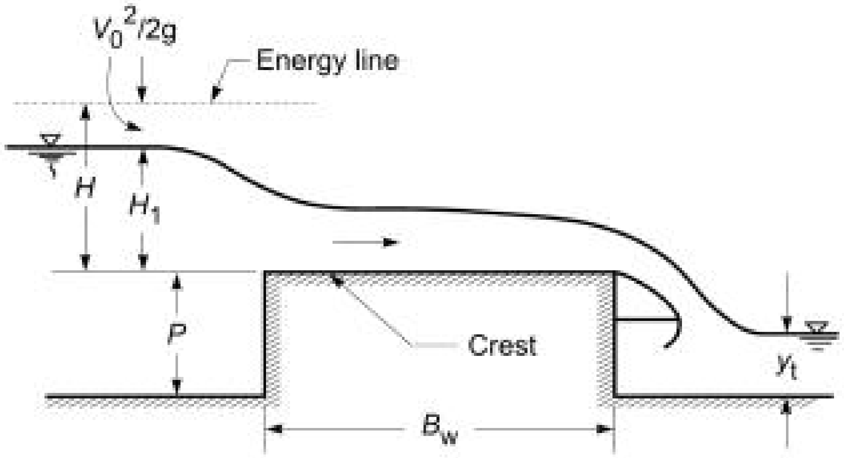

Fig. 12.5 *Broad Crested Weir*

The discharge over a broad crested weir is given by

$$Q = 1.705\ C_d\ LH^{3/2} \tag{12.34}$$

where L = length of the weir and H = energy head $= H_1 + \dfrac{V_0^2}{2g}$

The coefficient C_d depths on the geometry of the weir. For broad crested weirs with square entrance, C_d is about 0.85. For rounded entrance weirs C_d is about 0.98. It is usual to neglect the velocity of approach V_0.

[***Note:*** The topic of open-channel flows is indeed very vast and covers a very wide range of applications. The reader is reminded that the brief summary given above is but an introduction. There are books, treating in extensive detail, on open channel flow and the reader may refer to the following reference for further details.]

REFERENCE

12.1 Subramanya. K, *Flow in Open Channels,* Tata McGraw-Hill Pub. Co., New Delhi 2nd Ed., 11th Reprint, 2004.

WORKED EXAMPLES

A. Uniform Flow

12.1 A rectangular channel 2.5 m wide carries water at a depth of 1.2 m. The bed slope of the channel is 0.0036. Calculate the average shear stress on the boundary.

Solution:

Area $A = By = 2.5 \times 1.2 = 3.00\ \text{m}^2$

Wetted perimeter $P = B + 2y = 2.5 + (2 \times 1.2) = 4.9$ m

Hydraulic radius $R = A/P = \dfrac{3.00}{4.90} = 0.612$ m

Average boundary shear stress

$$\begin{aligned} \tau_0 &= \gamma R S_0 \\ &= (998 \times 9.81) \times 0.612 \times 0.0036 \\ &= \mathbf{21.58\ Pa} \end{aligned}$$

12.2 A trapezoidal channel has a bed width of 2.0 m and side slopes of 1.5 horizontal : 1 vertical. The channel has a longitudinal slope of 1/4000. If the Manning's coefficient of the channel boundary is 0.018, calculate the mean velocity and discharge in the channel for a depth of 1.4 m.

Solution: Referring to Fig. 12.6

Area
$$\begin{aligned} A &= (B + my)y \\ &= (2.0 + 1.5 \times 1.4) \times 1.4 = 5.74\ \text{m}^2 \end{aligned}$$

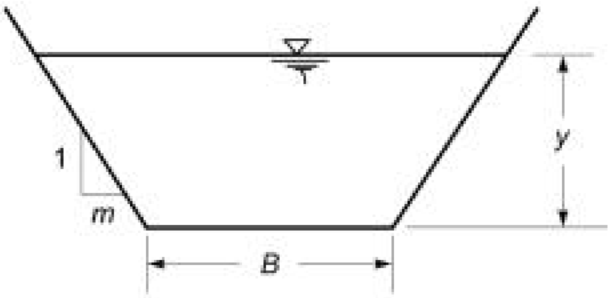

Fig. 12.6 *Trapezoidal channel*

Wetted perimeter
$$\begin{aligned} P &= (B + 2\sqrt{m^2+1}\cdot y) \\ &= (2.0 + 2\sqrt{(1.5)^2+1} \times 1.4) \\ &= 7.048\ \text{m} \end{aligned}$$

Hydraulic radius $R = A/P = \dfrac{5.74}{7.048} = 0.814$ m

Mean velocity, by Manning's formula

$$\begin{aligned} V &= \frac{1}{n} R^{2/3} S_0 \\ &= \frac{1}{0.018} \times (0.814)^{2/3} \times \left(\frac{1}{4000}\right)^{1/2} \\ &= \mathbf{0.766\ m/s} \end{aligned}$$

Discharge
$$\begin{aligned} Q &= AV \\ &= 5.74 \times 0.766 = \mathbf{4.40\ m^3/s} \end{aligned}$$

12.26 If y_1 and y_2 are alternate depths in a rectangular channel show that

$$y_c^3 = \frac{2y_1^2 y_2^2}{(y_1 + y_2)}$$

and hence the specific energy $E = \dfrac{y_1^2 + y_1 y_2 + y_2^2}{(y_1 + y_2)}$.

Solution: Let q = discharge intensity. Then

$$q = V_1 y_1 = V_2 y_2$$

As y_1 and y_2 are alternative depths, the specific energy

$$E = y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$= y_1 + \frac{q^2}{2g y_1^2} = y_2 + \frac{q^2}{2g y_2^2}$$

Hence $$(y_1 - y_2) = \frac{q^2}{2g}\left(\frac{1}{y_2^2} - \frac{1}{y_1^2}\right)$$

$$2(y_1 - y_2) = \frac{(y_1^2 - y_2^2)}{y_1^2 y_2^2} \cdot \frac{q^2}{g}$$

But in a rectangular channel $\dfrac{q^2}{g} = y_c^3$

Hence $$y_c^3 = \frac{2(y_1 - y_2)\, y_1^2 y_2^2}{(y_1^2 - y_2^2)}$$

$$\boldsymbol{y_c^3 = \frac{2y_1^2 y_2^2}{(y_1 + y_2)}}$$

Specific energy $$E = y_1 + \frac{1}{2}\cdot\left(\frac{q^2}{g}\right)\frac{1}{y_1^2}$$

$$= y_1 + \frac{1}{2y_1^2} \times \frac{2y_1^2 y_2^2}{(y_1 + y_2)}$$

$\therefore$ $$\boldsymbol{E = \frac{y_1^2 + y_1 y_2 + y_2^2}{(y_1 + y_2)}}$$

12.27 For a triangular channel having a vertex angle of 120°, calculate the critical depth for a discharge of 3.0 m^3/s.

Solution: Referring to Fig. 12.14

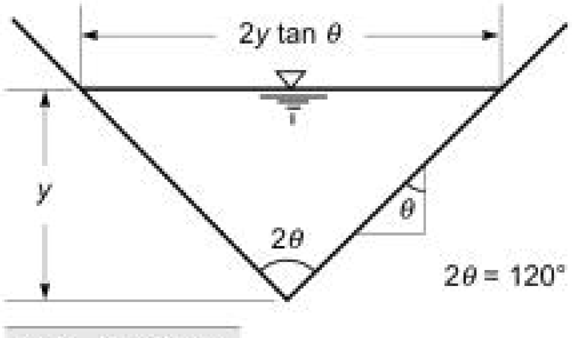

Fig. 12.14

Let y_c = Critical depth.

Top width $T_c = 2y_c \tan\theta$

Area $A_c = y_c^2 \tan\theta$

At critical depth $$\frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

$$= \frac{y_c^6 \tan^3\theta}{2y_c \tan\theta} = \frac{1}{2}\, y_c^5 \tan^2\theta$$

$$y_c = \left(\frac{2Q^2}{g}\,\frac{1}{\tan^2\theta}\right)^{1/5}$$

$$= \left(\frac{2 \times (3.0)^2}{9.81 \times (\tan 60°)^2}\right)^{1/5}$$

$$= \mathbf{0.906\ m}$$

12.28 Calculate the critical depth corresponding to a discharge of 6.0 m^3/s in (a) rectangular channel of width 3.0 m, (b) triangular channel of side slope 1.5 horizontal : 1 vertical (c) trapezoidal channel of bottom width 2.0 m and side slope 1.0 horizontal : 1.0 vertical

Solution:

(a) Rectangular channel:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$q = \frac{6.0}{3.0} = 2.0\ \text{m}^3/\text{s/m}$$

$$y_c = \left(\frac{(2.0)^2}{9.81}\right)^{1/3} = \mathbf{0.742\ m}$$

(b) In a triangular channel of side slope m horizontal : 1 vertical

$$A = my^2$$
$$T = 2my$$

At critical depth

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c} = \frac{m^3 y_c^6}{2my_c}$$

$$y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5}$$

$$= \left[\frac{2 \times (6.0)^2}{9.81 \times (1.5)^2}\right]^{1/5} = \mathbf{1.267\ m}$$

(c) In a trapezoidal channel of side slope m horizontal : 1 vertical,

$$A = (B + my)y \quad \text{and} \quad T = (B + 2my)$$

$$= \frac{(2.136 - 0.250)^3}{4 \times 2.136 \times 0.25} = 3.141 \text{ m}$$

12.40 A stationary hydraulic jump occurs in a rectangular channel with the initial and sequent depths being equal to 0.20 m and 1.20 m respectively. Estimate

(i) the discharge per unit width and

(ii) the energy loss

Solution:

(i) $$\frac{y_2}{y_1} = \frac{1}{2} [-1 + \sqrt{1 + 8F_1^2}]$$

$$\frac{1.20}{0.20} = \frac{1}{2} [-1 + \sqrt{1 + 8F_1^2}]$$

$$F_1^2 = 21 \quad \text{and} \quad F_1 = 4.583$$

$$F_1 = \frac{V_1}{\sqrt{9.81 \times 0.2}} = 4.583$$

$$V_1 = 6.419 \text{ m/s}$$

Discharge per unit width

$$q = V_1 y_1 = 6.419 \times 0.2 = \mathbf{1.284\ m^3/s/m}$$

(ii) The energy loss E_L is given by

$$E_L = \frac{(y_2 - y_1)^3}{4 y_1 y_2} = \frac{(1.20 - 0.20)^3}{4 \times 0.20 \times 1.20}$$

$$= \mathbf{1.042\ m}$$

12.41 If, in a hydraulic jump occurring in a horizontal rectangular channel, the Froude number before the jump is 10.0 and the energy loss is 3.20 m estimate the (i) sequent depths (ii) the discharge intensity and (iii) the Froude number after the jump.

Solution:

$$F_1 = 10.0 \quad \text{and} \quad E_L = 3.20 \text{ m}$$

The sequent depth ratio

$$\frac{y_2}{y_1} = \frac{1}{2} [-1 + \sqrt{1 + 8F_1^2}]$$

$$= \frac{1}{2} (-1 + \sqrt{1 + 8 \times (10)^2}) = 13.651$$

Energy loss $E_L = \dfrac{(y_2 - y_1)^3}{4 y_1 y_2}$

$$\frac{E_L}{y_1} = \frac{(y_2/y_1 - 1)^3}{4\,(y_2/y_1)}$$

$$\therefore \quad \frac{3.20}{y_1} = \frac{(13.651 - 1)^3}{4 \times 13.651} = 37.08$$

(i) y_1 = depth before the jump = $\dfrac{3.20}{37.08}$ = **0.0863 m**

y_2 = depth after the jump = 13.651 × 0.0863

= **1.178 m**

(ii) $$F_1 = \frac{V_1}{\sqrt{g y_1}}$$

$$10.0 = \frac{V_1}{\sqrt{9.81 \times 0.0863}}$$

$$V_1 = 9.201 \text{ m/s}$$

Discharge intensity

$$q = V_1 y_1 = 9.201 \times 0.0863$$

$$= \mathbf{0.7941\ m^3/s/m}$$

(iii) Froude number after the jump = F_2

$$F_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{q}{y_2 \sqrt{g y_2}} = \frac{0.7941}{1.178 \sqrt{9.81 \times 1.178}}$$

$$= \mathbf{0.1983}$$

12.42 For a hydraulic jump in a rectangular channel the velocity and depth after the jump are known to be 0.80 m/s and 1.75 m respectively. Calculate the depth before the jump, the energy loss and the power dissipated per metre width.

Solution:

$$F_2 = \frac{V_2}{\sqrt{g y_2}} = \text{Froude number after the jump}$$

$$= \frac{0.80}{\sqrt{9.81 \times 1.75}} = 0.1931$$

The sequent depth ratio is given in terms of F_2 as

$$\frac{y_1}{y_2} = \frac{1}{2} (-1 + \sqrt{1 + 8F_2^2})$$

$$= \frac{1}{2} (-1 + \sqrt{1 + 8 \times (0.1931)^2}) = 0.0697$$

$$y_1 = 0.0697 \times 1.75 = \mathbf{0.122\ m}$$

Energy loss $E_L = \dfrac{(y_2 - y_1)^3}{4 y_1 y_2}$

$$= \frac{(1.750 - 0.122)^3}{4 \times 0.122 \times 1.750} = \mathbf{5.054\ m}$$

Power dissipated

$$\mathbf{P} = \gamma Q E_L$$

= 9790 × (0.80 × 1.75) × 5.054 W/m

= 69270 W/metre width

= **69.27 kW/metre width**

12.43 Given the energy loss and Froude number after the jump as 9.00 m and 0.12 respectively estimate

Table 12.6 *Dimensions of Efficient Channels–Problem 12.42*

Prob.	Q (m^3/s)	B_e (m)	y_e (m)	m	V (m/s)	P_e (m)	A_e (m^2)	n	S_0
(i)	15			2.0				0.015	0.0004
(ii)				1.5	1.5			0.020	0.0009
(iii)				1.5		12.0		0.020	0.0006
(iv)				1.0			5.0	0.015	0.0004
(v)			1.2	1.5				0.018	0.001
(vi)		1.5		1.0				0.014	0.0004
(vii)	6.0			0				0.015	0.0008
(viii)	3.0	0						0.015	0.0008

Answers to Problem 12.42

Prob.	Q (m^3/s)	B_e (m)	y_e (m)	m	V (m/s)	P_e (m)	A_e (m^2)	n	S_0
(i)	15.0	0.991	2.10	2.0	1.376	10.383	10.90	0.015	0.0004
(ii)	1.256	0.84	0.20	1.5	1.50	0.842	0.837	0.020	0.0009
(iii)	30.94	1.726	2.85	1.5	1.551	12.0	19.952	0.020	0.0006
(iv)	5.87	1.370	1.654	1.0	1.175	6.047	5.0	0.015	0.0004
(v)	3.79	0.727	1.20	1.5	1.250	5.053	3.032	0.018	0.001
(vi)	8.01	1.50	1.811	1.0	1.337	6.621	5.994	0.014	0.0004
(vii)	6.0	1.190	2.380	0	2.118	9.522	2.833	0.015	0.0008
(viii)	3.0	0	1.544	1.0	1.059	4.366	2.832	0.015	0.0008

E. Specific Energy and Critical Depth

12.43 A circular channel 1.20 m in diameter is laid on a slope of 0.008. The Manning's n for the channel can be taken as 0.018. Calculate the specific energy when the depth of flow is 0.60 m.

(*Ans.* $E = 0.853$ m)

12.44 A rectangular channel 1.5 m wide carries a discharge of 9.0 m^3/s at a depth of 2.0 m. Calculate the (a) specific energy and (b) depth alternate to the given depth of 2.0 m.

(*Ans.* $E = 2.4587$ m;
$y_2 = 1.214$ m)

12.45 A rectangular channel 2.0 m wide carries a discharge of 6.0 m^3/s. If the specific energy is 1.8 m calculate the alternate depths.

(*Ans.* $y_1 = 0.625$ m; $y_2 = 1.626$ m)

12.46 In a rectangular channel 2.0 m wide the two alternate depths to a given specific energy are 0.30 m and 1.2 m respectively. Calculate the discharge.

(*Ans.* $Q = 2.446$ m^3/s)

12.47 For a constant specific energy of 3.0 m, what maximum flow may occur in a rectangular channel of 4.5 m bed width?

(*Ans.* $Q_m = 39.87$ m^3/s)

12.48 What is the lowest possible specific energy for a water discharge of 12 m^3/s to flow through a 3.0 m wide rectangular channel?

(*Ans.* $E_{min} = 1.766$ m)

12.49 A rectangular channel 12.0 m wide carries 200 m^3/s. Find the critical depth and critical velocity. What slope will produce this critical velocity in the channel if $n = 0.02$?

(*Ans.* $S_0 = 4.682 \times 10^{-3}$;
$y_c = 3.048$ m; $V_c = 5.468$ m/s)

12.50 What is the maximum discharge that may be carried by a 3.2 m wide rectangular channel at a specific energy of 1.8 m?

(*Ans.* $Q_m = 13.18$ m^3/s)

12.51 Fill in the missing data in Table 12.7, connected with critical depth computation in a rectangular channel.

Table 12.7 *Data on Calculation of y_c in a Rectangular Channel–Problem 12.51*

Prob.	Q (m^3/s)	B (m)	y_c (m)	E_c (m)
(i)			2.6	0.40
(ii)	10.48		0.70	
(iii)	6.0	2.0		
(iv)		5.0		1.2

Answers to Problem 12.51

Prob.	Q (m^3/s)	B (m)	y_c (m)	E_c (m)
(i)	5.15	2.6	0.40	0.60
(ii)	10.48	4.0	0.70	1.05
(iii)	6.0	2.0	0.917	1.376
(iv)	14.0	5.0	0.80	1.2

the formation of the jump at the toe. Manning's $n = 0.02$ and $S_o = 0.001$. Assume the channel to be a wide rectangular channel. Calculations may be performed by Direct step method taking only one step.

[*Ans:* $x = 83$ m]

[***Hint:*** Depth after the jump (y_2) = Normal depth in the channel (y_o)]

12.101 A rectangular channel is 3.0 m wide and has a bed slope of 0.001. A sluice gate in this channel issues out a flow with a depth of 0.3 m at the *vena contracta*. If a hydraulic jump takes place downstream of the gate to the normal depth of 2.0 m in the channel, estimate the location of the jump. Assume Manning's $n = 0.015$.

(*Ans.* $x = 56.5$ from *vena contracta*)

12.102 A weir is 3.0 m long and has a head of 1.2 m over the weir crest. The height of the weir crest above the channel bed is 0.8 m. Neglecting the velocity of approach, estimate the discharge if the weir is

(a) a broad crested weir with well rounded entrance.

(b) a broad crested weir with square entrance.

(c) a sharp crested weir.

[*Ans.* (i) $Q = 6.59$ m^3/s; (ii) $Q = 5.715$ m^3/s; (iii) $Q = 8.513$ m^3/s]

12.103 A broad crested weir is 2.5 m long and passes a discharge of 3.5 m^3/s under a head of 0.9 m. Neglecting the velocity of approach, estimate its coefficient of discharge.

(*Ans.* $C_d = 0.962$ m)

Substituting $V_1 = (A_2/A_1)V_2$ and simplifying, the ideal discharge (i.e. without any loss of head) is given by

$$\boxed{Q_t = \frac{A_2}{\sqrt{1-(D_2/D_1)^4}}\sqrt{2g\Delta h}}$$

Introducing the coefficient of discharge C_d to account for the losses and hence for the variation of Q_t from the actual discharge Q,

$$\boxed{Q = C_d\frac{A_2}{\sqrt{1-(D_2/D_1)^4}}\sqrt{2g\Delta h}} \qquad (13.10)$$

As the coefficient of contraction C_c for a venturimeter is 1.0, $C_d = C_v$.

The inlet head loss H_{Li} between the inlet and the throat is

$$\boxed{H_{Li} = \frac{V_2^2}{2g}\left(\frac{1}{C_d^2}-1\right)[1-(D_2/D_1)^4]} \qquad (13.11)$$

Also, it can be shown that

$$\boxed{H_{Li} = (1-C_d^2)\,\Delta h} \qquad (13.12)$$

The total head loss H_t due to the introduction of a venturimeter in a pipe is

$$H_t = H_{Li} + \text{(Loss in the expanding portion of the tube)} \qquad (13.13)$$

Usually D_2/D_1 is between 1/4 to 3/4. The coefficient of discharge C_d for a given geometry is

$$C_d = \text{fn (Reynolds number, } D_2/D_1)$$

However, in the normal designed operating range of the meter the effect of Reynolds number is very little and thus for a given tube C_d is essentially a constant:

The venturimeter is used for measuring rates of flow in both incompressible and compressible fluids.

13.4.1 Venturimeter with a Differential U-tube Manometer

Very often the change in the piezometric head between the inlet and throat of a venturimeter is measured by a differential U-tube manometer. For the general situation shown in Fig. 13.6,

$$\left(\frac{p_1}{\gamma}+Z_1\right) + x + y = y\,\frac{S_m}{S_p} + x + \left(Z_2+\frac{p_2}{\gamma}\right)$$

$$\boxed{\left(\frac{p_1}{\gamma}+Z_1\right)-\left(\frac{p_2}{\gamma}+Z_2\right) = \Delta h = y\left(\frac{S_m}{S_p}-1\right)} \qquad (13.14)$$

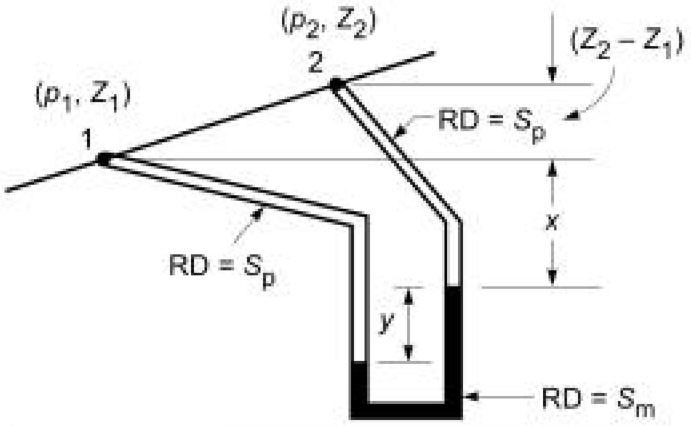

Fig. 13.6 *Differential Manometer*

In Eq. 13.14

S_m = relative density of manometric liquid

S_p = relative density of the fluid flowing in the pipe

y = reading of the differential manometer.

It is to be noted that the piezometric head difference Δh depends upon the gauge reading y regardless of the orientation of the venturimeter, whether it is horizontal, vertical or inclined, the same relation of Eq. 13.14, viz.

$$\Delta h = y\left(\frac{S_m}{S_p}-1\right)$$

holds goods.

[***Note:*** If an inverted U-tube differential manometer with $S_m < S_p$ is used then Δh is given by

$$\Delta h = y\left(1-\frac{S_m}{S_p}\right) \qquad (13.14(a)]$$

13.5 PITOT TUBE

If a small obstruction, in the form of a tube shown in Fig. 13.7, is inserted into a fluid so as to squarely face the flow, then at a point near the nose of the tube the velocity will be zero. This point is called a *stagnation*

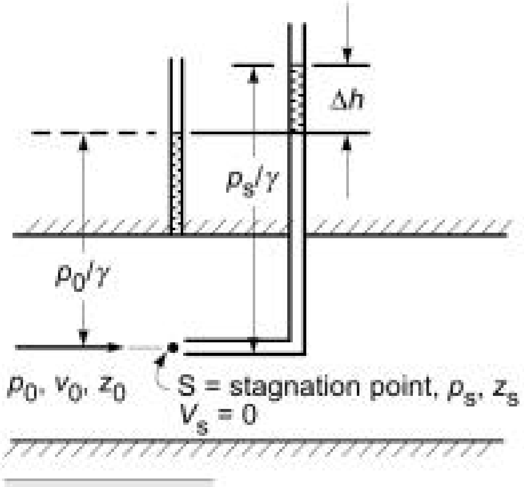

Fig. 13.7

13.7 Water at 20°C flows in a 20 cm diameter pipe in which a standard 10 cm diameter orifice meter is installed. The meter is provided with *vena contracta* taps. Calculate the discharge corresponding to a pressure head differential of 0.5 m. For water at 20°C, $\rho = 998$ kg/m^3 and $\mu = 0.001$ Pa.s. The flow coefficient K_0 of the meter for $D_1/D_2 = 0.5$ has the following variation with the pipe Reynolds number:

$\text{Re} = \dfrac{VD_1}{\nu}$	10^4	2×10^4	4×10^4	6×10^4	10^5	10^6
K_0	0.64	0.63	0.63	0.62	0.62	0.62

Solution:

Discharge $\quad Q = K_0 \cdot A_2 \sqrt{2 g \Delta H}$

$$A_2 = \text{Area of the orifice} = \frac{\pi}{4} \times (0.10)^2$$

$$= 7.854 \times 10^{-3}\ \text{m}^2$$

$$\Delta H = 0.5\ \text{m},\ \sqrt{2 g \Delta H} = \sqrt{2 \times 9.81 \times 0.5}$$

$$= 3.132\ \text{m/s}$$

As K_0 is not known to start with, a trial and error method is adopted. Let $K_0 = 0.62$

$$Q = 0.62 \times (7.854 \times 10^{-3}) \times 3.132$$

$$= \mathbf{0.01525\ m^3/s}$$

Velocity $\quad V = \dfrac{Q}{(\pi D_1^2/4)} = \dfrac{0.01525}{\left(\dfrac{\pi}{4} \times 0.20^2\right)} = 0.485$ m/s

Reynolds number

$$Re = \frac{VD_1}{\nu} = \frac{0.485 \times 0.20}{(0.001/998)} = 96806 = 10^5$$

For this value of Re, the flow coefficient $K_0 = 0.62$ which is the same as the assumed value. Hence no more iterations are required. The discharge $Q = 0.01525$ m^3/s = 15.25 L/s.

13.8 A liquid with a relative density of 0.70 flows through a 7.5 cm diameter pipe A 3.0 cm diameter ISA flow nozzle is inserted at a certain location to measure the flow. If the flow coefficient of the nozzle is 0.99, calculate the piezometric head difference across the nozzle for a discharge of 20 L/s. If the instrument is horizontal, what is the pressure difference across the nozzle?

Solution:

Discharge $\quad Q = K_f A_2 \sqrt{2 g \Delta H}$

$$Q = 0.002\ \text{m}^3/\text{s},\ K_f = 0.99$$

$$A_2 = \text{area of the nozzle} = \frac{\pi}{4} \times (0.03)^2$$

$$= 7.0686 \times 10^{-4}$$

$$0.002 = 0.99 \times 7.0686 \times 10^{-4} \sqrt{2 \times 9.81 \times \Delta H}$$

ΔH = piezometric head difference across the nozzle = 0.4163 m

$$\Delta H = \left(\frac{p_1}{\gamma} - \frac{p_2}{\gamma}\right) + (Z_1 - Z_2)$$

For a horizontal nozzle $Z_1 - Z_2 = 0$. Hence

$$\Delta p = (p_1 - p_2) = \gamma \Delta H$$

$$= 9.79 \times 0.70 \times 0.4163 = \mathbf{2.853\ kPa}$$

13.9 An external cylindrical mouthpiece is fitted to the side of a tank containing water. Obtain an expression for the minimum pressure developed in the mouthpiece if the coefficient of contraction is 0.62. At what head over the mouthpiece would cavitation occur in the mouthpiece?

Assume local atmospheric pressure = 96.48 kPa (abs) and vapour pressure = 4.0 kPa (abs)

Solution: A mouthpiece is a short tube fitted in place of an orifice. The flow in a mouthpiece first contracts to a *vena contracta* as in an orifice (Section C-C in Fig. 13.19 and then it expands to fill the tube of area a. The loss of head in the tube is essentially due to sudden expansion from the contracted section C-C to full pipe section (as at 2-2). Hence,

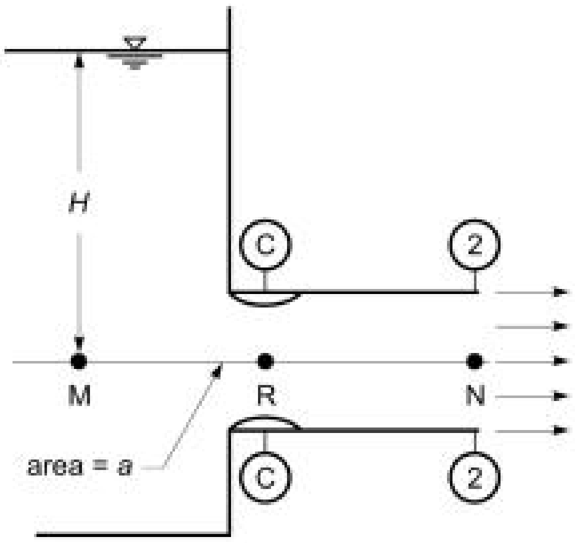

Fig. 13.19

Head loss $\quad H_L = (V_c - V_2)^2/2g$

But $\quad Q = V_c a_c = V_2 a$

Also $\quad a_c = C_c\, a$

Thus $\quad V_c = \dfrac{V_2}{C_c}$

$$H_L = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2 = \frac{V_2^2}{2g}\left(\frac{1}{0.62} - 1\right)^2$$

$$= 0.376\ \frac{V_2^2}{2g}$$

In the present case, $S_m = 13.6$, $S_p = 0.9$

$$\therefore \quad \Delta h = y\left(\frac{13.6}{0.9} - 1\right) = 14.11\, y$$

$$A_2 = \frac{\pi}{4} \times (0.1)^2 = 7.854 \times 10^{-3} \text{ m}^2$$

$$\frac{D_2}{D_1} = 0.5, \quad \sqrt{1-(D_2/D_1)^4} = 0.96825$$

$$Q = \frac{0.99 \times 7.854 \times 10^{-3}}{0.96825} \times \sqrt{2 \times 9.81 \times 14.11 y}$$

$$Q = 0.1336\sqrt{y}$$

(i) When $y = 9$ cm $= 0.09$ m, $Q = 0.1336\sqrt{0.09}$

$= 0.040$ m^3/s $=$ **40.0 L/s**

(ii) When $Q = 50$ L/s $= 0.050$ m^3/s

$$0.050 = 0.1336\sqrt{y}$$

$y = 0.14$ m $=$ **14 cm**

13.19 Determine the maximum discharge of water that can be carried without cavitation by a horizontal 100 mm × 50 mm venturimeter, which has a coefficient of discharge of 0.95. The inlet pressure is 10 kPa (gauge), the vapour pressure of water is 4 kPa (abs) and the local atmospheric pressure is 96.0 kPa (abs).

Solution: Cavitation occurs when the local pressure reaches the vapour pressure. Hence, for maximum discharge the pressure at throat $p_2 = p_v = 4.0$ kPa (abs)

Inlet pressure (abs)

= 10 kPa (gauge) + atmospheric pressure

$p_1 = 10.0 + 96.0 = 106.0$ kPa (abs)

For a horizontal venturimeter $Z_1 = Z_2$

$$\therefore \quad \Delta h = \left(\frac{p_1}{\gamma} + Z_1\right) - \left(\frac{p_2}{\gamma} + Z_2\right) = \frac{p_1 - p_2}{\gamma}$$

$$= \frac{106.0 - 4.0}{9.79} = 10.419 \text{ m}$$

The venturimeter discharge equation is

$$Q = \frac{C_d A_2}{\sqrt{1-(D_2/D_1)^4}} \times \sqrt{2 g \Delta h}$$

Here $C_d = 0.95$, $A_2 = \frac{\pi}{4} \times (0.05)^2 = 1.9635 \times 10^{-3}$ m^2

$$\frac{D_2}{D_1} = \frac{50}{100} = 0.5,$$

$$\sqrt{1-(D_2/D_1)^4} = \sqrt{1-(0.5)^4} = 0.96825$$

$$\therefore \quad Q = \frac{0.95 \times 1.9635 \times 10^{-3}}{0.96825} \times \sqrt{2 \times 9.81 \times 10.419}$$

$= 0.0275$ m^3/s

= 27.5 L/s (maximum discharge)

13.20 Show that the head loss between inlet and throat of a venturimeter can be expressed in terms of the throat velocity head as

$$H_{Li} = \left(\frac{1}{C_d^2} - 1\right)\left(1 - \left(\frac{D_2}{D_1}\right)^4\right)\frac{V_2^2}{2g}$$

Solution: First consider the situation without losses

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_{2t}^2}{2g} + Z_2$$

where V_{2t} = theoretical velocity at the throat

$$\text{But} \left(\frac{p_1}{\gamma} + Z_1 - \frac{p_2}{\gamma} - Z_2\right) = \Delta h$$

= difference in piezometric heads.

$$\text{Also} \qquad V_1 D_1^2 = V_2 D_2^2$$

$$\frac{V_{2t}^2}{2g}\left(1 - \left(\frac{D_2}{D_1}\right)^4\right) = \Delta h$$

$$\text{or} \qquad V_{2t} = \frac{1}{\sqrt{1-(D_2/D_1)^4}} \times \sqrt{2 g \Delta h}$$

Now let us consider the actual flow:

The actual discharge

$$Q = \frac{C_d A_2}{\sqrt{1-(D_2/D_1)^4}} \times \sqrt{2 g \Delta h} = V_2 A_2$$

$\therefore \qquad V_2$ = actual velocity at section 2

$$= \frac{C_d\sqrt{2g\Delta h}}{\sqrt{1-(D_2/D_1)^4}} = C_d V_{2t}$$

Hence here $C_d = C_v$ = coefficient of velocity

$$\text{Head loss} \quad H_{Li} = \frac{p_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} - \frac{p_2}{\gamma} - Z_2 - \frac{V_2^2}{2g}$$

$$= \Delta h + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$\text{Substituting } \Delta h = [1 - (D_2/D_1)^4]\,\frac{1}{C_d^2}\frac{V_2^2}{2g}$$

$$\text{and} \qquad \frac{V_1^2}{2g} = \frac{V_2^2}{2g}\left(\frac{D_2}{D_1}\right)^4$$

$$H_{Li} = \frac{V_2^2}{2g}\left[\frac{1}{C_d^2}\left\{1 - \left(\frac{D_2}{D_1}\right)^4\right\} + \left(\frac{D_2}{D_1}\right)^4 - 1\right]$$

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(a) static pressure
(b) dynamic pressure
(c) sum of the static and dynamic pressures
(d) piezometric head

13.26 A static tube is used to measure
(a) the velocity
(b) undisturbed fluid pressure
(c) the total head
(d) datum head

13.27 The pitot-static tube measures
(a) the dynamic pressure
(b) the static pressure
(c) the total head
(d) the difference in static and dynamic pressures

13.28 To measure static pressure in a pipe, one uses a pressure gauge connected to a
(a) pitot tube
(b) venturimeter
(c) orifice meter
(d) piezometer tapping.

13.29 Preston tube is a device to measure
(a) boundary pressure
(b) mean velocity of flow
(c) boundary shear stress
(d) turbulent velocity fluctuation

13.30 A Pitot-static tube indicates a differential head of 0.75 m of water between its two openings when inserted in a stream of water. If the coefficient of the tube is 0.99, the velocity in m/s, at the location of the tube is
(a) 4.43 (b) 0.78
(c) 3.84 (d) 3.80

13.31 A Pitot tube (coefficient = 1.0) is used to measure the velocity of air mass density 1.2 kg/m^3. If the head difference in a vertical U-tube fitted with water is 12 mm, then the velocity of air in m/s is
(a) 10 (b) 14
(c) 17 (d) 20

13.32 A hot-wire anemometer is used to measure essentially the
(a) wind speed over ground surfaces
(b) turbulent velocity fluctuations in a flow
(c) shear stress on a boundary
(d) drag force on a body

13.33 A laser-doppler anemometer (LDA) is a device to measure
(a) the turbulent velocity fluctuations in a flow
(b) shear stress at a boundary
(c) drag force on an airfoil
(d) surface tension of a fluid

13.34 The discharge per metre length over a suppressed rectangular weir of a crest height 10 cm above the bed under a head of 10 cm, in L/s/m, is
(a) 73 (b) 59
(c) 106 (d) 64

13.35 The discharge, in L/min, in a 90° V-notch having a $C_d = 0.58$ under a head of 0.10 m, is
(a) 260 (b) 310
(c) 130 (d) 173

13.36 The coefficient of discharge by Rehbock formula of a sharp crested suppressed rectangular weir of crest height P and passing a free flow with a head H_1 is $C_d =$

(a) $0.611 + 0.8 \dfrac{H_1}{P}$

(b) $0.611 + 0.075 \dfrac{H_1}{P}$

(c) $0.602 + 0.075 \dfrac{P}{H_1}$

(d) $0.675 + 0.0075 \dfrac{H_1}{P}$

13.37 The coefficient of discharge of a suppressed rectangular weir at the limit of application of Rehbock formula is
(a) 0.786 (c) 0.886
(c) 0.986 (d) 1.06

13.38 A suppressed sharp crested weir is 0.6 m high and discharges water at a head of 1.2 m. The coefficient of discharge of this weir is
(a) 0.611 (b) 0.701
(c) 0.736 (d) 0.761

13.39 The Rehbock formula for estimating the C_d of a suppressed rectangular weir is valid up to H_1/P
(a) ≤ 0.50 (b) ≤ 5.0
(c) ≤ 10.0 (d) > 20

13.40 The discharge over a suppressed sharp crested rectangular weir is given by

$Q = \dfrac{2}{3} C_d \sqrt{2g}\ L H_1^{3/2}$. Here H_1 is

(a) the difference in elevation between the upstream water surface and the energy line
(b) the difference in elevation between the energy line and the weir crest
(c) the difference in elevation of the water surface and the weir crest
(d) the difference in elevation between the water surface and the bottom of the channel.

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B. Venturimeter

13.19 A vertical venturimeter 15 cm × 10 cm installed in a pipe carrying water downwards shows the same pressure at the inlet and at the throat. The throat is 25 cm below the inlet. If C_d of the meter is 0.95, calculate the discharge in the pipe.

(*Ans.* Q = 18.4 L/s)

13.20 A horizontal venturimeter 9 cm × 4 cm is installed in a 9 cm water pipe. A differential water-mercury manometer reads 30 cm. If the coefficient C_d of the meter is 0.96, estimate the (i) discharge in the pipe and (ii) head loss in the converging section of the meter.

(*Ans.* Q = 10.6 L/s, H_L = 0.296 m)

13.21 A venturimeter is used for the measurement of discharge of water in a horizontal pipeline. The upstream diameter is 300 mm, the throat is of 150 mm diameter and the difference in pressure between the inlet and throat is 3 m head of water. If the loss of head through the converging section of the meter is one-eighth of the throat velocity head, calculate the discharge in the pipe.

(*Ans.* Q = 131.5 L/s)

13.22 Oil of relative density 0.9 flows in a 10 cm pipe. A 10 cm × 5 cm venturimeter (C_d = 0.97) in the pipe exhibits a reading of 12 cm in a mercury-oil differential manometer. Calculate the head loss in the inlet to throat region and estimate the discharge.

(*Ans.* H_L = 0.10 m, Q = 11.33 L/s)

13.23 Crude oil (relative density = 0.85) flows upwards at a volumetric rate of 60 L/s through a vertical venturimeter with an inlet diameter of 200 mm and a throat diameter of 100 mm. The coefficient, of discharge of the venturimeter is 0.98. The vertical distance between the pressure tappings is 300 mm.

(i) Determine the difference in the readings of pressure gauges connected to the inlet and throat sections.

(ii) If a differential U-tube mercury-oil manometer is used to connect the two tappings determine the manometer reading.

[*Ans.* (i) Δp = 26.66 kPa; (ii) y = 19.36 cm]

13.24 A 20 cm × 10 cm venturimeter with C_d = 0.96 carries 30 L/s of water. A differential gauge has an indicator liquid M and the manometer reading is 1.16 m. What is the relative density of the manometer liquid M?

(*Ans.* S_m = 1.75)

13.25 A 15 cm × 7.5 cm venturimeter has C_d = 0.96. If the loss of head between inlet and throat is 0.78 cm calculate the discharge

(*Ans.* Q = 6.12 L/s)

13.26 A 20 cm × 10 cm venturimeter (C_d = 0.96) carries water. A differential mercury water manometer shows a reading of 10 cm. (i) Calculate the discharge. (ii) If the loss of head in the expansion part of the meter is taken as eight times the velocity head in the pipe, calculate the total head loss due to the venturimeter.

[*Ans.* (i) Q = 38.7 L/s; (ii) H_L = 0.7178 m]

13.27 A 20 cm × 15 cm venturimeter is fixed vertically in a pipe carrying water. The throat is 45 cm above the inlet. If the inlet pressure is 103 kPa (abs), what is the maximum discharge that can be passed through the pipe without cavitation occurring in the meter? The vapour pressure of water = 3.5 kPa (abs). The C_d of the meter is 0.95.

(*Ans.* Q_{max} = 280 L/s)

13.28 A 15 cm diameter pipe has a discharge of 60 L/s of water and the pressure head at a section is 5 m (gauge). If a horizontal venturimeter is to be installed at this section, what is the minimum diameter to be adopted to ensure that the pressure head at the throat does not go below 2.3 m (abs)? The atmospheric pressure is 10.3 m. Assume C_d = 0.95.

(*Ans.* D_2 = 7.19 cm)

13.29 A venturimeter with a throat diameter of 12 cm is inserted in a pipe of 20 cm diameter. The pipe carries a liquid of relative density 0.65. The differential mercury-liquid U-tube manometer connected to the throat and the inlet records a readings of 4.0 cm. If the head loss between the inlet and the throat is 8% of the velocity head in the throat, estimate (a) discharge and (b) the coefficient of discharge of the venturimeter.

(*Ans.* (a) Q = 45.9 L/s; (b) C_d = 0.957)

C. Pitot Tube

13.30 A pitot tube is inserted into an air stream at 95.0 kPa (abs). If a mercury-air manometer indicates a positive gauge pressure with a reading of 20 mm, calculate the velocity of air. ρ_{air} = 1.22 kg/m^3 and P_{atmos} = 100 kPa. Assume the pitot tube coefficient C = 0.98.

(*Ans.* V = 109.8 m/s)

13.31 The velocity of an oil flow (RD = 0.90) was measured by a pitot-static tube. The instrument had a

14

Unsteady Flow

CONCEPT REVIEW

14.1 INTRODUCTION

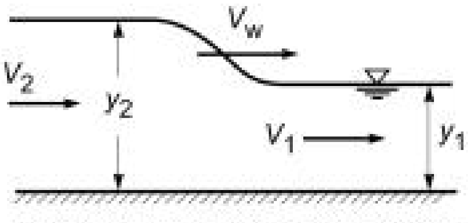
A flow is unsteady when flow parameters such as discharge, velocity, pressure vary with time. In unsteady flows additional forces due to time variation of velocity and momentum come into play and the problem becomes more difficult to solve. Typical examples of unsteady flow are: (i) emptying of a tank containing liquid through an orifice (Chapter 13), (ii) mass oscillation of a liquid column, (iii) gradual establishment of flow in a long pipe due to sudden opening of a valve, (iv) water hammer pressure phenomenon due to sudden closure of a valve in a pipeline, (v) surges in open channels and, (vi) passage of flood waves in rivers. In this chapter only three kinds of unsteady phenomenon are dealt with in detail. These are: (a) Surges in prismatic channels, (b) Water hammer phenomenon in pipes and (c) Establishment of flow in pipes due to sudden opening of a valve. The aspects relating to emptying of vessels through orifices or weirs is already dealt with in Chapter 13 and is not repeated here. For details on other types of unsteady flows the reader is referred to standard texts on the subject.

14.2 SURGES IN OPEN CHANNELS

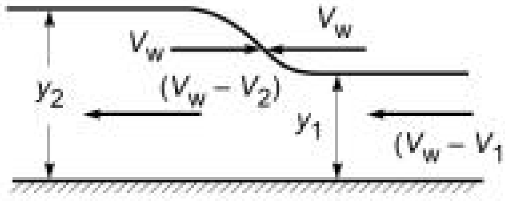
Whenever there is a sudden change in the discharge or depth or both in an open channel, a rapidly varied unsteady phenomenon, known as surge, develops. Such situations occur during sudden operation of a control gate. A surge producing an increase in depth is known as *positive surge* and the one which causes a decrease in the depth is known as *negative surge*. Positive surges have steep fronts, more like a hydraulic jump, and the shape of the wave does not change during its translation. They are also known as *moving hydraulic jumps*. These are relatively easy to analysis than negative surges. In this section only positive surges are considered.

14.2.1 Positive Surge Moving Downstream

Figure 14.1 shows a horizontal, frictionless rectangular channel in which a positive surge in moving downstream. Suffix 1 refers to the conditions before the arrival of the surge and suffix 2 refers to a section after the passage of the surge. The absolute velocity of the surge is V_w and is assumed constant. The unsteady flow situation can be simulated to an equivalent steady state flow by superimposing a velocity $(-V_w)$, (directed to the left in Fig. 14.1 (a)) at all sections [Fig. 14.1(b)]. The conditions are now



Positive surge moving downstream
(a)

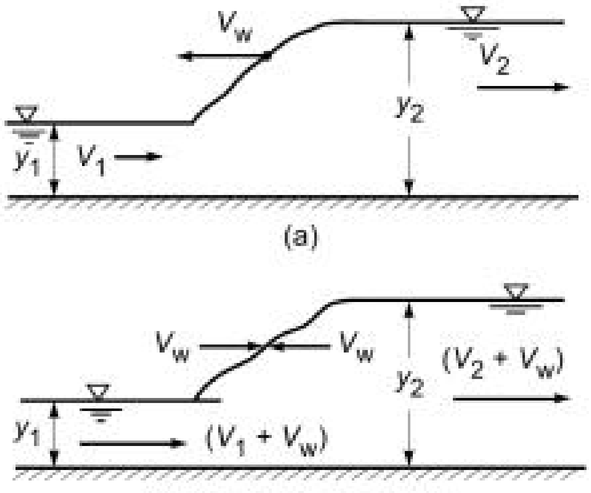


Simulated steady flow
(b)

Fig. 14.1

14.3 A 10 m wide rectangular channel supplies 35 m^3/s of water at a normal depth of 2.0 m to a turbine installation. Due to a major reduction in load, the supply to the turbine is cut down to 5 m^3/s only Determine the velocity of propagation of the surge wave and also the height of the surge.

Solution: A relative steady flow situation is simulated by superimposing a velocity V_w to the right on the system (Fig. 14.8).



(a)

(b) Simulated steady flow

Fig. 14.8 *Positive Surge Moving Upstream*

Here $y_1 = 2.0$ m

$$V_1 = \frac{35.0}{2.0 \times 10.0} \ 1.75 \text{ m/s}$$

Also $Q_2 = 10 \times y_2\, V_2 = 5.0$ m^3/s

$$V_2\, y_2 = 0.5 \text{ m}^3\text{/s/m}$$

$$V_2 = 0.5/y_2$$

Also, by the continuity equation

$$y_1\,(V_w + V_1) = y_2\,(V_w + V_2)$$

$$2.0\,(V_w + 1.75) = y_2\, V_w + 0.5$$

$$V_w\,(2.0 - y_2) = -3.0$$

$$V_w = \left(\frac{3.0}{y_2 - 2.0}\right)$$

From the surge equation, (Eq. 14.5),

$$\frac{(V_w + V_1)^2}{g y_1} = \frac{1}{2}\frac{y_2}{y_1}\left(\frac{y_2}{y_1} + 1\right)$$

Substituting for V_w,

$$\frac{\left(\dfrac{3.0}{y_2 - 2.0} + 1.75\right)^2}{9.81 \times 2.0} = \frac{1}{2}\frac{y_2}{2.0}\left(\frac{y_2}{2.0} + 1\right)$$

$$\left(\frac{3.0}{y_2 - 2.0} + 1.75\right)^2 = 2.4525\ y_2(y_2 + 2.0)$$

By trial and error, $y_2 = 2.763$ m.

$$V_2 = \frac{0.5}{2.763} = 0.181 \text{ m/s}$$

$$V_w = \frac{3.0}{y_2 - 2.0} = \frac{3.0}{2.763 - 2.0}$$

$$= 3.932 \text{ m/s in simulated flow.}$$

The surge moves *upstream* with a velocity.

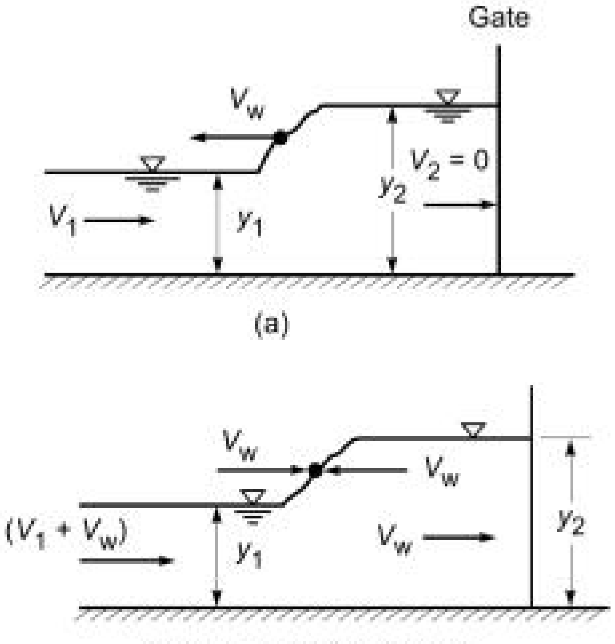
$$\mathbf{V_w = 3.932\ m/s}$$

$$\text{Height of surge} = y_2 - y_1 = 2.763 - 2.000$$

$$= \mathbf{0.763\ m.}$$

14.4 Water flows at a depth of 0.8 m at a velocity of 0.9 m/s in a wide rectangular channel in a reach immediately upstream of a control gate. The gate is suddenly closed completely. Determine the resulting surge conditions.

Solution: Let V_w = velocity of the surge.
Superimpose a velocity V_w to the right to simulate the steady flow conditions as in Fig. 14.9.



(a)

(b) Simulated steady flow

Fig. 14.9

Here $V_2 = 0$

$$y_1 = 0.8 \text{ m}$$

$$V_1 = 0.9 \text{ m/s}$$

By continuity equation,

$$y_1\,(V_1 + V_w) = y_2\,(V_2 + V_w)$$

$$0.8\,(0.9 + V_w) = y_2\,(0 + V_w)$$

$$V_w = \frac{0.72}{(y_2 - 0.8)}$$

By the surge equation, (Eq. 14.5),

$$\frac{(V_1 + V_w)^2}{g y_1} = \frac{1}{2}\frac{y_2}{y_1}\left(1 + \frac{y_2}{y_1}\right)$$

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reduces the velocity from 3.0 m/s to 0.5 m/s in 5 s, estimate the total pressure on the pipe walls at the valve.

For water:

$K = 2.10 \times 10^9$ Pa; for steel; $E = 2.11 \times 10^{11}$ Pa.

Solution: Velocity of pressure wave,

$$C = \sqrt{K/\rho}\left[\frac{1}{1+(D/t)(K/E)}\right]^{1/2}$$

$$= \left(\frac{2.10 \times 10^9}{998}\right)^{1/2}$$

$$\left[\frac{1}{1+\left(\frac{400}{10}\right)(2.10 \times 10^9/2.11 \times 10^{11})}\right]^{1/2}$$

$$= 1450.6 \times 0.8457 = 1227 \text{ m/s}$$

Critical time for rapid closure

$$T_0 = 2L/C = (2 \times 2500)/1227 = 4.076 \text{ s}$$

Actual time of closure $T = 5.0$ and $T/T_0 = 5.0/4.076 = 1.227$ it is a slow closure case.

For a rapid closure:

$$p_{\text{hr}} = \text{water hammer pressure} = -\rho C \Delta V$$

$$= -998 \times 1227 \times (0.5 - 3.0)$$

$$p_{\text{hr}} = 3.061 \times 10^6 \text{ Pa} = 3061 \text{ kPa}$$

At slow closure, pressure rise:

$$p_{\text{hs}} = \frac{T_0}{T}\, p_{\text{hr}} = \frac{4.076}{5.0} \times 3061 = 2495 \text{ kPa}$$

Total pressure at the valve end

$$p_{\text{t}} = p_{\text{static}} + p_{\text{hs}}$$

$$= (9.79 \times 250) + 2495 = \mathbf{4943\ kPa}$$

14.20 A valve at the downstream end of a 3500 m long pipeline is closed in 4.0 s. The pipe carries water flowing at a velocity of 0.8 m/s. Assuming the pipe to be rigid, calculate (i) the peak water hammer pressure produced by this closure and (ii) the length of the pipe subjected to the above peak pressure at the end of the closure time. [$K_{\text{water}} = 2.20 \times 10^3$ MPa].

Solution: As the pipe is rigid, velocity of pressure wave

$$C = \sqrt{K/\rho} = \left(\frac{2.2 \times 10^9}{998}\right)^{1/2} = 1484.7 \text{ m/s}$$

Critical time

$$T_0 = \frac{2L}{C} = \frac{2 \times 3500}{1484.7} = 4.715 \text{ s}$$

As T = time of closure = 4.0 s, the closure is rapid. Hence the water hammer pressure

$$p_{\text{h}} = \rho C V_0 = 998 \times 1484.7 \times 0.8 \text{ Pa}$$

$$= \mathbf{1.185\ MPa}$$

Length of the pipe, from the valve end, affected by this peak pressure, during the closure time.

$$x_0 = L - \frac{CT}{2}$$

$$= 3500 - \frac{1484.7 \times 4.0}{2} = \mathbf{530.6\ m}$$

14.21 A 2800 m long pipeline conveys water at a velocity of 2.0 m/s. If a valve at the downstream end is closed suddenly, calculate the maximum pressure rise. Sketch the variation of the water hammer pressure with time at (i) the valve end (ii) midpoint B of the pipe and (iii) at point C 2100 m upstream of the valve. The pipe starts from a lake at the upstream end. Assume the pipe to be rigid. For water, take $K = 1.956 \times 10^9$ Pa.

Solution: $$C = \sqrt{K/\rho} = \left(\frac{1.956 \times 10^9}{998}\right)^{1/2} = 1400 \text{ m/s}$$

Critical time $T_0 = \dfrac{2L}{C} = \dfrac{2 \times 2800}{1400} = 4$ s

Period of pressure fluctuation: $2T_0 = 8$ s

Water hammer pressure

$$p_{\text{h}} = \rho C V_0 = 998 \times 1400 \times 2.0 \text{ Pa}$$

$$= \mathbf{2.794\ MPa}$$

(i) Figure 14.16 (a) shows the variation of the water hammer pressure p_{h}, at the valve V with the static pressure at V as datum.

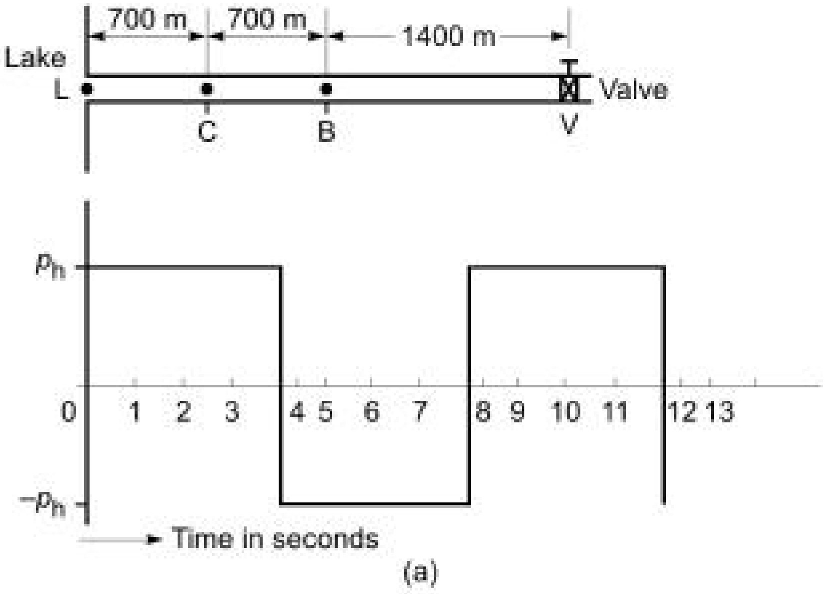

Fig. 14.16(a) *Variation of p_h at Valve V*

Time taken for p_{h} to reach the lake = 2800/1400 = 2 s

Time taken for the reflected wave to reach V = 2 s

Hence from 0 to 4 s, the pressure at V will be + p_{h} and from 4 to 8 s, it will be − p_{h} and from 8 to 12 s, it will be + p_{h}

The gas constant R is given by

$$\boxed{R = \frac{8314}{M_g}\ \text{J/(kg}\cdot\text{K)}} \qquad (15.2)$$

where M_g = molecular weight of gas. For air $M_g = 28.97$ and R = 287 J/(kg·K).

Further $$\boxed{R = c_p - c_v} \qquad (15.3)$$

where c_p = specific heat at constant pressure
and c_v = specific heat at constant volume.
The ratio $c_p/c_v = k$ is an important thermodynamic property of a gas. For air and other diatomic gases $k = 1.4$. From Eq. 15.3

$$\boxed{c_v = \frac{R}{k-1}} \qquad (15.4a)$$

and $$\boxed{c_p = \frac{kR}{k-1}} \qquad (15.4b)$$

Note that the units of R, c_p and c_v are J/(kg·K).
Since 1 Joule (J) = N·m = (kg·m/s^2) (m) the unit J/(kg·K) = m^2/(s^2·K).
If R is to be expressed in heat units, the relevant conversion is 1 kcal = 4812 J.
Thus for air

$$R = 287\ \text{J/(kg}\cdot\text{K)} = \frac{287}{4812} = 0.0596\ \text{kcal/(kg}\cdot\text{K)}$$

Table 15.1 gives the values of M_g, ρ, R, c_v, c_p and k for some common gases.
(2) Another fundamental equation for a perfect gas is

$$\boxed{\frac{p}{\rho^k} = pv^k = \text{constant}} \qquad (15.4)$$

Thus by Eqs 15.4 and 15.1, between two sections 1 and 2,

$$\frac{p_1}{\rho_1^k} = \frac{p_2}{\rho_2^k} \quad \text{and} \quad \left(\frac{T_2}{T_1}\right) = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \qquad (15.5)$$

15.3 BASIC DEFINITIONS

Internal energy, u, is the energy of unit mass of fluid due to molecular activity. Change of internal energy

$$\boxed{u_2 - u_1 = c_v\,(T_2 - T_1)} \qquad (15.6)$$

Thus $$\boxed{c_v = \left(\frac{\partial u}{\partial T}\right)_{v\,=\,\text{constant}}} \qquad (15.7)$$

Enthalpy, h

The energy possessed by a unit mass of a gas by virtue of its absolute temperature under which it exists is known as enthalpy. It is represented as a sum of pressure per unit mass (p/ρ) and internal energy per unit mass (u) as

$$\boxed{h = u + p/\rho} \qquad (15.8)$$

For a perfect gas the change in enthalpy

$$\Delta h = c_p\Delta T (15.9)$$

Thus $$\boxed{c_p = \left(\frac{\partial h}{\partial T}\right)_{p\,=\,\text{constant}}} \qquad (15.10)$$

Entropy, s

Entropy is defined as a measure of the availability of energy for conversion into mechanical work. The entropy change ds for a perfect gas is

$$T\text{d}s = \text{d}h - \frac{\text{d}p}{\rho} \qquad (15.11)$$

Table 15.1 *Properties of Common Gases at 1 atm. and 20°C*

Gas	*Formula*	*Mol. Weight* M_g	ρ *kg/m*^{3}	R *J/(kg·K)*	c_p *J/(kg·K)*	c_V *J/(kg·K)*	k
Air		29	1.205	287	1003	716	1.40
Carbon dioxide	CO_2	44	1.84	188	858	670	1.28
Carbon monoxide	CO	28	1.16	297	1040	743	1.40
Oxygen	O_2	32	1.33	260	909	649	1.40
Nitrogen	N_2	28	1.16	297	1040	743	1.40
Chlorine	Cl_2	71	2.946	117	461	344	1.34
Methane	CH_4	16	0.668	520	2250	1730	1.30
Helium	He	4	0.166	2077	5220	3143	1.66
Hydrogen	H_2	2	0.0839	4120	14450	10330	1.40

$$\rho_0 = \frac{p_0}{RT_0} = \frac{300{,}000}{2077 \times 263}$$

$$= 0.549 \text{ kg/m}^3$$

Under conditions of maximum flow, critical conditions will prevail at the nozzle throat. At critical condition

$$M_1 = M_1^* = 1.0 \qquad \text{Hence } C_1^* = V_1^*$$

$$\frac{p_1^*}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)} = \left(\frac{2}{1+1.66}\right)^{1.66/(1.66-1)}$$

$$= (0.7519)^{2.515} = 0.488$$

$$p_1^* = 0.488 \times 300 = 146.43 \text{ kPa (abs)}$$

The receiver pressure p_2 should be less than or equal to $p_1^* = 146.43$ kPa (abs) for maximum rate of flow condition. Hence maximum downstream receiver pressure possible $= p_1^* = 146.43$ kPa (abs)

$$\frac{T_1^*}{T_0} = \left(\frac{2}{k+1}\right) = (0.7519)$$

$$T_1^* = 263 \times 0.7519 = 197.7 \text{ K}$$

$$\frac{\rho_1^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{1/(k-1)} = (0.7519)^{1/0.66} = 0.649$$

$$\rho_1^* = 0.549 \times 0.649 = 0.3565 \text{ kg/m}^3$$

$$V_1^* = C_1^* = \text{Speed of sound} = \sqrt{kRT_1^*}$$

$$= \sqrt{1.66 \times 2077 \times 197.7} = 825.6 \text{ m/s}$$

Area of nozzle

$$A_1 = \frac{\pi}{4} \times (0.02)^2 = 3.1416 \times 10^{-4} \text{ m}^2$$

Maximum mass rate of flow under the flow conditions prevailing

$$\dot{m}_{\max} = \rho_1^* A_1 V_1^*$$

$$= 0.3565 \times 3.1416 \times 10^{-4} \times 825.6$$

$$= \mathbf{0.0925 \text{ kg/s}}$$

E. Energy Equation in Isentropic flow

15.20 A duct conveys an isentropic flow of air. At section 1, velocity, pressure and temperature are 100 m/s, 200 kPa (abs) and 70°C respectively. At another downstream section (Section 2) the velocity is 250 m/s. Calculate (i) the temperature at section 2, (ii) Mach number and pressure at section 2 and (iii) the densities at sections 1 and 2 [Take $k = 1.4$ and $R = 287$ J/(kg·K)]

Solution:

$$T_1 = 273 + 70 = 343 \text{ K}$$

$$V_1 = 100 \text{ m/s and } V_2 = 250 \text{ m/s}$$

$$p_1 = 200 \text{ kPa (abs)}$$

$$c_p = \frac{k}{k-1} R = \frac{1.4}{0.4} \times 287 = 1004.5$$

(i) $V_2^2 - V_1^2 = 2c_p(T_1 - T_2)$

$$(250)^2 - (100)^2 = 2 \times 1004.5\,(343 - T_2)$$

$$T_2 = 316.87 \text{ K} = \mathbf{43.87°C}$$

Sonic speed at section $2 = C_2 = \sqrt{kRT_2}$

$$C_2 = \sqrt{1.4 \times 287 \times 316.87}$$

$$= 356.82 \text{ m/s}$$

(ii) Mach number at Section $2 = M_2 = \dfrac{V_2}{C_2}$

$$= \frac{250}{356.82} = \mathbf{0.70}$$

(iii) At Section 1, $V_1 = 100$ m/s

Sonic speed $= C_1 = \sqrt{kRT_1} = \sqrt{1.4 \times 287 \times 343}$

$$= 371.24 \text{ m/s}$$

Mach number $M_1 = V_1/C_1 = \dfrac{100}{371.24} = 0.2694$

Stagnation pressure $= p_{01}$

$$p_{01} = p_1\left(1 + \frac{k-1}{2} M_1^2\right)^{k/(k-1)}$$

$$= 200\,[1 + 0.2 \times (0.2694)^2]^{3.5} \text{ kPa (abs)}$$

$$= 210.3 \text{ kPa (abs)} = p_{02}$$

At Section 2:

$$V_2 = 250 \text{ m/s}$$

$$M_2 = 0.70$$

Stagnation pressure $= p_{02}$

$$\frac{p_{02}}{p_2} = \left(1 + \frac{k-1}{2} M_2^2\right)^{k/(k-1)}$$

$$\frac{p_{02}}{p_2} = [1 + 0.2 \times (0.7)^2]^{3.5} \text{ kPa (abs)}$$

$$= 1.3871$$

$$p_2 = \frac{210.3}{1.3871} = \mathbf{151.6 \text{ kPa (abs)}}$$

Densities:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{200{,}000}{287 \times 343} = \mathbf{2.0317 \text{ kg/m}^3}$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{151600}{287 \times 316.87} = \mathbf{1.667 \text{ kg/m}^3}$$

[***Note:***

1. Check on calculation —

$$\rho_{01} = \text{Stagnation density at 1}$$

$$= \rho_1\left(1 + \frac{k-1}{2} M_1^2\right)^{1/(k-1)}$$

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You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

are: (i) centrifugal pumps, (ii) mixed flow pumps, and (iii) axial flow pumps. This classification is based on the direction of flow of the fluid in the impeller. Figure 16.6 shows a typical centrifugal pump and a typical casing.

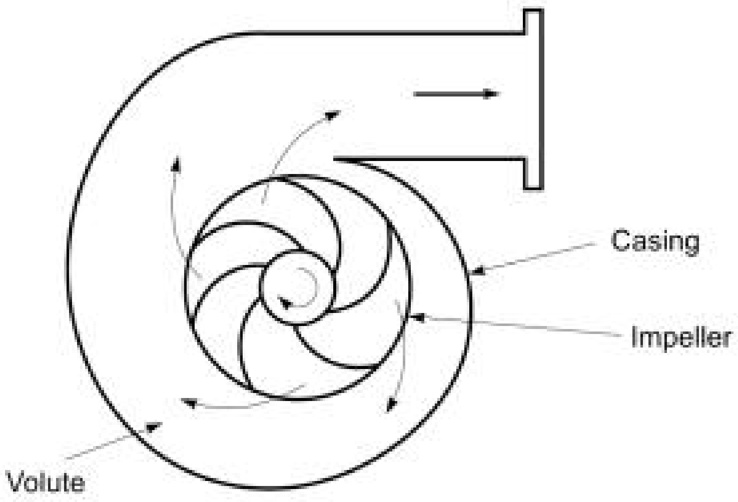

Fig. 16.6 *Centrifugal Pump*

Net Head Developed, H

In a pump the mechanical energy through the shaft and impeller is converted to fluid energy. The difference between the total energy heads at the intake and discharge flanges of the pump is denoted as net head H developed in the pump. The intake end is commonly called the *suction end* and the discharge end as the *delivery end*. Denoting these by suffixes s and d respectively (see Fig. 16.7) the net head $H = H_d - H_s$

or
$$H = \left(\frac{p_d}{\gamma} + Z_d + \frac{V_d^2}{2g}\right) - \left(\frac{p_s}{\gamma} + Z_s + \frac{V_s^2}{2g}\right) \quad (16.30)$$

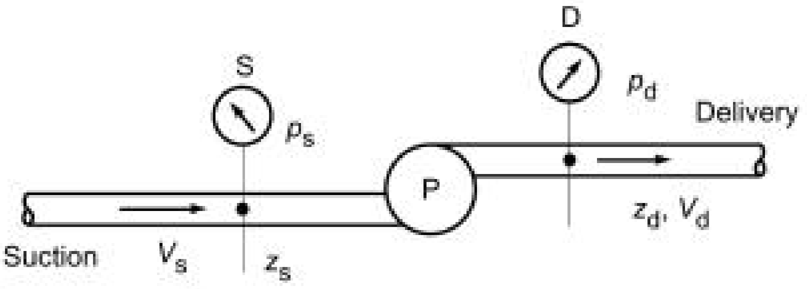

Fig. 16.7

Velocity Triangles

Velocity triangles at inlet and outlet of an impeller are shown in Fig. 16.8 (a), (b) and (c). Three kinds of vane configurations viz. (i) backward curved vanes (ii) radial vane and (iii) forward curved vanes are shown. The backward curved vane pumps are the most common. Unless the data are explicitly clear about any other types, the backward curved blade with (β_1 and β_2) < 90° is assumed in all problems.

Notations

The following notations are used in connection with flow in pump impellers:

Inlet	*Outlet*
V_1 = absolute velocity	V_2 = absolute velocity
v_1 = relative velocity	v_2 = relative velocity
u_1 = peripheral velocity	u_2 = peripheral velocity
α_1 = inclination of V_1 with u_1 direction = direction of absolute velocity at inlet	α_2 = inclination of V_2 with u_2 direction = direction of absolute velocity at exit
β_1 = inclination of v_1 with u_1 direction = blade angle at inlet	β_2 = inclination of v_2 with u_2 direction = blade angle at exit

Note that all angles are measured with respect to *positive direction* of u. If some angles are obtuse their complementary angles are used with a prime. For example: $\beta_1' = 180 - \beta_1$ and $\beta_2' = 180 - \beta_2$. The suffixes 1 and 2 are used for inlet and outlet conditions respectively.

Torque

The torque acting on the fluid is

$$T = \rho\, Q\, (r_2 V_2 \cos \alpha_2 - r_1 V_1 \cos \alpha_1)$$
$$= \rho\, Q\, (r_2 V_{u2} - r_1 V_{u1}) \quad (16.31)$$

where Q = discharge

and $V_u = V \cos \alpha$ = whirl velocity.

The discharge Q itself is given in terms of flow velocity $V_f = V \sin \alpha$

as
$$Q = \pi D_2 b_2 V_{f2} = \pi D_1 b_1 V_{f1} \quad (16.32)$$

where b = width of the impeller at the given radius.

Power P

The power transmitted by the impeller to water is

$$\mathbf{P} = T\omega = \rho\, Q\, (u_2 V_{u2} - u_1 V_{u1})$$
$$= \gamma Q H' \quad (16.33)$$

In Eq. 16.33 H' = theoretical head $= \frac{1}{g}(u_2 V_{u2} - u_1 V_{u1})$ (16.34)

This is also known as *manometric head*. Now H' = head transferred from the shaft = head supplied to machine. If H = actual head delivered to water (= net head developed), then $H' = H + h_L$ where h_L = losses. The losses are composed of shock losses at entrance and exit to the blades, frictional losses in blade passages and circulation in the passages.

Hydraulic Efficiency, η_H

$$\eta_H = \frac{H}{H'} = \frac{gH}{u_2 V_{u2} - u_1 V_{u1}} = 1 - \frac{h_1}{H'} \quad (16.35)$$

For radial entry, i.e. $V_{u1} = 0$, $\boxed{\eta_H = \frac{gH}{u_2 V_{u2}}}$ (16.36)

The hydraulic efficiency is also known as *manometric efficiency*.

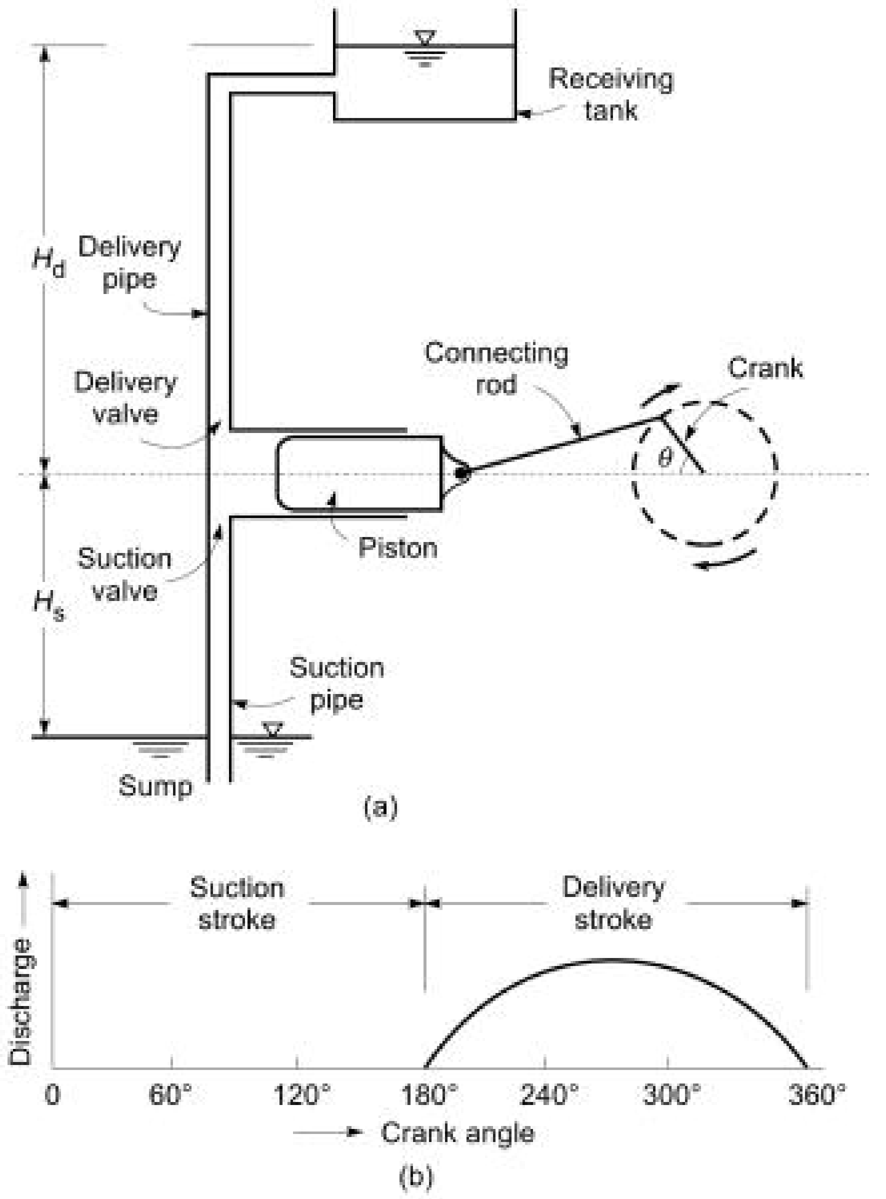
Receiving tank
H_d
Delivery pipe
Delivery valve
Connecting rod
Crank
θ
Piston
Suction valve
H_s
Suction pipe
Sump
(a)
Discharge
Suction stroke
Delivery stroke
0
60°
120°
180°
240°
300°
360°
Crank angle
(b)

Fig. 16.12 *Single Acting Reciprocating Pump (a) Schematic Sketch (b) Variation of Discharge with Crank Angle*

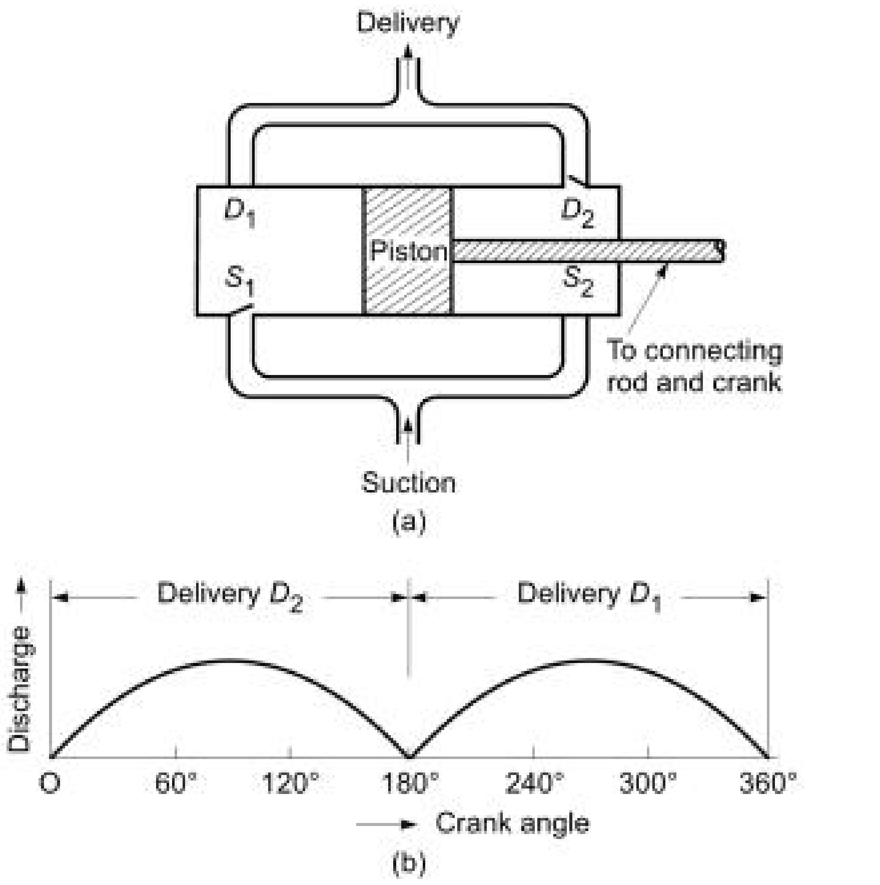
Delivery
D_1
D_2
Piston
S_1
S_2
To connecting rod and crank
Suction
(a)
Discharge
Delivery D_2
Delivery D_1
O
60°
120°
180°
240°
300°
360°
Crank angle
(b)

Fig. 16.13 *Double Acting Reciprocating Pump (c) Schematic Sketch (d) Variation of Discharge with Crank Angle*

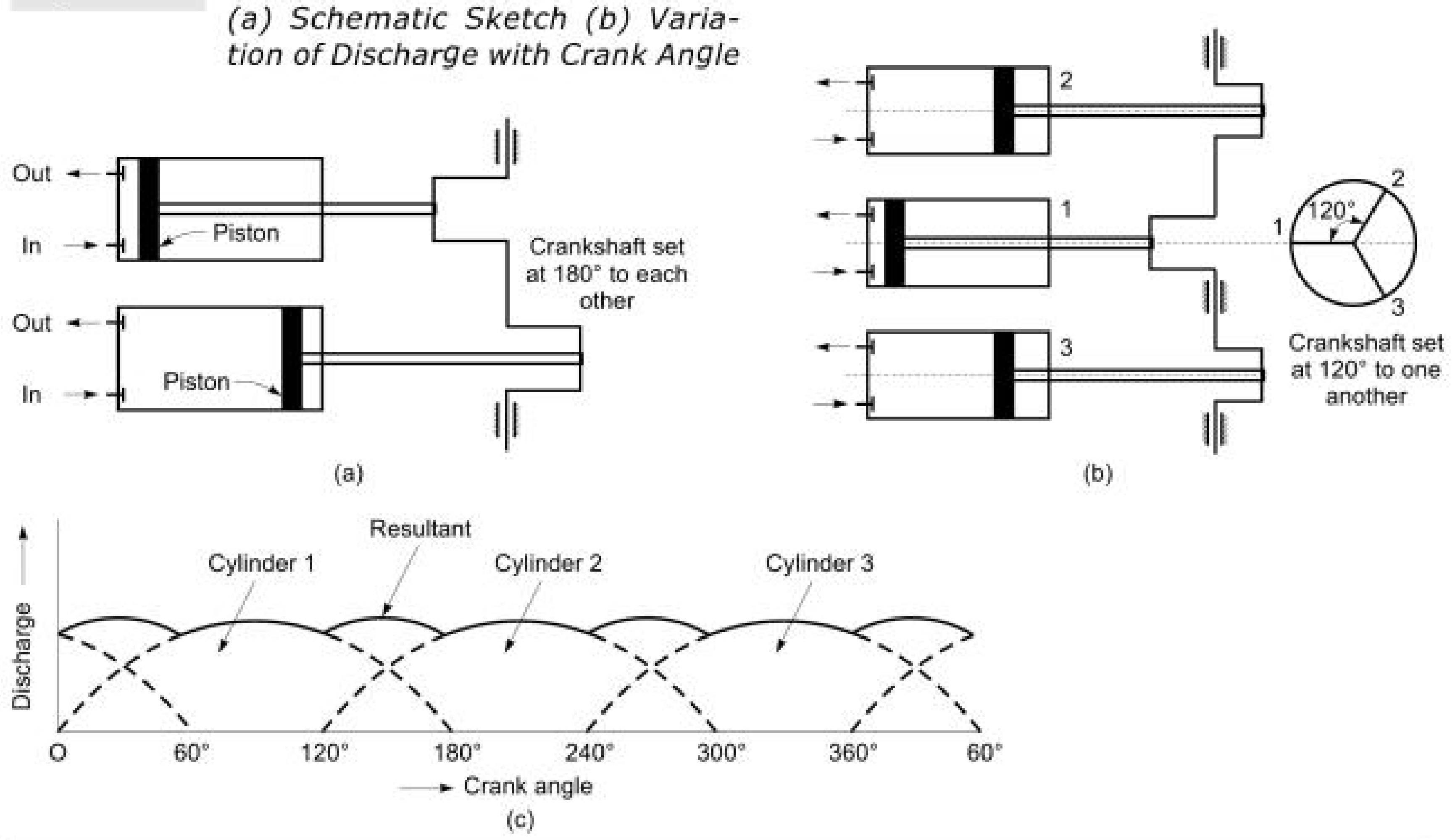
Out
In
Piston
Out
In
Piston
Crankshaft set at 180° to each other
(a)
2
1
3
1
2
3
120°
Crankshaft set at 120° to one another
(b)
Discharge
Resultant
Cylinder 1
Cylinder 2
Cylinder 3
O
60°
120°
180°
240°
300°
360°
60°
Crank angle
(c)

Fig. 16.14 *Multi Throw Reciprocating Pump (a) Two-throw Pump (b) Three-throw Pump (c) Variation of Discharge in a Three-throw Pump*

Discharge:

$$\frac{Q_m}{N_m D_m^3} = \frac{Q_p}{N_p D_p^3}$$

$$Q_m = \text{Model discharge} = Q_p\left(\frac{N_m}{N_p}\right)\left(\frac{D_m}{D_p}\right)^3$$

$$= 18.685\left(\frac{1073.3}{300}\right)\left(\frac{1}{8}\right)^3$$

$$= \mathbf{0.13056\ m^3/s}$$

Power: $\dfrac{\mathbf{P}_m}{N_m^3 D_m^3} = \dfrac{\mathbf{P}_p}{N_p^3 D_p^5}$

$$\mathbf{P}_m = \text{Model power} = \mathbf{P}_p\left(\frac{N_m}{N_p}\right)^3\left(\frac{D_m}{D_p}\right)^5$$

$$= 6750\left(\frac{1073.3}{300}\right)^3\left(\frac{1}{8}\right)^5 = \mathbf{9.433\ kW}$$

Specific speed Since the model and the prototype are similar we expect them to have the same specific speed.

Hence $$N_{sm} = \frac{N_m\sqrt{\mathbf{P}_m}}{H_m^{5/4}} = \frac{1073.3\sqrt{9.433}}{(9)^{5/4}}$$

$$= \mathbf{211.5\ rpm}$$

$$N_{sp} = \frac{N_p\sqrt{\mathbf{P}_p}}{H_p^{5/4}} = \frac{300\sqrt{6760}}{(45)^{5/4}} = 211.5\text{ rpm}$$

It is seen that $N_{sm} = N_{sp}$, as expected. This is a check on the calculations.

16.19 A turbine develops 8,000 kW when running at 100 rpm. The head on the turbine is 30 m. If the head is reduced to 18 m, determine the speed and power developed by the turbine.

Solution: For geometrically similar turbines, the unit speed

$$N_u = \frac{N}{\sqrt{H}}$$

$$\therefore \quad \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$N_2 = N_1\sqrt{H_2/H_1} = 100\sqrt{18/30}$$

$$= \mathbf{77.46\ rpm}$$

Unit power $\mathbf{P}_u = \dfrac{\mathbf{P}}{H^{3/2}}$

$$\frac{\mathbf{P}_1}{H_1^{3/2}} = \frac{\mathbf{P}_2}{H_2^{3/2}}$$

$$\mathbf{P}_2 = \mathbf{P}_1 \times \left(\frac{H_2}{H_1}\right)^{3/2}$$

$$= 8000 \times \left(\frac{18}{30}\right)^{3/2} = \mathbf{3718\ kW}$$

16.20 A Francis turbine produces 6750 kW at 300 rpm under a net head of 45 m with an overall efficiency of 85%. What would be revolutions per minute, discharge and brake power of the same turbine under a net head of 60 m under homologous conditions?

Solution: Power developed

$$\mathbf{P} = \eta_o\,\gamma Q H$$

$$6750 = 0.85 \times \frac{(9.81 \times 998)}{1000} \times Q \times 45$$

Discharge $Q = 18.03$ m³/s.

For calculating N, Q and $\mathbf{P}$ at $H = 60$ m the unit relationships are used.

$$N_u = \frac{N}{\sqrt{H}}$$

$$\therefore \quad \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

or $$N_2 = N_1\sqrt{H_2/H_1} = 300\sqrt{60/45} = \mathbf{346.4\ rpm}$$

$$Q_u = \frac{Q}{\sqrt{H}}$$

$$\therefore \quad Q_2 = Q_1\sqrt{H_2/H_1} = 18.03\sqrt{60/45} = \mathbf{20.82\ m^3/s}$$

$$\mathbf{P}_u = \frac{\mathbf{P}}{H^{3/2}}$$

$$\therefore \quad \mathbf{P}_2 = \mathbf{P}_1\left(\frac{H_2}{H_1}\right)^{3/2} = 6750\left(\frac{60}{45}\right)^{3/2} = \mathbf{10{,}392\ kW}$$

E. Rotodynamic Pumps

16.21 A centrifugal pump has an impeller of outer diameter 30 cm. The vane tips are radial at the outlet. For a rotative speed of 1450 rpm calculate the net head developed. Assume manometric efficiency = 0.82.

Solution: Refer to Fig. 16.26.

From the outlet velocity triangle

$$V_{f2} = v_2 \quad \text{and} \quad V_2 \cos\alpha_2 = V_{u2} = u_2$$

$$u_2 = \pi D_2 N/60 = \pi \times 0.30 \times 1450/60 = 22.78\text{ m/s}$$

Hence, manometric efficiency

$$\eta_H = \frac{gH}{u_2 V_{u2}} = \frac{gH}{u_2^2}$$

Draw the characteristics of the pump and find the best efficiency point (BEP). What is the specific speed of the pump? Estimate the shut off head.

Solution: Plots of Q vs H and Q vs η are plotted on the same graph sheet as shown in Fig. 16.34 from which the BEP is $\eta_{\max} = 73.5$ with $Q = 30.3$ L/s and $H = 30$ m. The specific speed N_s is calculated for the Best Efficiency Point (Design Point).

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}} = \frac{1500\sqrt{0.0303}}{(30)^{3/4}} = 20.37$$

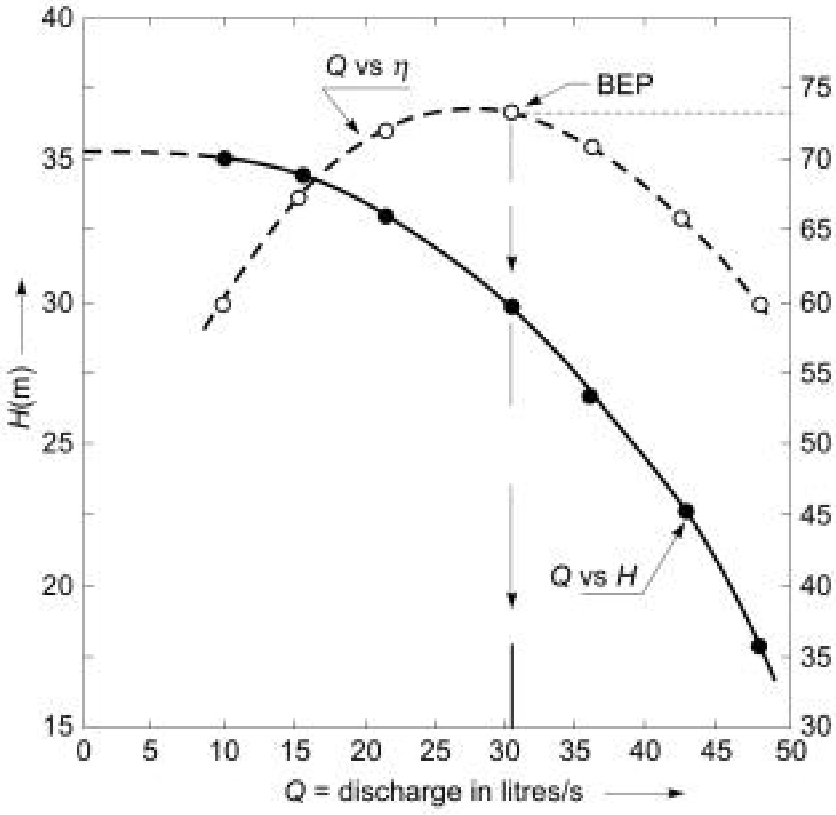

Fig. 16.34 *Pump Characteristics*

By extrapolating the Q vs H curve, the shut off head which corresponds to $Q = 0$ is found to be 35.5 m.

16.36 Two centrifugal pumps A and B are available for use in a pipe flow system and their characteristics are as follows:

Pump A		*Pump B*	
Discharge Q (m^3/s)	*Head* H (*m*)	*Discharge* Q (m^3/s)	*Head* H (*m*)
0	40.0	0	45
0.1	38.0	0.1	43
0.2	29.0	0.2	38
0.25	23.0	0.3	28
0.30	16.0	0.4	14

Determine the head-capacity curves for these pumps when both pumps are connected in (i) parallel and (ii) in series.

Solution:

(i) *Pumps in parallel:* The individual head-discharge curves for pump A and B are plotted in Fig. 16.35(a). When the pumps are connected in parallel, the discharge gets added up for a given head. The head-discharge data are obtained as in Table 16.2(a) and plotted in Fig. 16.35(a).

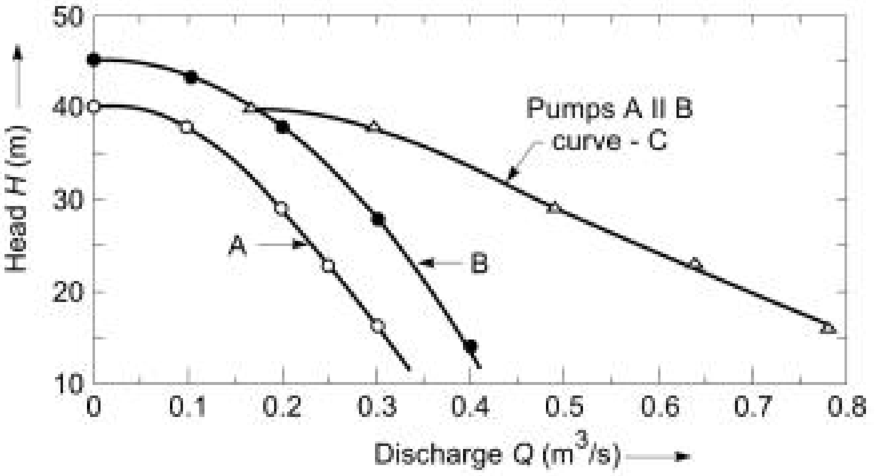

Fig. 16.35(a) *Pumps in Parallel*

Table 16.2(a) *Pumps A and B in Parallel*

Head	*Discharge from A alone* Q_a (m^3/s)	*Discharge from B alone* Q_b (m^3/s)	*combined discharge* $Q_t = Q_a + Q_b$ (m^3/s)
45	0	0	0
43	0	0.1	0.1
40	0	0.17*	0.17
38	0.1	0.20	0.30
29	0.2	0.29*	0.49
23	0.3	0.34*	0.64
16	0.4	0.38*	0.78

Values of Q_b indicated by an asterisk (*) are interpolated values of curve B from Fig. 16.35(a). Values of Q_t and corresponding H are plotted in Fig. 16.35 (a) to get curve C which is the head-discharge relationship for pumps A and B operating in parallel.

(ii) *Pumps in series:* When the pumps are connected in series the heads of individual pumps for a given discharge get added up. Thus the combined head $H_t = H_a + H_b$ for a given discharge. Table 16.2(b) is prepared from the individual *H-Q* relationship shown in Fig. 16.35(a).

Table 16.2(b) *Pumps A and B in Series*

Discharge Q (m^3/s)	*Head(m)* H_a *(for pump A alone)*	*Head (m)* H_b *(for pump B alone)*	*Combined head* H_t *(m)* ($H_t = H_a + H_b$)
0	40	45	85
0.1	38	43	81
0.2	29	38	67
0.25	23	34*	57
0.30	16	28	44
0.35	10*	22*	32

* Interpolated values from curves in Fig. 16.35(a).

16.46 The maximum permissible suction lift for centrifugal pump in precice (at sea level and 30° C) is

(a) 12 m (b) 10 m
(c) 6 m (d) 3 m

16.47 Cavitation parameter σ defined in terms of NPSH (net positive suction head) and net head H as, $\sigma =$

(a) $\dfrac{\text{NPSH}}{\sqrt{H}}$ (b) $\dfrac{H}{\text{NPSH}}$
(c) $\dfrac{H}{\sqrt{\text{NPSH}}}$ (d) $\dfrac{\text{NPSH}}{H}$

16.48 In a pump installation the local atmospheric pressure is 9.8 m of water, vapour pressure head is 0.4 m (abs), height of the pump above sump water level is 5 m. For a head loss in the suction side of 0.6 m, the NPSH is

(a) 3.8 m (b) 5.0 m
(c) 4.0 m (d) 15.8 m

16.49 Water is to be lifted by a net head of 180 m. Identical pumps with specific speed of 30 and rotative speed of 1450 rpm capable of discharging 200 L/s are available. The number of pumps required is

(a) 1 (b) 2
(c) 3 (d) 4

16.50 Water is pumped through a pipeline to a height of 10 m at the rate of 0.1 m^3/s. Frictional and other minor losses are 5 m. Pumping power required, in kW, is

(a) 14.7 (b) 9.80
(c) 20.0 (d) 13.3

Reciprocating Pumps

16.51 In a reciprocating pump without air vessel, the friction head in the delivery pipe is maximum at the crank angle $\theta =$

(a) 0° (b) 90°
(c) 135° (d) 180°

16.52 In a reciprocating pump without air vessels, the acceleration head in the suction pipe is maximum at the crank angle $\theta =$

(a) 0° (b) 90°
(c) 135° (d) 180°

16.53 In a single acting reciprocating pump without air vessel, the average velocity in the delivery pipe is given by $V_s =$

(a) $\left(\dfrac{A}{A_s}\right)\omega r$ (b) $\left(\dfrac{A}{A_s}\right)\dfrac{\omega r}{\pi}$
(c) $\left(\dfrac{A}{A_s}\right)\pi\omega r$ (d) $\left(\dfrac{A}{A_s}\right)\dfrac{\pi\omega r}{2}$

where ω = angular velocity of the crank of radius = r

16.54 In a single reciprocating pump without air vessel, the ratio of the average friction head to the maximum fraction head in the delivery pipe is given by $h_{\text{fda}}/h_{\text{fdm}}$ =

(a) 1/2 (b) 1/3
(c) 2/3 (d) 3/2

16.55 The indicator diagram of a reciprocating pump is a plot of

(a) Work done *vs* stroke length
(b) Acceleration head *vs* stroke length
(c) Angular displacement *vs* stroke length
(d) Pressure head *vs* stroke length

16.56 An air vessels in the delivery side of a reciprocating pump

(a) maintains steady discharge output
(b) prevents cavitation in the system
(c) enables suction head to be increased
(d) enables the pump to run at higher speed

PROBLEMS

A. Turbines

Velocity Triangle Relationships

16.1 An inward flow reaction turbine has a runner of outer diameter 1.2 m and inner diameter 0.6 m. The blades occupy 5% of the peripheral area and the widths of the blades are 25 cm and 30 cm at the inlet and outlet respectively. If a discharge of 3.0 m^3/s enters radially, determine the flow velocities at the inlet and outlet of the runner.

(*Ans.* V_{f1} = 3.35 m/s; V_{f2} = 5.58 m/s)

16.2 A Francis turbine has a wheel of outer diameter = 1.25 m and inner diameter = 0.6 m. The runner blades are radial at inlet and the discharge is radial at outlet. If the flow enters the vanes at 10° and the velocity of flow is 3.0 m/s calculate the speed of the runner and the vane angle at the outlet.

(*Ans.* N = 260 rpm; β_2 = 20.18°)

16.3 A Francis turbine has wheel diameters of 0.9 m at entrance and 0.45 m at exit. The runner blades are radial at entrance and the guide vanes are at 12°.

The variation of H_t vs Q is shown plotted as curve D in Fig. 16.35(b) and represents the head-discharge curve for pumps A and B in series.

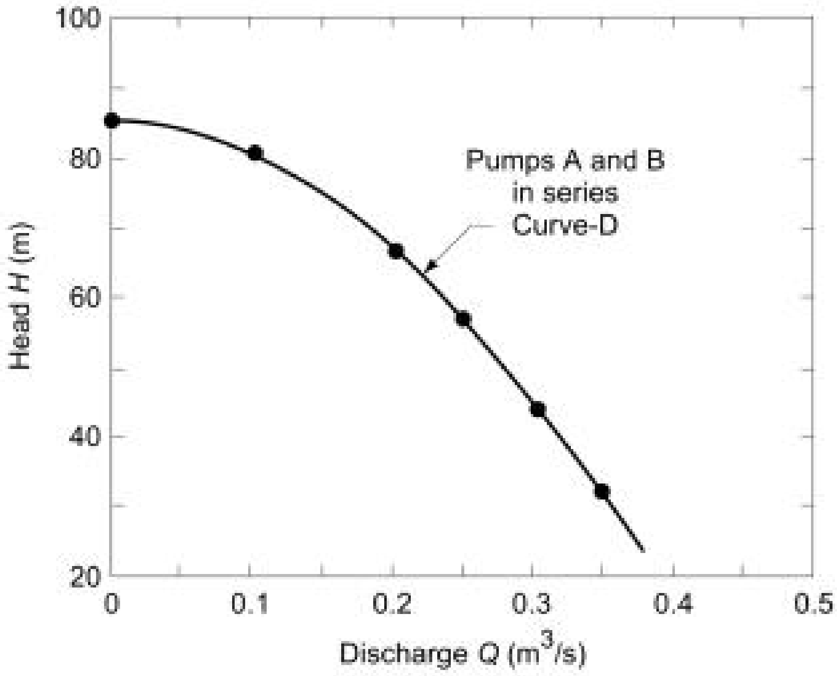

Fig. 16.35(b) *Pumps in Series*

16.37 A pump can delivery discharge of 0.10 m^3/s to a head of 30 m. The critical cavitation number σ_c for the pump is found to be 0.12. The pump is to be installed at a location where the barometric pressure is 96.0 kPa (abs) and the vapour pressure is 3.0 kPa (abs). Assuming an intake pipe friction of 0.3 m, determine the minimum value of NPSH. What would be the maximum allowable elevation above the sump water surface at which the pump can be located?

Solution:

(i) By Eq. 16.53 $\sigma_c = \dfrac{(\text{NPSH})_{\min}}{H}$

$$0.12 = \frac{(\text{NPSH})_{\min}}{30}$$

Minimum NPSH = **3.6 m**

(ii) $$\text{NPSH} = \frac{(p_{\text{atm}})_{\text{abs}}}{\gamma} - \frac{p_v}{\gamma} - Z_s - h_L$$

where Z_s = elevation of the pump above the sump water surface. $(Z_s)_{\max}$ corresponds to σ_c. Hence

$$(Z_s)_{\max} = \frac{(p_{\text{atm}})_{\text{abs}}}{\gamma} - \frac{p_v}{\gamma} - h_L - (\text{NPSH})_{\min}$$

$$= \frac{96.0}{9.79} - \frac{3.0}{9.79} - 0.3 - 3.6 = \mathbf{5.6\ m}$$

H. Reciprocating Pumps

16.38 A single acting reciprocating pump has a 15 cm piston with a crank of radius 15 cm. The delivery pipe is 10 cm diameter. At a speed of 60 rpm, 310 litres/s of water is lifted to a total height of 15 m. Find the slip, coefficient of discharge and theoretical power in kW required to drive the pump.

Solution: Theoretical discharge $Q_t = \dfrac{ALN}{60}$

$$A = \frac{\pi}{4}(0.15)^2 = 0.01767 \text{ m}^2$$

$$L = 2r = 2 \times 0.15 = 0.30 \text{ m}; \quad N = 60 \text{ rpm}$$

$$Q_t = \frac{0.01767 \times 0.30 \times 60}{60} = 0.0053 \text{ m}^3/\text{s} = 318\ l/\text{min}$$

$$Q_a = \text{actual discharge} = 310\ l/\text{min}$$

$$\text{Slip} = \frac{(318 - 310) \times 100}{318} = \mathbf{2.52\ \%}$$

Coefficient of discharge = 310/318 = **0.975**

Total head = $H_t = (H_s + H_d) = 15$ m

Power = $\mathbf{P}_t = \rho Q_a H_t = 9800 \times 0.0053 \times 15 = 778$ W

= **0.778 kW**

16.39 A three-throw pump has a 20 cm diameter cylinder and a stroke of 40 cm. The pump is required to deliver 5000 litres/minute of water at a static head of 80 m. Friction losses can be taken as 2.0 m in suction pipe and 18 m in the delivery pipe. Assume a slip of 2% and pump efficiency of 90%. The velocity head in the delivery pipe can be neglected as too small. Determine the speed of the pump and the power required.

Solution:

$$A = \frac{\pi}{4}(0.2)^2 = 0.3142 \text{ m}^2, \quad L = 0.4 \text{ m},$$

$$Q_a = (50000/(60 \times 1000) = 0.0833 \text{ m}^3/\text{s}$$

For a three-throw pump

$$Q_t = \frac{3ALN}{60}$$

$$Q_t = \frac{0.3142 \times 0.40 \times N}{60} = 6.283 \times 10^{-4} \text{ N}$$

Since, Slip = 2%, $\dfrac{Q_t - Q_a}{Q_t} = 0.02$

$$\frac{(6.28 \times 10^{-4} N - 0.0833)}{6.28 \times 10^{-4} N} = 0.02$$

$$N = \mathbf{135.4\ rpm}$$

Total head $= H_t = (H_s + h_{fs}) + (H_d + h_{fd})$

$= 80 + 2 + 18 = 100$ m

Power = $\mathbf{P} = \gamma Q_a H_t/\eta = (9800 \times 0.00833 \times 100)/0.90$

= 778 W = **90730 W**

= **90.73 kW**

16.40 A single acting reciprocating pump has a 20 cm piston with a crank of radius 40 cm. The delivery

in the delivery side by fitting an air vessel very near the cylinder on the delivery pipe.

Take Darcy-Weisbach friction factor $f = 0.02$.

Solution:

Without Air Vessel

$$A = \frac{\pi}{4}(0.15)^2 = 0.0177 \text{ m}^2, \quad L = 0.3 \text{ m},$$

For a single acting pump

$$Q_t = \frac{ALN}{60} = \frac{0.0177 \times 0.3 \times 60}{60} = 0.00531 \text{ m}^3\text{/s}$$

$$\omega = \frac{2\pi \times 60}{60} = 6.283 \text{ rad/s}, \; r = 0.30/2 = 0.150 \text{ m}$$

Maximum friction head

$$h_{fdm} = \frac{fL_d}{2gd_d}\left(\frac{A}{A_d}\omega r\right)^2$$

$$= \frac{0.02 \times 30}{2 \times 9.81 \times 0.075} \times (4 \times 6.283 \times 0.15)^2$$

$$= 5.795 \text{ m}$$

Time averaged friction head

$$h_{fda} = (2/3)\, h_{fdm} = (2/3) \times 5.795 = 3.863 \text{ m}$$

With Air Vessel: The velocity is steady in the delivery pipe with an average value of

$$V_d = Q_t/A_d = 0.00531/0.004425 = 1.2 \text{ m/s}$$

Friction head: $h_{fdl} = \dfrac{fL}{2gd_d} V_d^2$

$$= \frac{0.02 \times 30}{2 \times 9.81 \times 0.075} \times (1.2)^2 = 0.587 \text{ m}$$

Friction power saved due to air vessel in delivery pipe, in kW,

$$\mathbf{P}_f = \frac{\gamma Q_t}{1000}(h_{fda} - h_{fdl})$$

$$= (9800/1000) \times 0.00531 \times (3.863 - 0.587)$$

$$= \mathbf{0.17 \text{ kW}}$$

16.50 A double acting reciprocating pump has the following data:

Cylinder diameter = 20 cm

Stroke = 40 cm

Diameter of suction pipe = 10.0 cm

Crank speed = 90 rpm

If an air vessel is fitted on the suction side, calculate the flow into or out of the air vessel when the crank angle is (i) 45° and (ii) 150°

Solution:

$$A = \frac{\pi}{4}(0.2)^2 = 0.0314 \text{ m}^2, \quad L = 0.4 \text{ m},$$

$$\omega = \frac{2\pi \times 90}{60} = 9.425 \text{ rad/s}, \; r = 0.40/2 = 0.20 \text{ m}$$

$$(A/A_s) = (20/10)^2 = 4, \; A_s = A/4 = 0.007854 \text{ m}^2$$

Since the pump is double acting

$$Q_t = \frac{2ALN}{60} = \frac{2 \times 0.0314 \times 0.4 \times 90}{60}$$

$$= 0.03768 \text{ m}^3\text{/s}$$

v_s = Instantaneous velocity of unsteady flow in suction pipe between air vessel and the cylinder

$$= \frac{A}{A_s}\omega r \sin\theta = 4 \times 9.425 \times 0.20 \sin\theta$$

$$= 7.54 \sin\theta$$

Discharge in this part of suction pipe at any instant

$$Q_s = v_s A_s = 7.54 \sin\theta \times 0.007854 = 0.05922 \sin\theta$$

Discharge into/out of air vessel:

If $Q_t < Q_s$, then flow goes out of the air vessel

If $Q_t > Q_s$, then flow goes into the air vessel

Crank angle θ	Q_t m^3/s	Q_s m^3/s	*Remarks*
45°	0.03768	0.04187	Flow going out of air vessel = **0.00419 m³/s**
150°	0.03768	0.02961	Flow going into air vessel = **0.00807 m³/s**

16.51 Following data pertain to a single acting reciprocating pump installation:

Cylinder diameter = 15 cm

Stroke length = 20 cm

Static suction head = 4.0 m

Diameter of suction pipe = 7.5 cm

Suction pipe length = 8.0 m

Crank speed = 60 rpm

Delivery pipe diameter = 7.5 cm

Length of delivery pipe = 30.0 m

Static delivery head = 25.0 m

Darcy–Weisbach friction factor $f = 0.025$.

Air vessels are provided very near to the cylinder both on the suction and the delivery sides. Assume Darcy-Weisbach friction factor $f = 0.025$. Estimate the power required to drive the pump by assuming a pump efficiency of 80%.

Solution:

$$A = \frac{\pi}{4}(0.15)^2 = 0.01767 \text{ m}^2, \quad L = 0.2 \text{ m},$$

$$\omega = \frac{2\pi \times 60}{60} = 6.283 \text{ rad/s}, \; r = 0.20/2 = 0.10 \text{ m}$$

$$(A/A_s) = (15/7.5)^2 = 4,$$

$$A_d = A_s = A/4 = 0.004179 \text{ m}^2$$

$$Q_t = \frac{ALN}{60} = \frac{0.01767 \times 0.2 \times 60}{60} = 0.003534 \text{ m}^3\text{/s}$$

Since the air vessels are very near to the cylinder, it is assumed that the entire suction an delivery pipes will have steady flow.

the speed and power developed in the model. Assuming an overall efficiency of 0.90 for both the model and prototype calculate the discharges in the model and prototype.

(*Ans.* (i) $N_m = 197.18$ rpm; $\mathbf{P}_m = 2.739$ kW;
(ii) $Q_p = 1.7024$ m^3/s; $Q_m = 0.1036$ m^3/s)

16.27 A 1 : 5 model of a turbine develops 2.0 kW of power at 400 rpm under a head of 3.0 m. What is the specific speed of the runner? Assuming an overall efficiency of 0.85 for both the model and prototype, calculate the rotational speed, power and discharge of the prototype when working under a head of 20 m.

(*Ans.* $N_s = 143.3$; $N_p = 206.6$ rpm;
$\mathbf{P}_p = 861.2$ kW; $Q_p = 5.165$ m^3/s)

16.28 In a small hydro development a Kaplan turbine runs under a head of 2.1 m. It has a runner of 3.5 m diameter and develops 600 kW at 80 rpm. Assuming an overall efficiency of 80%, estimate the discharge and specific speed of the machine. If a 1.5 m diameter homologous turbine is to be tested at a head of 3.0 m, What are the rotational speed, discharge and power of that unit?

(*Ans.* $Q_p = 36.48$ m^3/s; $N_s = 775$;
$N_m = 156.2$ rpm; $Q_m = 5.607$ m^3/s; $\mathbf{P}_m = 64.6$ kW)

16.29 A turbine is to operate under a head of 25 m at 200 rpm. The discharge is 9.0 m^3/s. If the eficiency is 90%, determine the performance of the turbine under a head of 20 m.

(*Ans.* $N_2 = 178.9$ rpm;
$Q_2 = 8.05$ m^3/s; $\mathbf{P}_2 = 1418.6$ kW)

16.30 A Kaplan turbine operates under a head of 15.2 m, has a speed of 75 rpm and develops 50 MW of power. The overall efficiency of the turbine is 82%. Calculate the specific speed, unit speed, unit discharge and unit power.

(*Ans.* $N_s = 558.8$; $N_u = 19.24$;
$Q_u = 105.10$; $\mathbf{P}_u = 843.7$)

B. Rotodynamic Pumps

Velocity Triangle Relationships

16.31 A centrifugal pump having an overall efficiency of 70% delivers 1500 L/min of water against a static head of 20 m. The suction and delivery pipes are of 20 cm diameter and has a combined total length of 1000 m. Assuming $f = 0.02$ estimate the power input required.

(*Ans.* $\mathbf{P}_s = 8.122$ kW)

16.32 A centrifugal pump runs at 800 rpm and delivers 5000 L/min against a head of 7 m. The impeller has an outer diameter of 25 cm and a width of 5 cm at the outlet. If the backward curved vane at the outlet makes an angle of 45°, determine the manometric efficiency. What is the specific speed of the pump?

(*Ans.* $\eta_H = 78.5\%$; $N_s = 53.65$)

16.33 A centrifugal pump has an impeller of diameter 30 cm whose width at exit is 6.0 cm. The velocity of flow through the impeller is constant at 3.0 m./s. The impeller vanes are radial at the outer periphery. If the rotational speed is 1000 rmp and the manometric efficiency is 80%, calculate (i) the head produced and (ii) discharge.

(*Ans.* (i) $H = 20.12$ m; (ii) $Q = 169.6$ L/s)

16.34 A centrifugal pump has vanes which are radial at the outer periphery. The impeller has an outer diameter of 20 cm and a width of 3 cm at that diameter. If the diacharge is 1800 L/min and the net head produced is 3.5 m, calculate the (i) rotational speed of the impeller and (ii) magnitude and direction of absolute velocity at exit. Manometric efficiency can be assumed as 0.85.

(*Ans.* $N = 607$ rpm;
(ii) $V_2 = 6.552$ m/s, $\alpha_2 = 14.06°$)

16.35 A centrifugal pump impeller is 40 cm in outer diameter and 2.5 cm wide at the exit, and its blade angle is 30°. When run at a speed of 2100 rpm, the flow rate through the pump is 80 L/s.

(i) Calculate the radial, relative and absolute velocities at the impeller exit.

(ii) If there is no inlet whirl, what would be the head added to the water by the impeller?

(*Ans.* (i) 2.55 m/s, 5.10 m/s
and 39.62 m/s. (ii) 177.18 m)

16.36 A centrifugal pump running at 1200 rpm delivers water at a net head of 10.0 m. At the outlet of the impeller the vane angle is 30° with the peripheral velocity. The impeller has an outer diameter of 25 cm and a width of 5 cm at the outlet. Estimate the discharge (Assume manometric efficiency = 85%).

(*Ans.* $Q = 190$ L/s)

16.37 A centrifugal pump impeller rotates at a speed of 1000 rpm. The external and internal diameters of the impeller are 0.40 m and 0.20 m respectively. The vanes make an angle of 35° with the tangential direction of rotation at the outlet. If the radial velocity of flow through the impeller is constant at 1.75 m/s, find the (i) angle of vanes at the inlet, (ii) absolute velocity and its direction at outlet and (iii) net head of the pump. (Assume manometric efficiency = 80%)

(*Ans.* (i) $\beta_1 = 9.5°$; (ii) $V_2 = 18.52$ m/s,
$\alpha_2 = 5.4°$; (iii) $H = 31.5$ m)

Codes:	A	B	C	D
(a)	2	1	4	3
(b)	1	2	3	4
(c)	1	2	4	3
(d)	2	1	3	4

33. Match List I (Name of instrument) with List II (Variable measured) and select the correct answer using the code given below the lists.

List I	List II
A. Hot-wire anemometer	1. Boundary shear stress
B. Orifice meter	2. Discharge
C. Pitot tube	3. Velocity
D. Preston tube	4. Pressure
	5. Turbulence

Codes:	A	B	C	D
(a)	2	3	4	1
(b)	5	2	3	4
(c)	2	5	1	3
(d)	5	2	3	1

34. Match List I with List II and select the correct answer using the code given below the lists.

List I (*Shape of the weir*)	List II (*Value of the exponent in the equation* $Q = KH^n$)
A. Rectangular weir	1. 1.0
B. Triangular weir	2. 2.0
C. Sutro weir	3. 1.5
D. Parabolic weir	4. 2.5

Codes:	A	B	C	D
(a)	4	2	1	3
(b)	2	3	4	1
(c)	3	4	2	1
(d)	3	4	1	2

35. Which of the following statements are true in relation to water hammer phenomenon?

1. It causes surface erosion in pipes.
2. The pressure rise is given by ρ CV for sudden closure of valve.
3. It is accompanied by serious cavitation.
4. The volume modulus of fluid is the relevant fluid property.
5. It is governed by the Reynolds number of the flow.

Select the correct answer using the codes given below.

Codes: (a) 3 and 5 (b) 2 and 4
(c) 2 and 5 (d) 1, 2, 3 and 4

36. Match List I with List II and select the correct answer using the code given below the lists.

List I	List II
A. Positive surge traveling upstream	1. Occurs on upstream of gate that is partly closed suddenly.
B. Positive surge traveling downstream	2. Occurs on downstream of gate that is partly closed suddenly.
C. Negative surge traveling upstream	3. Occurs on upstream of gate that is opened suddenly.
D. Negative surge traveling downstream	4. Occurs on the downstream that is opened suddenly.

Index

Absolute pressure 18
— values 370
— viscosity 2
Acceleration 69
Adiabatic 4
— process 372
Aeration of nappe 341
Air vessel 403
Alternate depths 273
Ambient pressure 373
Anemometer 208
Aneroid barometer 18, 24
Angular momentum 119
— velocity 120
Atmospheres 18
Axial flow pumps 396

Backwater curve 271
Barometer 18
Bars 18
Bernoulli equation 93
Best efficiency point 419
—section 273
Bingham fluid 12
Blasius equation 212
Body forces 117
Borda's internal mouthpiece 338
Boundary forces 117
— layer 81, 175
Bourdon gauge 18, 24
Break in the grade 295
Broad-crested weir 277
Buckingham Pi theorem 140
Bulk modulus of elasticity 4, 5
Buoyancy 20

Capillarity 4
Cartesian coordinates 70
Cauchy-Reimann equations 88
Cavitation 5, 47, 99, 399, 402
Centi stoke 3
Centipoise 3
Centre of buoyancy 20
— of pressure 19
Centrifugal pumps 396
Chezy coefficient 272
— formula 272
Choking condition 375
Cipolletti weir 319, 341
Circulation 71
Coefficient of surface tension 3
— of viscosity 2
Colebrook equation 213
Commercial pipes 213
Compressibility 5
Conjugate functions 92
Contaminant Propagation 237
Continuity 372
— equation 158
Continuum 1
— principle 1
Contracted weirs 318
Control volume 117
Convective acceleration 78
Converging nozzle 373
Creeping motion 160, 195
Critical depth 273
— Reynolds number 158, 177, 210
— slope 287, 310
— time 351
Cup anemometer 200
Current meter 319

Darcy law 172
— Weisbach equation 159, 162, 211, 272
Datum 17

Delivery end 396
Density 1
Dilatant fluid 12
Dimensional analysis 139
Direct step method 276, 298
Displacement thickness 175
Distorted models 142
Doublet 73
Draft tube 106, 115
Drag 194
Dynamic Similarity 140
— viscosity 2

Effective roughness 213
Efficient section 273
Energy line 94
— loss 93, 291
— slope 232
Engineering system 139
Enthalpy 371
Entropy 371
Equation of State 370
Equilibrium boundary layer flow 210
Equipotential lines 72
Equivalent Length 232
— sand grain roughness 213
Establishment of flow 349
Euler equation 93, 373
— number 141

Fall velocities 195
Flood waves 349
Flow coefficient 315
— net 72, 111
— nozzle 316
— ratio 393
— velocity 392
— work 93
Fluid flow machines 391
Form drag 194
Francis formula 319
— turbine 391
Free flow 318
Friction factor 159
Frictional drag 194
Froude number 141, 274
Full supply depth 283

Gas constant 371
Gauge pressure 17
Geometric similarity 140
Gradually varied flow 271

Haaland equation 213
Hagen–poiseuille equation 159
Head loss 94
Homologous pumps 398
Hoop stress 20
Hydraulic efficiency 392
— grade line 94
— jump 274, 291, 349
Hydrodynamically rough 178, 211
— smooth 178, 211
Hydrometer 39
Hydrostatic pressure distribution 17
Hypersonic 370

Ideal fluid 12, 93
Impeller 395
Impulse turbine 393
Indicator diagram 402
Internal energy 371
Inverted U-tube manometer 330
Irrotational 71
— flow 93
— vortex 103
Isentropic 4
— flow 372
Isothermal 4
— process 372

Kaplan turbine 393, 408
Karman vortex trail 206
Karman's coefficient 210
— model 210
Kinematic Similarity 140
— viscosity 3
Kinematics 69
Kinetic energy 93
— correction factor 95

Laminar flow 118, 158
— sublayer 177, 210
Laplace equation 72
Lapse rate 19
Lift 194
Line source 73
Linear momentum equation 117
Local acceleration 70, 78

Mach angle 373
— cone 373
— number 141, 318
Magnus effect 197
Manning's formula 272
Manometers 18, 19
Manometric efficiency 396
— head 396

Mechanical efficiency 392
Meta centre 21
Metacentric height 21
Method of repeating variables 140
Minor losses 232
Mixed flow pumps 396
Model scales 141
Modular limit 304
Moment of momentum equation 117
Momentum correction factor 118
— thickness 176
Moody diagram 213
— equation 213
Most efficient section 273
Mouthpiece 323

Navier–Stokes equations 158
Negative surge 349
Neutral equilibrium 21
Newton 1
Newtonian fluids 2
Newton's law of viscosity 2
Nominal thickness 175
Non-newtonian 13
— uniform flow 69
Normal convective acceleration 70
— depth 272
— shock 375, 383
— shock wave 375
Notches 318
NPSH 399

Open channel flow 271
Orifices 314

Path line 69
Pelton turbines 119
— wheel 409
Perfect gas 13, 18, 370
— equation 4
Piezometric head 21, 53, 94
Pipe network 237
Pipes in Parallel 234
— in series 234
Pitot tube 220, 318
— static tube 318
Poise 3
Positive displacement machine 391
— surge 349
Potential energy 93
— vortex 104
Prandtl's model 210
Pressure 2
— drag 194
Preston tube 340
Profile drag 194
Pseudoplastic 13
— fluid 12
Pumps in Series 399

Raleigh's method 140
Rapid closure 351
Rapidly varied flow 271
Reaction turbine 115
Reciprocating pump 399
Rehbock formula 318
Relative density 2
Reynolds number 141
Rheological behaviour 13, 436
Rigid body motion 21
Rotameter 320
Rotational motion 71
Rotodynamic machines 120
— pump 395
Rough boundary 212

Separation 179
— phenomenon 179
Shape factor 176
— number 392, 398
Sharp crested 318
Shear velocity 210
Shock 373
Shut off head 418
SI units 1
Similitude 140
— scale ratios 141
Simple harmonic motion 401
Sink 73
Siphon 98, 110, 234
Skin friction 150
Slip 401
Slow closure 352
Sluice gate 128
Small orifice 314
Smooth boundary 212
Sonic boom 379
Sonic velocity 372
Source 73
Specific energy 273
— gravity 2
— speed 392
— weight 2
Speed factor 392
Stagnation point 204, 317
Stagnation pressure 111
— values 373
Stall 198
Stalling 179

Standard lined canal sections 283
— trapezoidal section 284
— triangular section 283
Static tube 340
Steady flow 69
Stoke 3, 161
— equation 161
— law 195
Streak line 69
Stream line 69
— tube 70
Streamlined body 205
Strength of source 73
Strouhal number 196
Submerged flow 129, 318
Suction end 396
Supersonic 370
Suppressed weir 318
Surface tension 3
Surge tank 366
Surges 349
Swamee and Jain equation 213
Swirl velocity 391

Tailwater level 319
Tangential convective acceleration 70
Thin plate weirs 318
Thoma number 399
Three-reservoir problem 236
Tidal bores 350
Time line 69
Torque 119
Total drag coefficient 194
— energy 93
Transitions 288
Transonic flow 370
Trapezoidal weir 319
Triangular weir 319
Turbine 105
Turbo machine 391
Turbulent boundary layer 175

Uniform flow 69, 73, 207
Unit quantities 394
Unsteady flow 69
Unsteady flows 349

Vacuum pressures 18
Vapour pressure 5, 99
Vector 69
Velocity potential 72
Vena contracta 297, 314
Venturimeter 316
Villemonte formula 319
Viscometer 161
Viscosity 2
Vortex 73
— shedding 196
— strength 73
Vorticity 71

Wake 179
Water hammer 349, 350
Wave resistance 150
Weber number 141
Weirs 318
Whirlpool 111

Zero pressure gradient flow 176